

# Association Rules Fundamentals



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# Association rules

- Objective
  - extraction of frequent correlations or pattern from a transactional database

Tickets at a supermarket counter

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk
...	...

- Association rule
  - diapers  $\Rightarrow$  beer
    - 2% of transactions contains both items
    - 30% of transactions containing diapers also contains beer



# Association rule mining

- A collection of transactions is given

- a transaction is a set of items
- items in a transaction are  
*not ordered*

- Association rule

$$A, B \Rightarrow C$$

- $A, B$  = items in the rule body
- $C$  = item in the rule head

- The  $\Rightarrow$  means co-occurrence

- *not* causality

- Example

- coke, diapers  $\Rightarrow$  milk

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk
...	...



# Transactional formats

- Association rule extraction is an *exploratory technique* that can be applied to any data type
- A transaction can be any set of items
  - Market basket data
  - Textual data
  - Structured data
  - ...



# Transactional formats

- Textual data

- A document is a transaction
- Words in a document are items in the transaction



- Data example

- Doc1: algorithm analysis customer data mining relationship
- Doc2: customer data management relationship
- Doc3: analysis customer data mining relationship social

- Rule example

customer, relationship  $\Rightarrow$  data, mining



# Transactional formats

- Structured data

- A table row is a transaction
- Pairs (attribute, value) are items in the transaction

- Data example

Refund	Marital Status	Taxable Income	Cheat
No	Married	< 80K	No

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



- Transaction

Refund=no, MaritalStatus=married, TaxableIncome<80K, Cheat=No

- Rule example

Refund=No, MaritalStatus=Married  $\Rightarrow$  Cheat = No



# Definitions

- **Itemset** is a set including one or more items
  - Example: {Beer, Diapers}
- **k-itemset** is an itemset that contains k items
- **Support count** (#) is the frequency of occurrence of an itemset
  - Example:  $\#\{\text{Beer, Diapers}\} = 2$
- **Support** is the fraction of transactions that contain an itemset
  - Example:  $\text{sup}(\{\text{Beer, Diapers}\}) = 2/5$
- **Frequent itemset** is an itemset whose support is greater than or equal to a *minsup* threshold

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk



# Rule quality metrics

- Given the association rule

$$A \Rightarrow B$$

- A, B are itemsets
- *Support* is the fraction of transactions containing both A and B

$$\frac{\#\{A,B\}}{|T|}$$

- |T| is the cardinality of the transactional database
  - a priori probability of itemset AB
  - rule frequency in the database
- *Confidence* is the frequency of B in transactions containing A

$$\frac{\text{sup}(A,B)}{\text{sup}(A)}$$

- conditional probability of finding B having found A
  - “strength” of the “ $\Rightarrow$ ”





# Rule quality metrics: example

- From itemset {Milk, Diapers} the following rules may be derived
- Rule: Milk  $\Rightarrow$  Diapers
  - support  
 $\text{sup} = \#\{\text{Milk, Diapers}\} / \#\text{trans.} = 3/5 = 60\%$
  - confidence  
 $\text{conf} = \#\{\text{Milk, Diapers}\} / \#\{\text{Milk}\} = 3/4 = 75\%$
- Rule: Diapers  $\Rightarrow$  Milk
  - same support  
 $s = 60\%$
  - confidence  
 $\text{conf} = \#\{\text{Milk, Diapers}\} / \#\{\text{Diapers}\} = 3/3 = 100\%$

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk



# Association rule extraction

- Given a set of transactions  $T$ , association rule mining is the extraction of the rules satisfying the constraints
  - support  $\geq$  *minsup* threshold
  - confidence  $\geq$  *minconf* threshold
- The result is
  - complete (*all* rules satisfying both constraints)
  - correct (*only* the rules satisfying both constraints)
- May add other more complex constraints



# Association rule extraction

- Brute-force approach
  - enumerate all possible permutations (i.e., association rules)
  - compute support and confidence for each rule
  - prune the rules that do not satisfy the *minsup* and *minconf* constraints
- Computationally *unfeasible*
- Given an itemset, the extraction process may be split
  - first generate frequent itemsets
  - next generate rules from each frequent itemset
- Example
  - Itemset  
    {Milk, Diapers} sup=60%
  - Rules  
    Milk  $\Rightarrow$  Diapers (conf=75%)  
    Diapers  $\Rightarrow$  Milk (conf=100%)



# Association rule extraction

## (1) Extraction of frequent itemsets

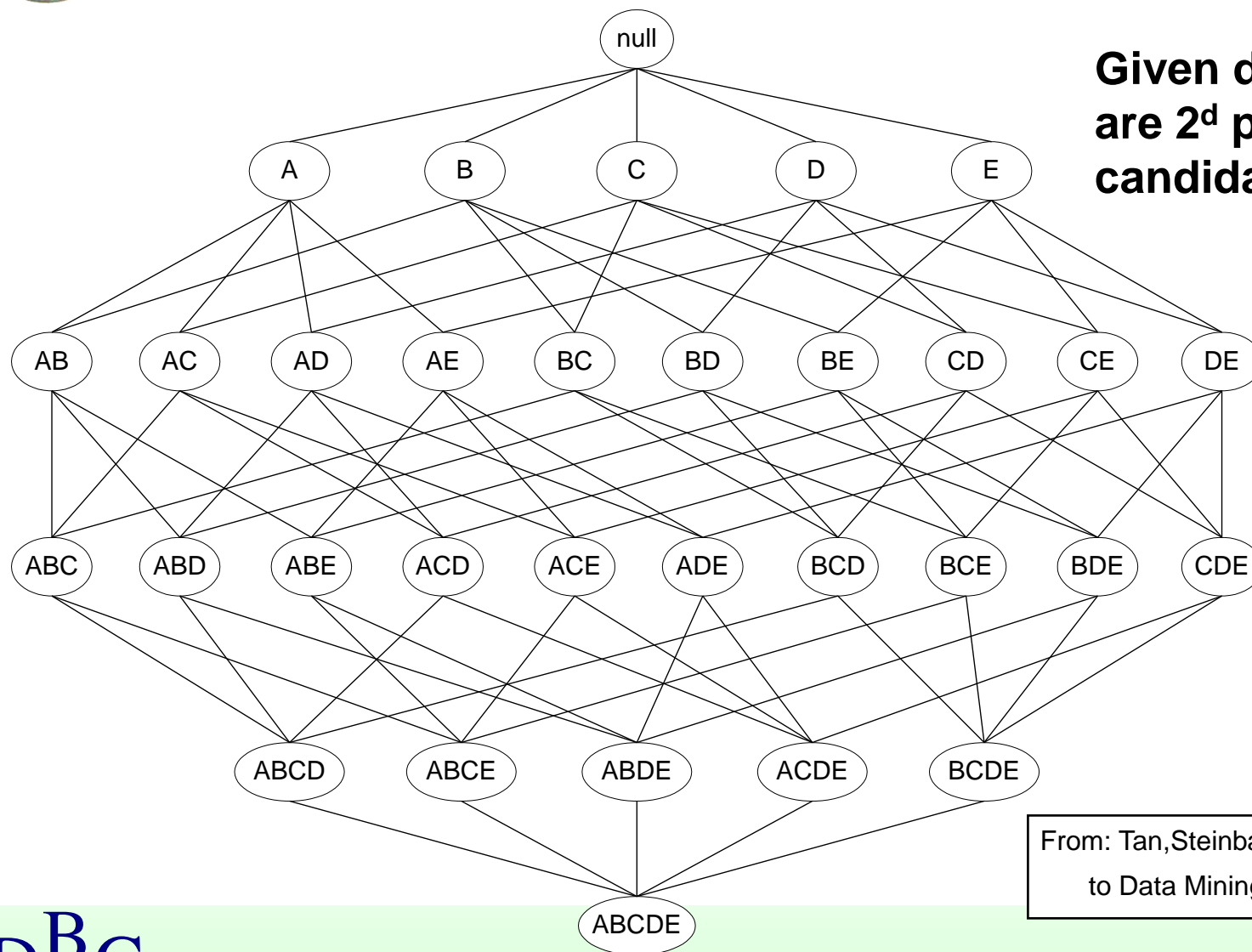
- many different techniques
  - level-wise approaches (Apriori, ...)
  - approaches without candidate generation (FP-growth, ...)
  - other approaches
- most computationally expensive step
  - limit extraction time by means of support threshold

## (2) Extraction of association rules

- generation of all possible binary partitioning of each frequent itemset
  - possibly enforcing a confidence threshold



# Frequent Itemset Generation



**Given d items, there are  $2^d$  possible candidate itemsets**

From: Tan, Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006



# Frequent Itemset Generation

- Brute-force approach
  - each itemset in the lattice is a *candidate* frequent itemset
  - scan the database to count the support of each candidate
    - match each transaction against every candidate
  - Complexity  $\sim O(|T| 2^d w)$ 
    - $|T|$  is number of transactions
    - $d$  is number of items
    - $w$  is transaction length



# Improving Efficiency

- Reduce the **number of candidates**
  - Prune the search space
    - complete set of candidates is  $2^d$
- Reduce the **number of transactions**
  - Prune transactions as the size of itemsets increases
    - reduce  $|T|$
- Reduce the **number of comparisons**
  - Equal to  $|T| \cdot 2^d$
  - Use efficient data structures to store the candidates or transactions



# The Apriori Principle

*"If an itemset is frequent, then all of its subsets must also be frequent"*

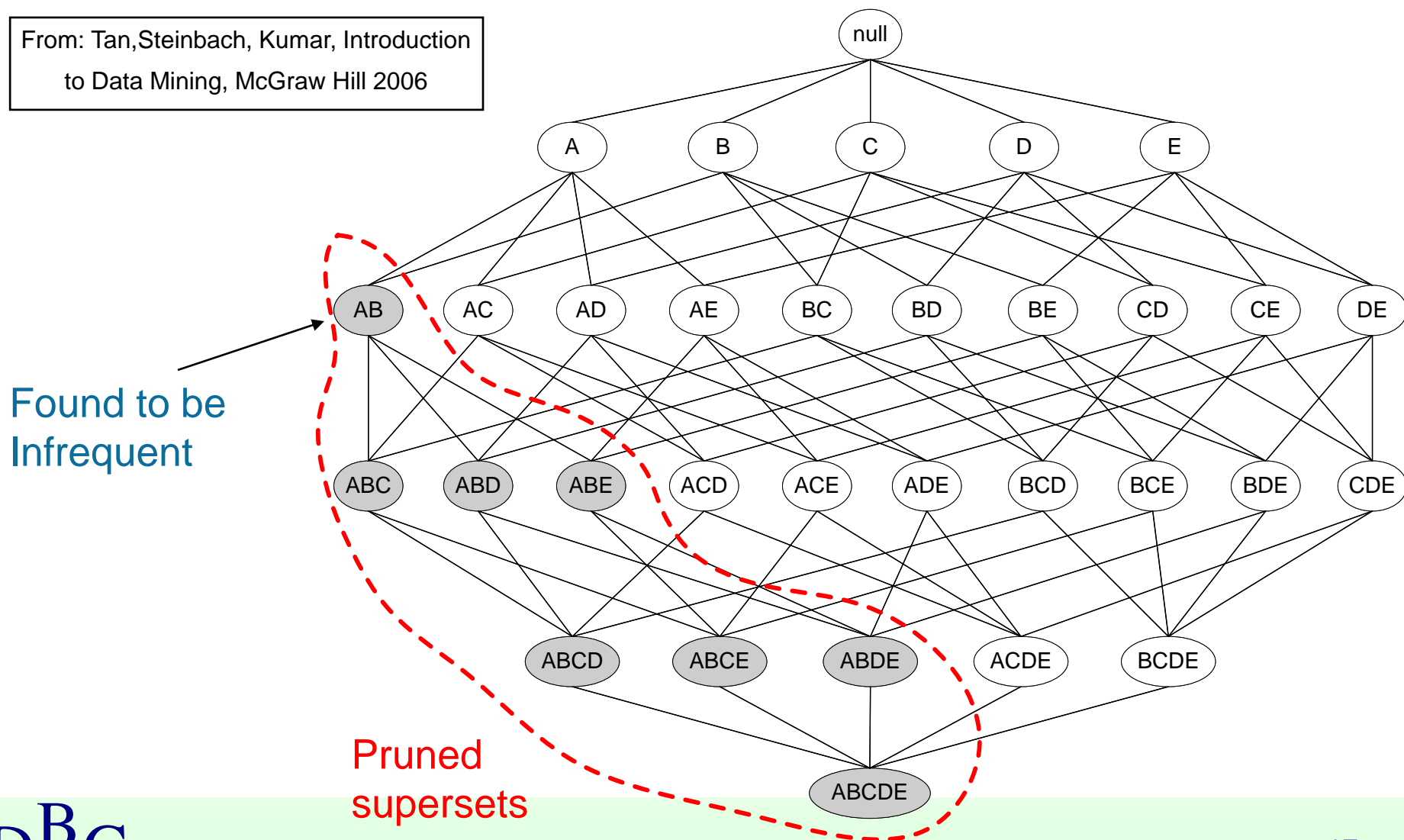
- The support of an itemset can never exceed the support of any of its subsets
- It holds due to the antimonotone property of the support measure
  - Given two arbitrary itemsets A and B  
if  $A \subseteq B$  then  $\text{sup}(A) \geq \text{sup}(B)$
- It reduces the number of candidates





# The Apriori Principle

From: Tan, Steinbach, Kumar, Introduction  
to Data Mining, McGraw Hill 2006





# Apriori Algorithm [Agr94]

- Level-based approach
  - at each iteration extracts itemsets of a given length  $k$
- Two main steps for each level
  - (1) Candidate generation
    - Join Step
      - generate candidates of length  $k+1$  by joining frequent itemsets of length  $k$
    - Prune Step
      - apply Apriori principle: prune length  $k+1$  candidate itemsets that contain at least one  $k$ -itemset that is not frequent
  - (2) Frequent itemset generation
    - scan DB to count support for  $k+1$  candidates
    - prune candidates below minsup



# Apriori Algorithm [Agr94]

## ■ Pseudo-code

$C_k$ : Candidate itemset of size  $k$

$L_k$ : frequent itemset of size  $k$

$L_1 = \{\text{frequent items}\};$

**for** ( $k = 1; L_k \neq \emptyset; k++$ ) **do**

**begin**

$C_{k+1}$  = candidates generated from  $L_k$ ;

**for each** transaction  $t$  in database **do**

increment the count of all candidates in  $C_{k+1}$   
that are contained in  $t$

$L_{k+1}$  = candidates in  $C_{k+1}$  satisfying *minsup*

**end**

**return**  $\cup_k L_k$ ;



# Generating Candidates

- Sort  $L_k$  candidates in lexicographical order
- For each candidate of length  $k$ 
  - Self-join with each candidate sharing same  $L_{k-1}$  prefix
  - Prune candidates by applying Apriori principle
- Example: given  $L_3 = \{abc, abd, acd, ace, bcd\}$ 
  - Self-join
    - $abcd$  from  $abc$  and  $abd$
    - $acde$  from  $acd$  and  $ace$
  - Prune by applying Apriori principle
    - $acde$  is removed because  $ade, cde$  are not in  $L_3$
    - $C_4 = \{abcd\}$



# Apriori Algorithm: Example

## Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

$\text{minsup} > 1$



# Generate candidate 1-itemsets

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

1<sup>st</sup> DB  
scan



$C_1$

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

$\text{minsup} > 1$

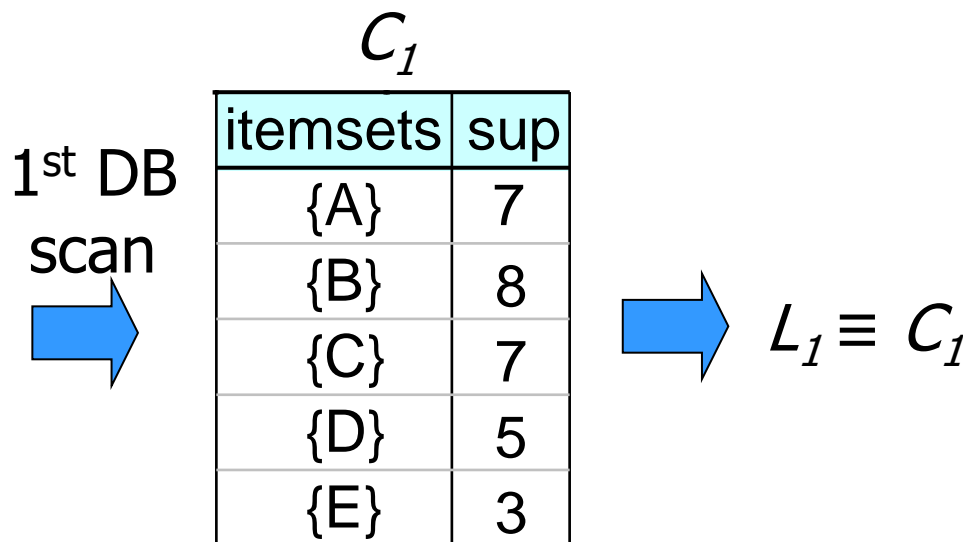


# Prune infrequent candidates in $C_1$

## Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup > 1



- All itemsets in set  $C_1$  are frequent according to minsup > 1



# Generate candidates from $L_1$

$L_1$			
itemsets	sup		$C_2$
{A}	7	➔	itemsets
{B}	8		{A,B}
{C}	7		{A,C}
{D}	5		{A,D}
{E}	3		{A,E}
			{B,C}
			{B,D}
			{B,E}
			{C,D}
			{C,E}
			{D,E}





# Count support for candidates in $C_2$

$L_1$

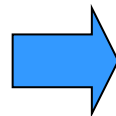
itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3



$C_2$

itemsets
{A,B}
{A,C}
{A,D}
{A,E}
{B,C}
{B,D}
{B,E}
{C,D}
{C,E}
{D,E}

2<sup>nd</sup>  
DB  
scan

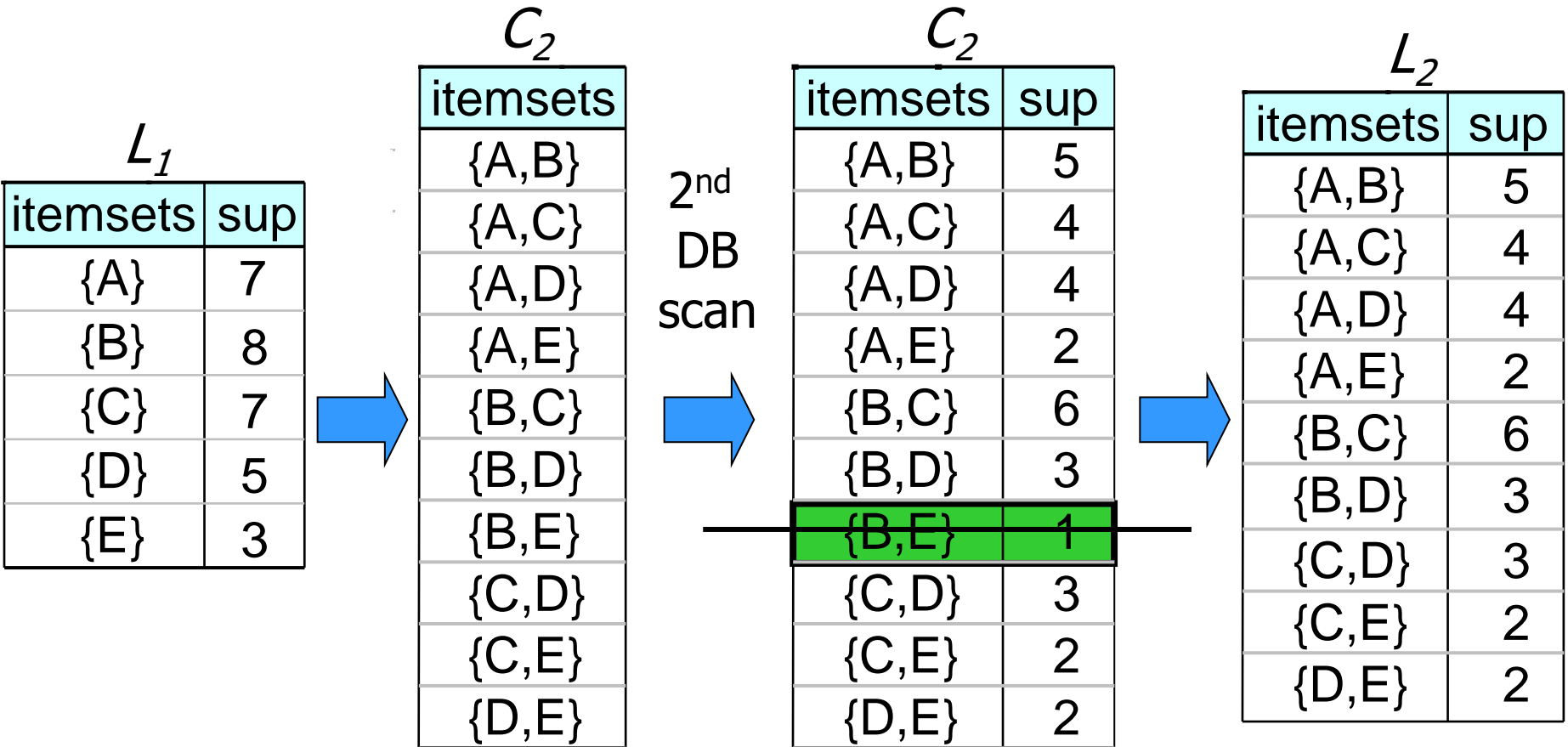


$C_2$

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{B,E}	1
{C,D}	3
{C,E}	2
{D,E}	2



# Prune infrequent candidates in $C_2$

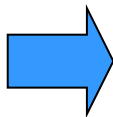




# Generate candidates from $L_2$

$L_2$

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2



$C_3$

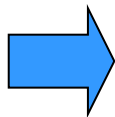
itemsets
{A,B,C}
{A,B,D}
{A,B,E}
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}



# Apply Apriori principle on $C_3$

 $L_2$ 

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

 $C_3$ 

itemsets
{A,B,C}
{A,B,D}
<del>{A,B,E}</del>
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

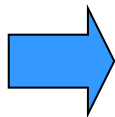
- Prune {A,B,E}
  - Its subset {B,E} is infrequent ({B,E} is not in  $L_2$ )



# Count support for candidates in $C_3$

$L_2$

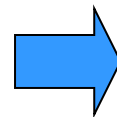
itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2



$C_3$

itemsets
{A,B,C}
{A,B,D}
<del>{A,B,E}</del>
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

3<sup>rd</sup>  
DB  
scan



$C_3$

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,C,E}	1
{A,D,E}	2
{B,C,D}	2
{C,D,E}	1



# Prune infrequent candidates in $C_3$

 $L_2$ 

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

 $C_3$ 

itemsets
{A,B,C}
{A,B,D}
<del>{A,B,E}</del>
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

3<sup>rd</sup>  
DB  
scan

 $C_3$ 

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
<del>{A,C,E}</del>	<del>1</del>
{A,D,E}	2
{B,C,D}	2
<del>{C,D,E}</del>	<del>1</del>

 $L_3$ 

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

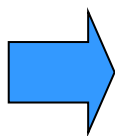
- {A,C,E} and {C,D,E} are actually infrequent
  - They are discarded from  $C_3$



# Generate candidates from $L_3$

$L_3$

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2



$C_4$

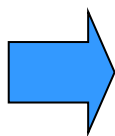
itemsets
{A,B,C,D}



# Apply Apriori principle on $C_4$

$L_3$

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2



$C_4$

itemsets
{A,B,C,D}

- Check if {A,C,D} and {B,C,D} belong to  $L_3$ 
  - $L_3$  contains all 3-itemset subsets of {A,B,C,D}
  - {A,B,C,D} is potentially frequent

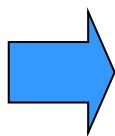




# Count support for candidates in $C_4$

$L_3$

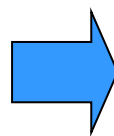
itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2



$C_4$

itemsets
{A,B,C,D}

4<sup>th</sup> DB  
scan

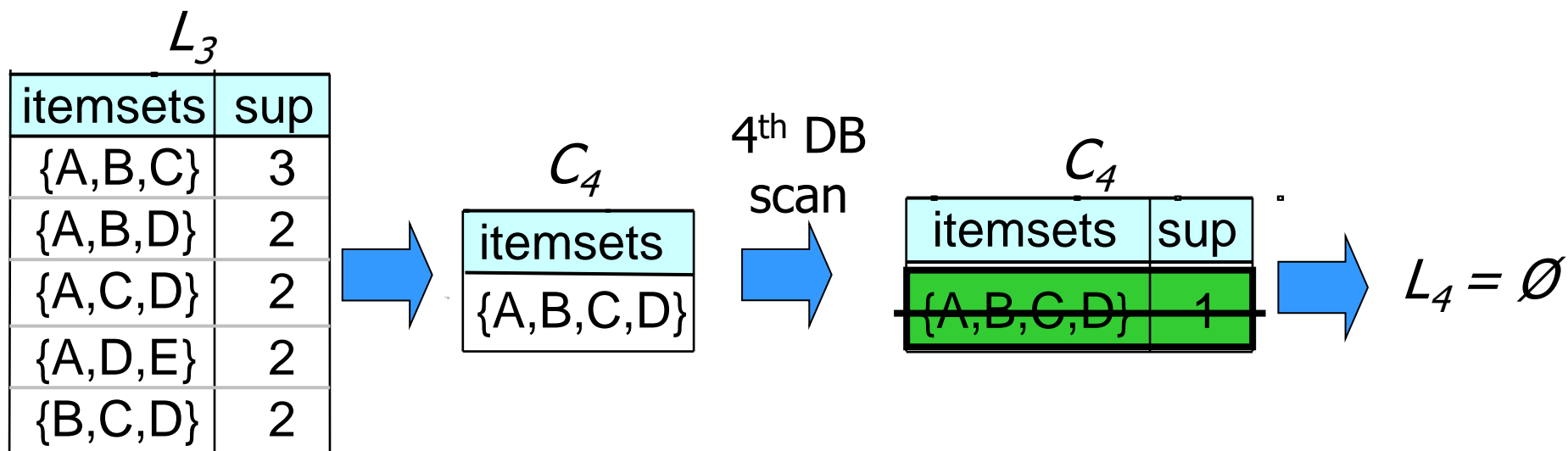


$C_4$

itemsets	sup
{A,B,C,D}	1



# Prune infrequent candidates in $C_4$



- {A,B,C,D} is actually infrequent
  - {A,B,C,D} is discarded from  $C_4$

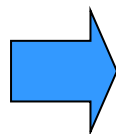


# Final set of frequent itemsets

## Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup > 1



$L_1$

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

$L_3$

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

$L_2$

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2



# Counting Support of Candidates

- Scan transaction database to count support of each itemset
  - total number of candidates may be large
  - one transaction may contain many candidates
- Approach [Agr94]
  - candidate itemsets are stored in a *hash-tree*
    - *leaf* node of hash-tree contains a list of itemsets and counts
    - *interior* node contains a hash table
  - subset function finds all candidates contained in a transaction
    - match transaction subsets to candidates in hash tree



# Performance Issues in Apriori

- Candidate generation
  - Candidate sets may be huge
    - 2-itemset candidate generation is the most critical step
    - extracting long frequent itemsets requires generating all frequent subsets
- Multiple database scans
  - $n + 1$  scans when longest frequent pattern length is  $n$



# Factors Affecting Performance

- Minimum support threshold
  - lower support threshold increases number of frequent itemsets
    - larger number of candidates
    - larger (max) length of frequent itemsets
- Dimensionality (number of items) of the data set
  - more space is needed to store support count of each item
  - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
  - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
  - transaction width increases in dense data sets
  - may increase max length of frequent itemsets and traversals of hash tree
    - number of subsets in a transaction increases with its width



# Improving Apriori Efficiency

- Hash-based itemset counting [Yu95]
  - A  $k$ -itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
- Transaction reduction [Yu95]
  - A transaction that does not contain any frequent  $k$ -itemset is useless in subsequent scans
- Partitioning [Sav96]
  - Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB



# Improving Apriori Efficiency

- Sampling [Toi96]
  - mining on a subset of given data, lower support threshold + a method to determine the completeness
- Dynamic Itemset Counting [Motw98]
  - add new candidate itemsets only when all of their subsets are estimated to be frequent





# FP-growth Algorithm [Han00]

- Exploits a main memory compressed representation of the database, the FP-tree
  - high compression for dense data distributions
    - less so for sparse data distributions
  - complete representation for frequent pattern mining
    - enforces support constraint
- Frequent pattern mining by means of FP-growth
  - recursive visit of FP-tree
  - applies divide-and-conquer approach
    - decomposes mining task into smaller subtasks
- Only two database scans
  - count item supports + build FP-tree



# FP-tree construction

## Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

$\text{minsup} > 1$

- (1) Count item support and prune items below minsup threshold
- (2) Build Header Table by sorting items in decreasing support order

## Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



# FP-tree construction

## Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

$\text{minsup} > 1$

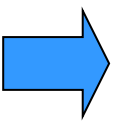
- (1) Count item support and prune items below minsup threshold
- (2) Build Header Table by sorting items in decreasing support order
- (3) Create FP-tree
  - For each transaction  $t$  in DB
    - order transaction  $t$  items in decreasing support order
      - same order as Header Table
    - insert transaction  $t$  in FP-tree
      - use existing path for common prefix
      - create new branch when path becomes different



# FP-tree construction

Transaction

TID	Items
1	{A,B}

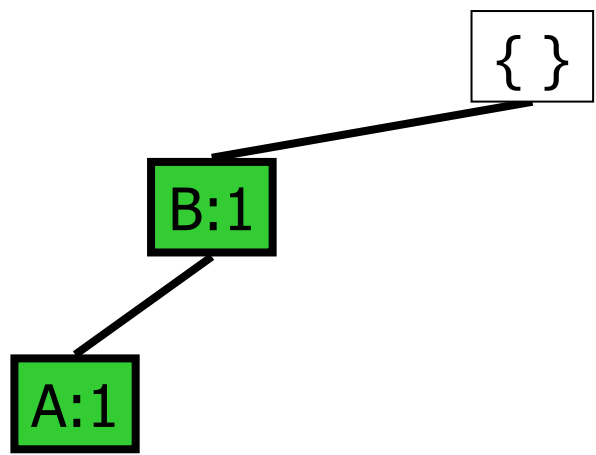


Sorted transaction

TID	Items
1	{B,A}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



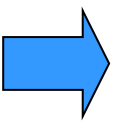
FP-tree



# FP-tree construction

Transaction

TID	Items
2	{B,C,D}

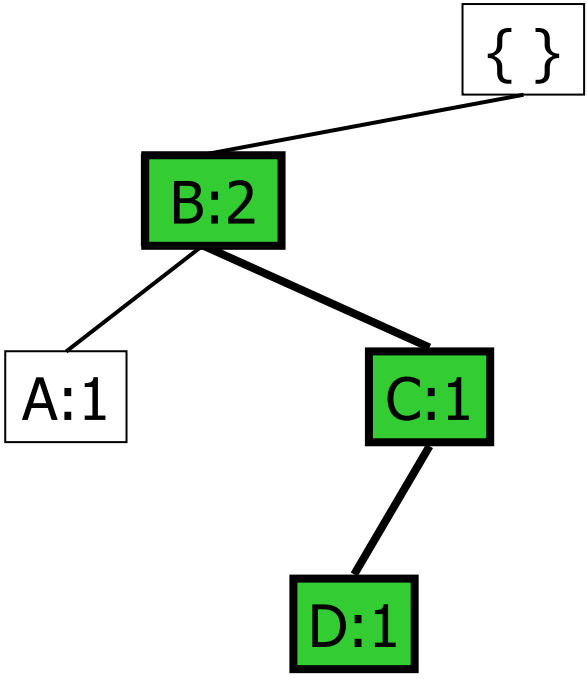


Sorted transaction

TID	Items
2	{B,C,D}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



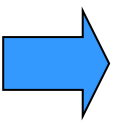
FP-tree



# FP-tree construction

Transaction

TID	Items
3	{A,C,D,E}

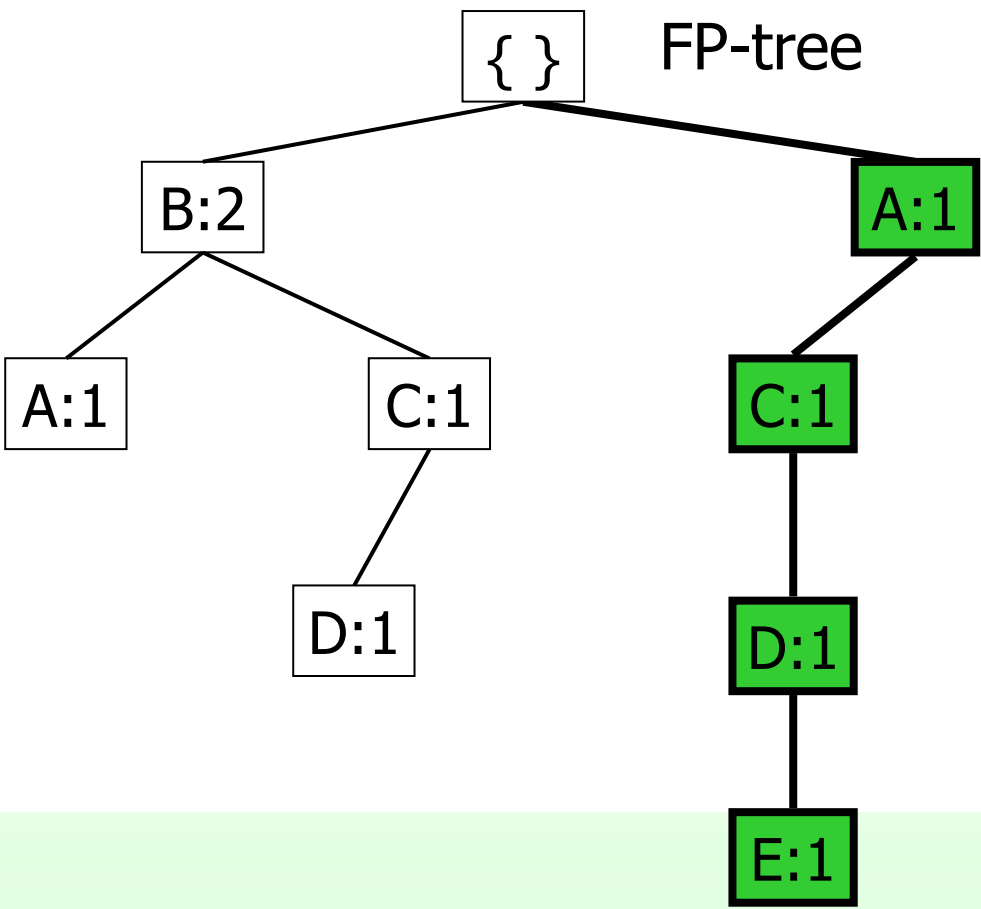


Sorted transaction

TID	Items
3	{A,C,D,E}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

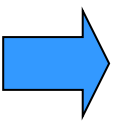




# FP-tree construction

Transaction

TID	Items
4	{A,D,E}

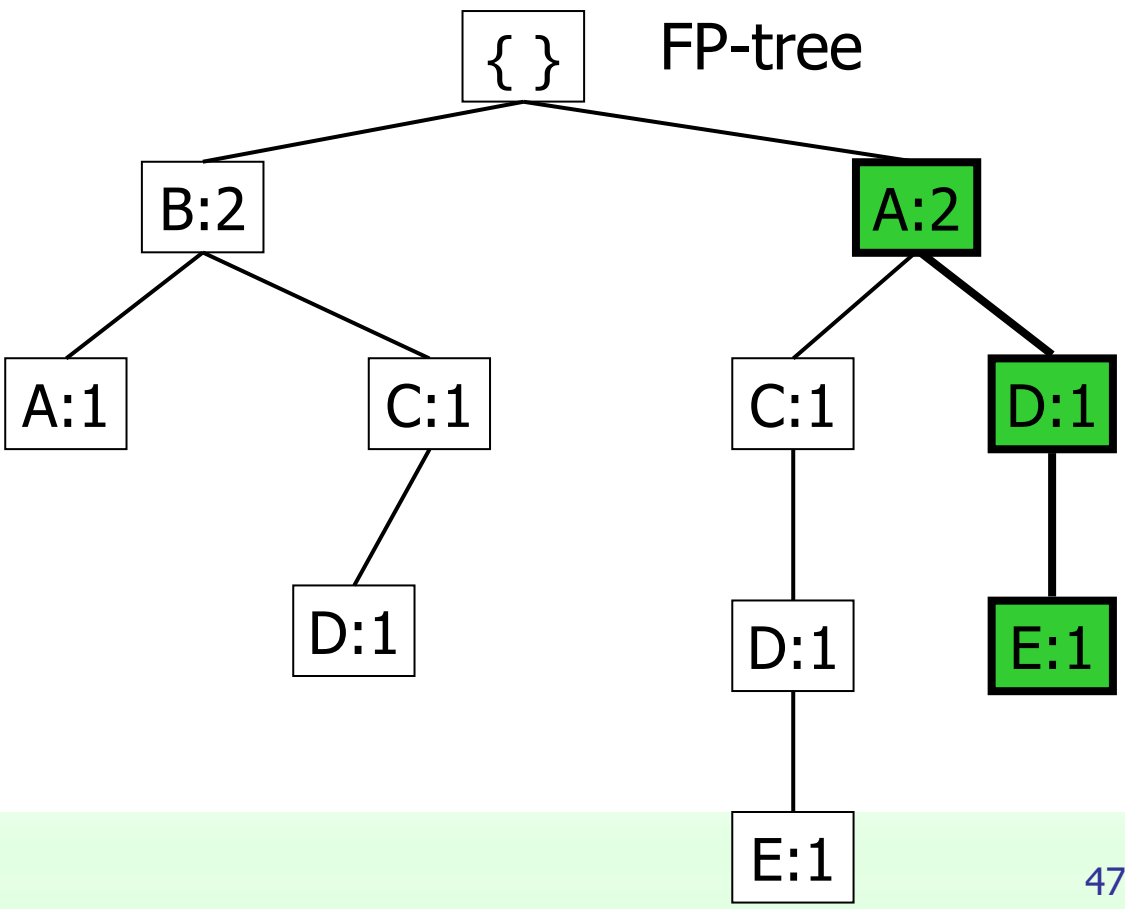


Sorted transaction

TID	Items
4	{A,D,E}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

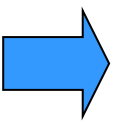




# FP-tree construction

Transaction

TID	Items
5	{A,B,C}

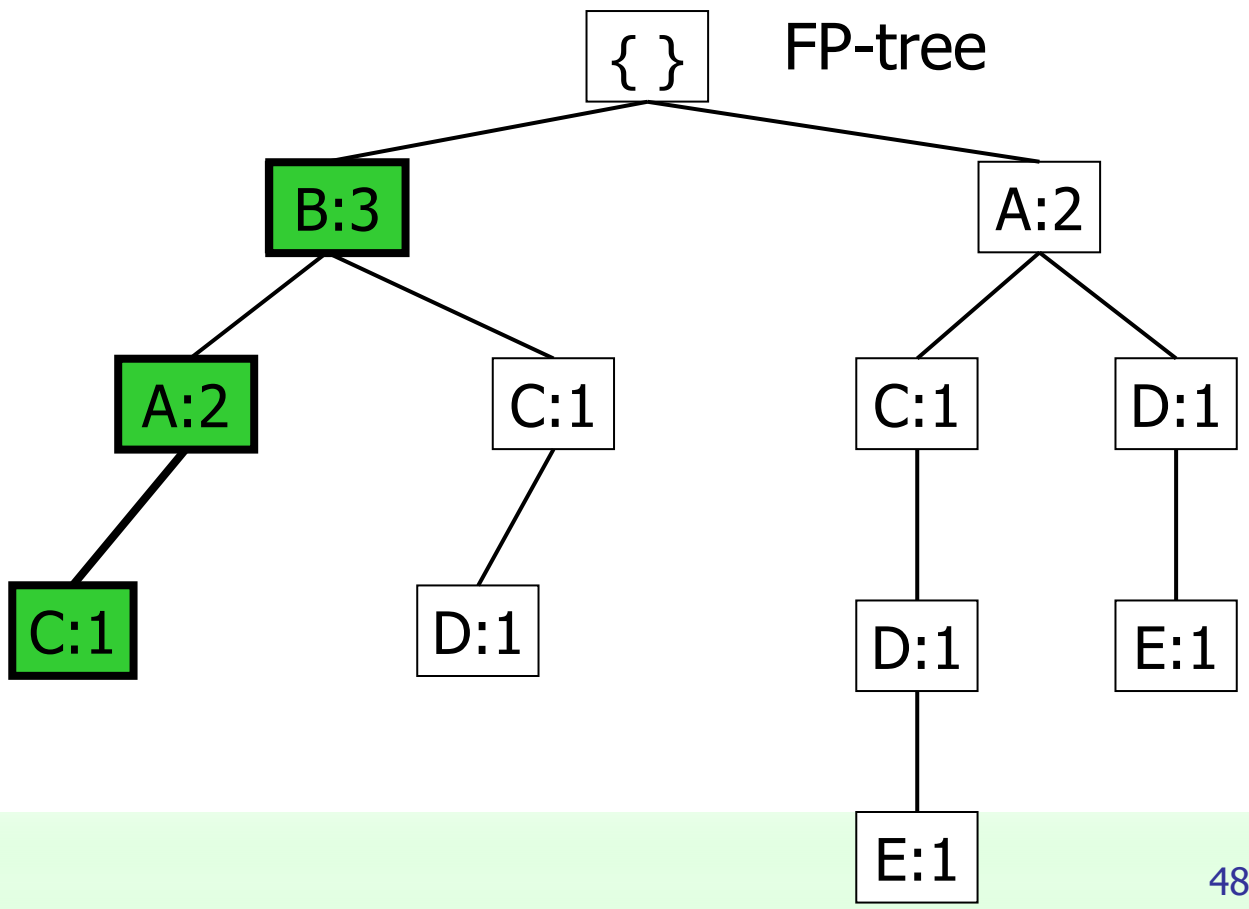


Sorted transaction

TID	Items
5	{B,A,C}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

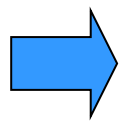






# Transaction

TID	Items
6	{A,B,C,D}

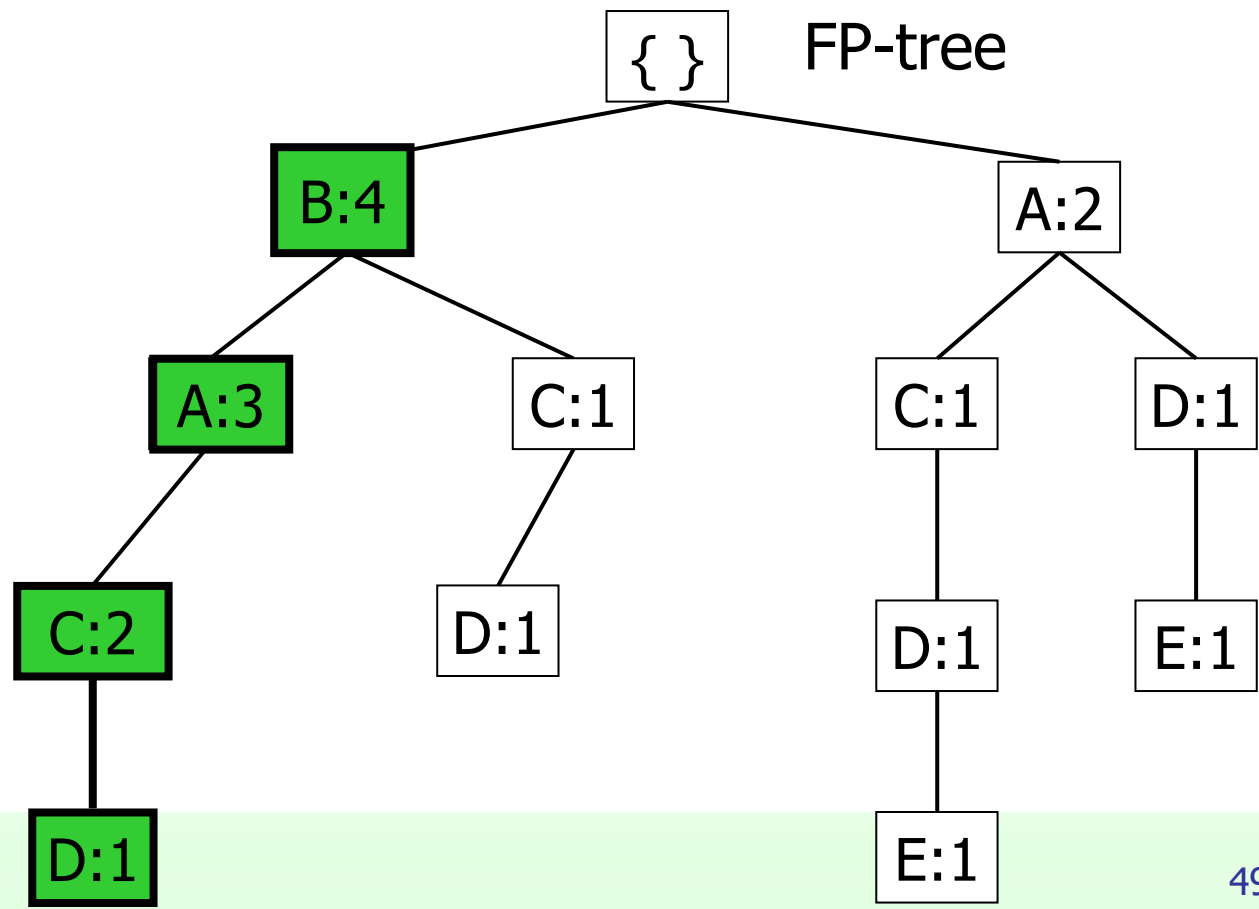


# Sorted transaction

TID	Items
6	{B,A,C,D}

# Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

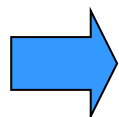




# FP-tree construction

Transaction

TID	Items
7	{B,C}

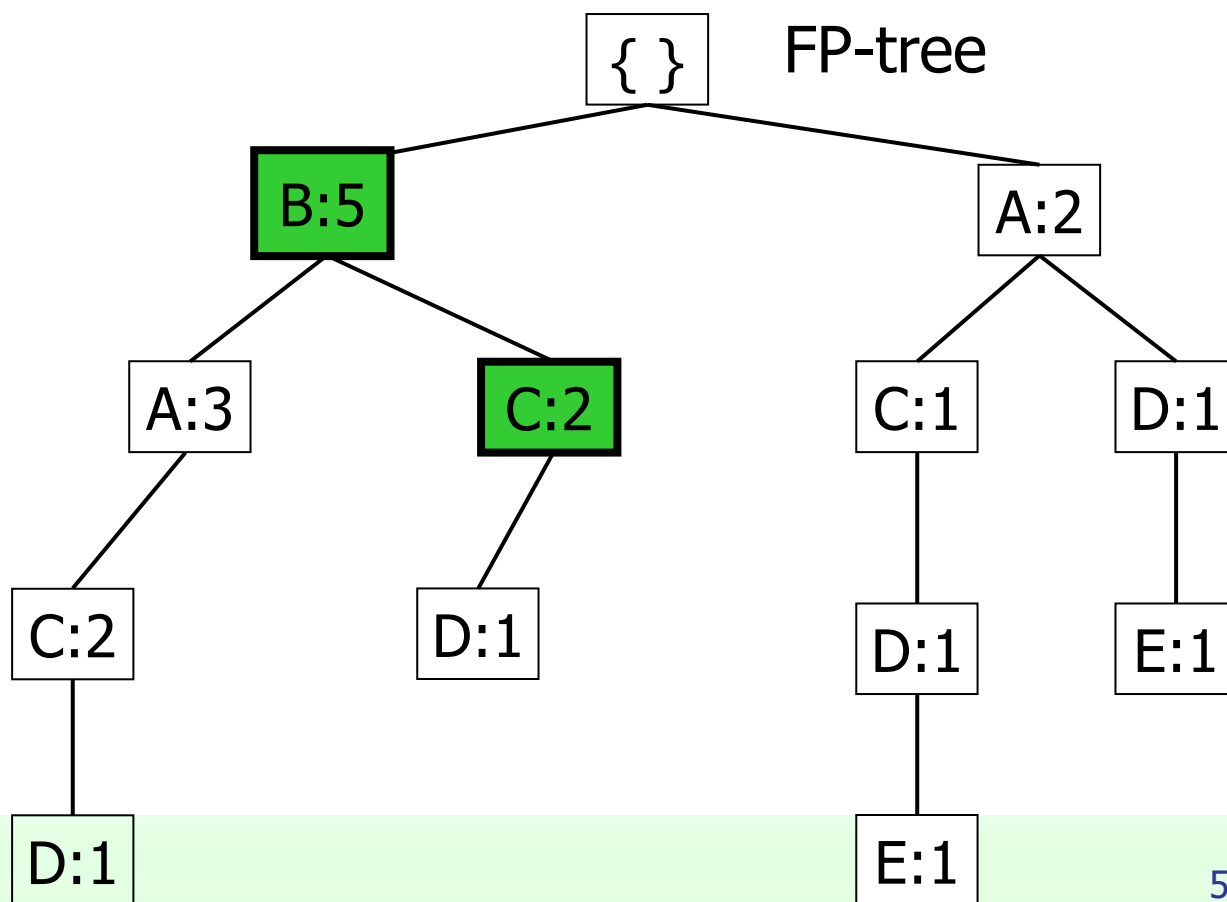


Sorted transaction

TID	Items
7	{B,C}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

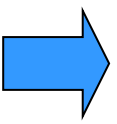




# FP-tree construction

Transaction

TID	Items
8	{A,B,C}

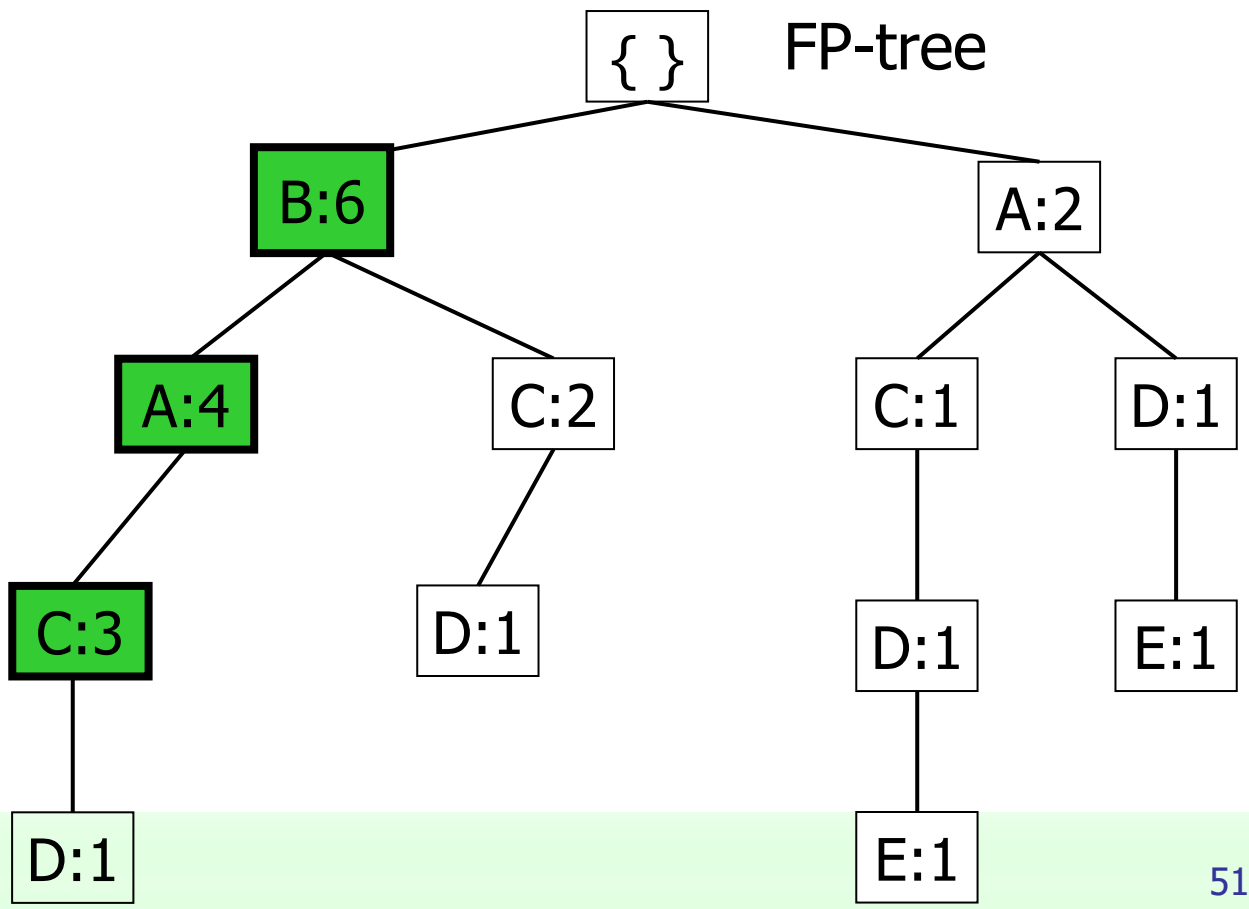


Sorted transaction

TID	Items
8	{B,A,C}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



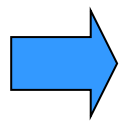


# FP-tree construction

Transaction

Sorted transaction

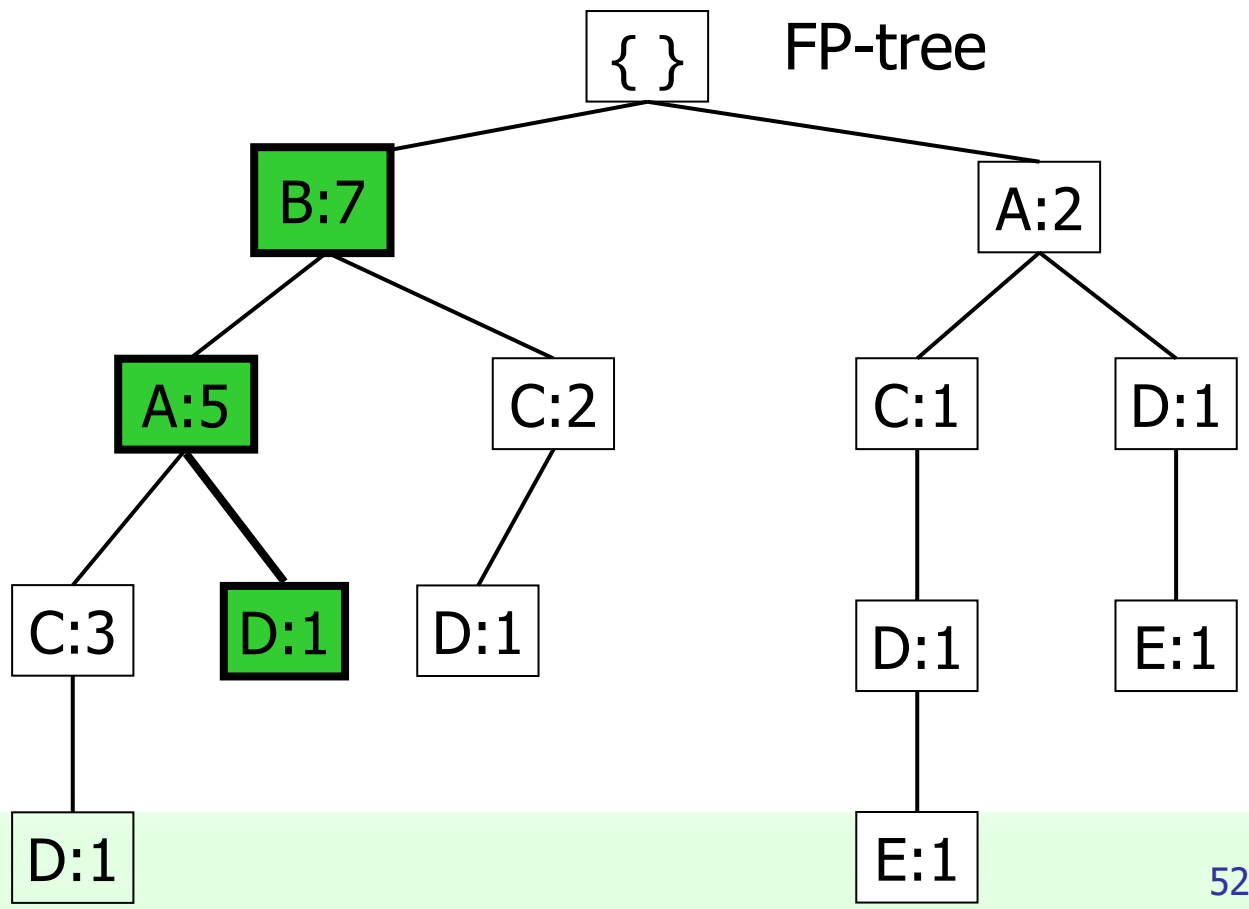
TID	Items
9	{A,B,D}



TID	Items
9	{B,A,D}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

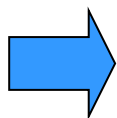




# FP-tree construction

Transaction

TID	Items
10	{B,C,E}

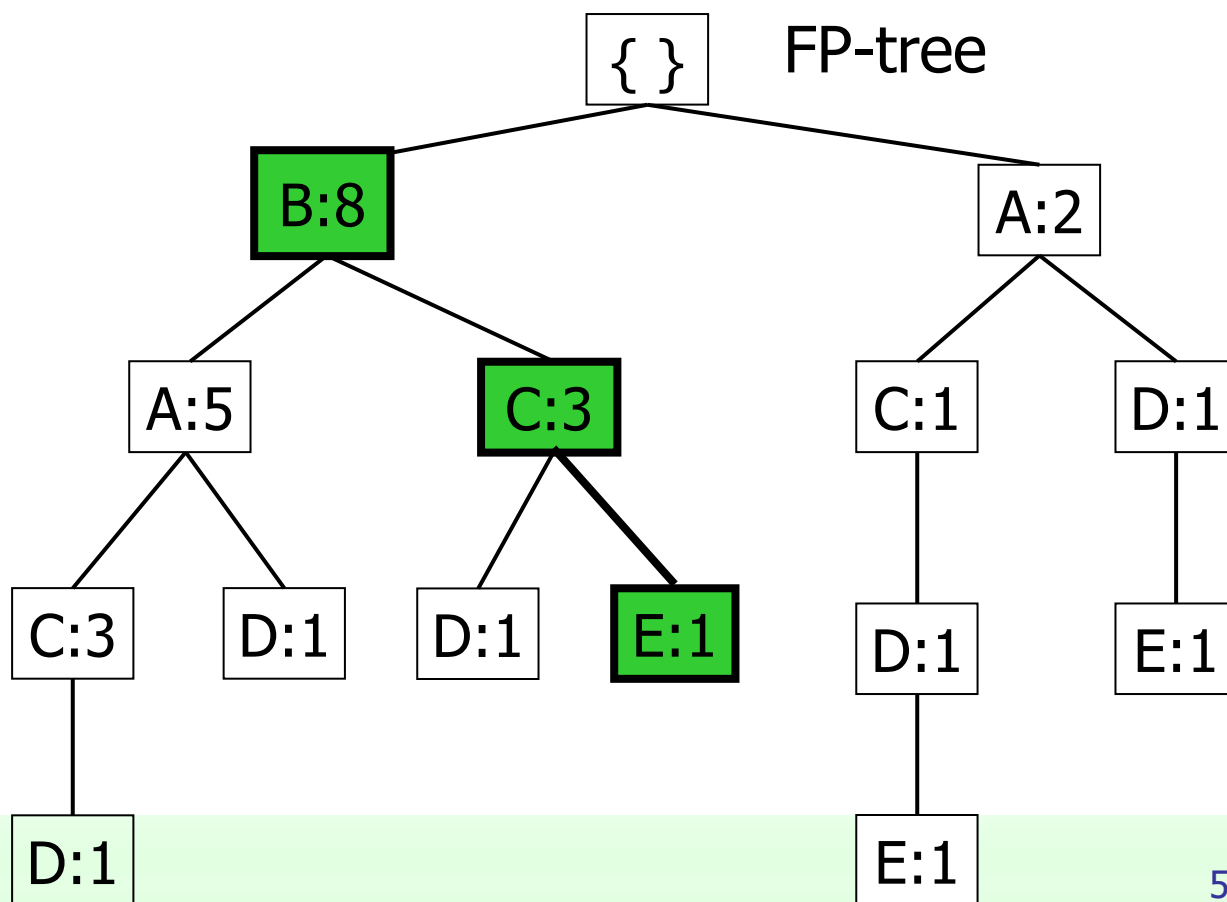


Sorted transaction

TID	Items
10	{B,C,E}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

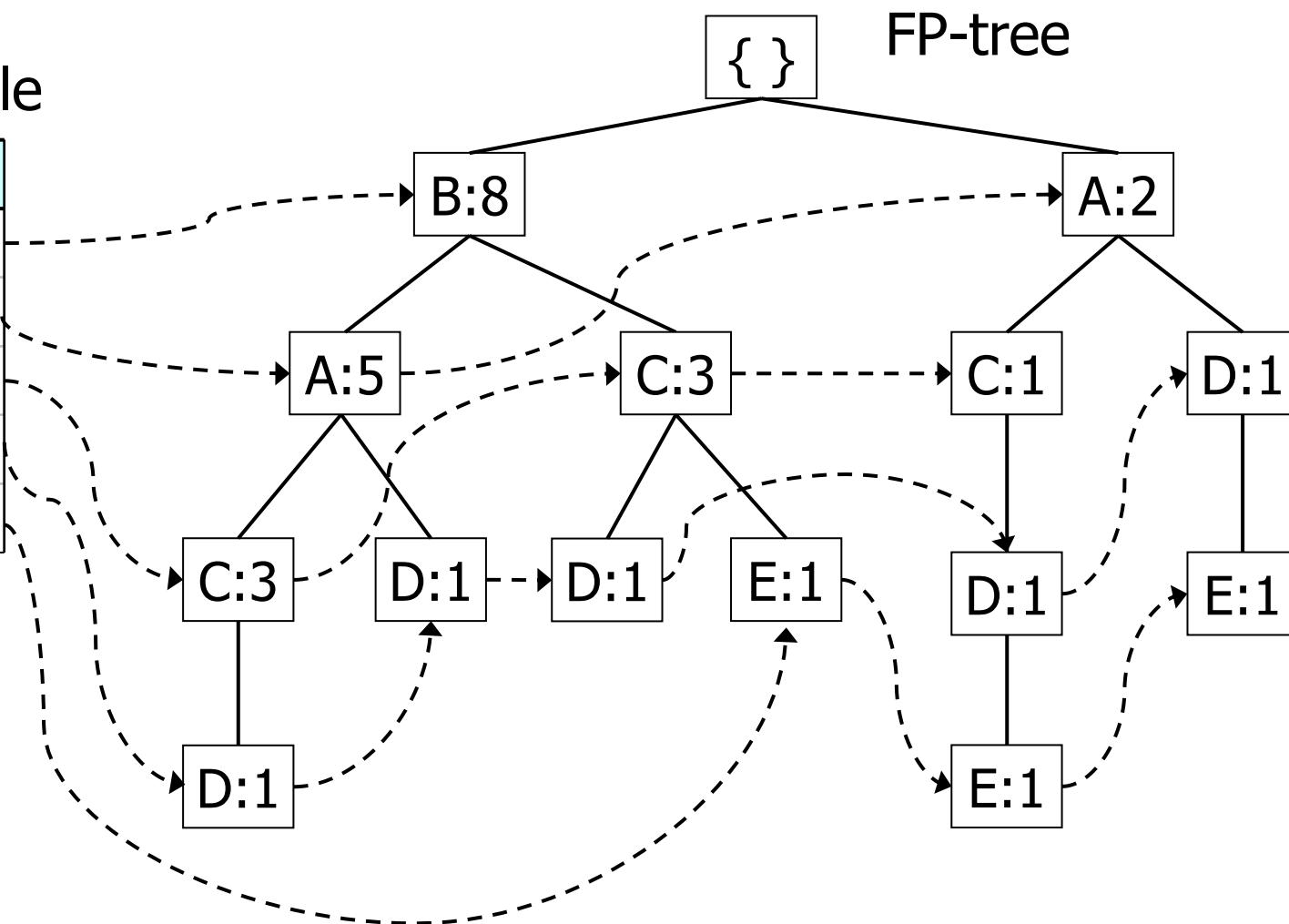




# Final FP-tree

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3





# FP-growth Algorithm

- Scan Header Table from lowest support item up
- For each item  $i$  in Header Table extract frequent itemsets including item  $i$  and items preceding it in Header Table
  - (1) build Conditional Pattern Base for item  $i$  (i-CPB)
    - Select prefix-paths of item  $i$  from FP-tree
  - (2) recursive invocation of FP-growth on i-CPB

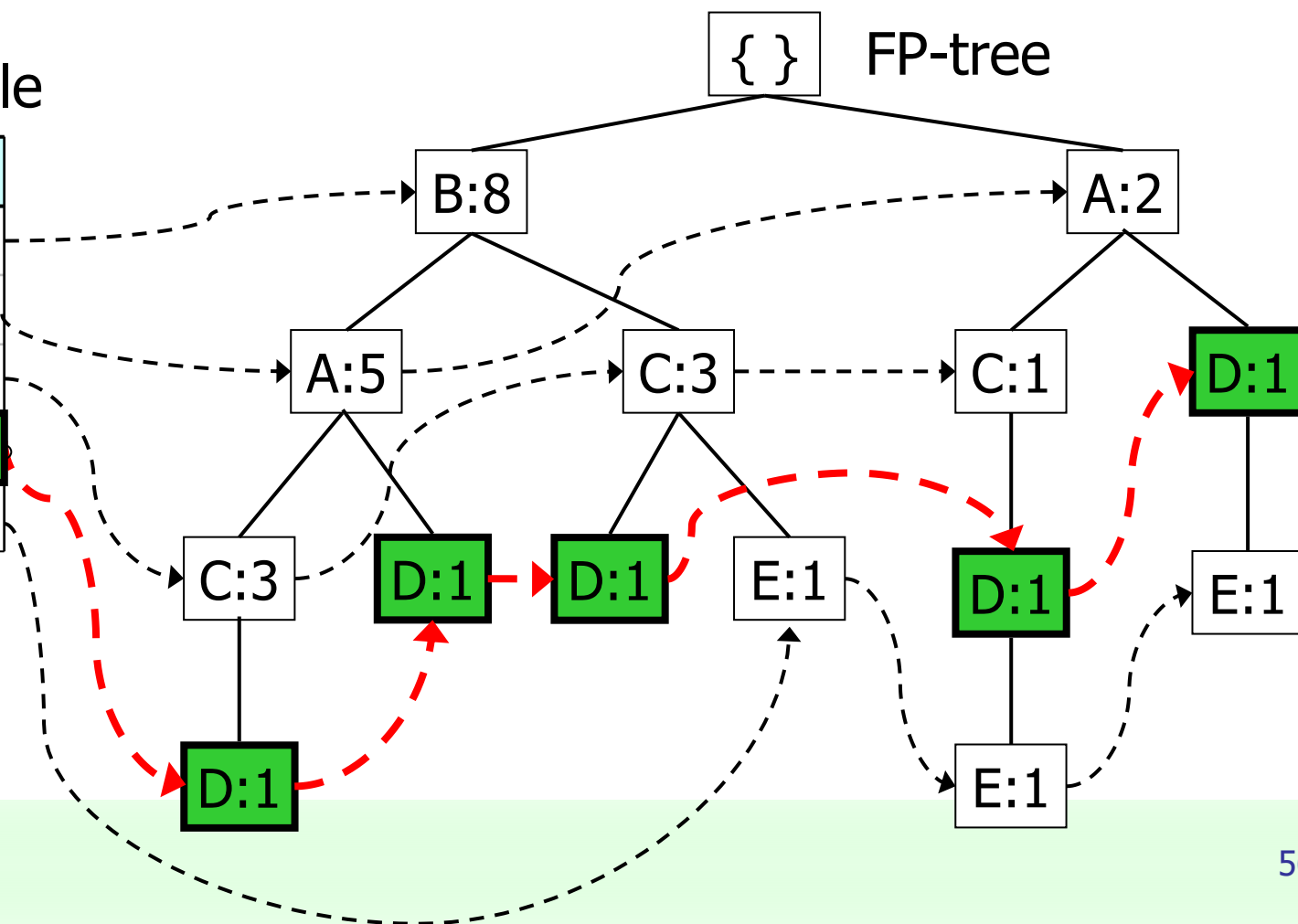


# Example

- Consider item D and extract frequent itemsets including
  - D and supported combinations of items A, B, C

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
<b>{D}</b>	<b>5</b>
{E}	3







# Conditional Pattern Base of D

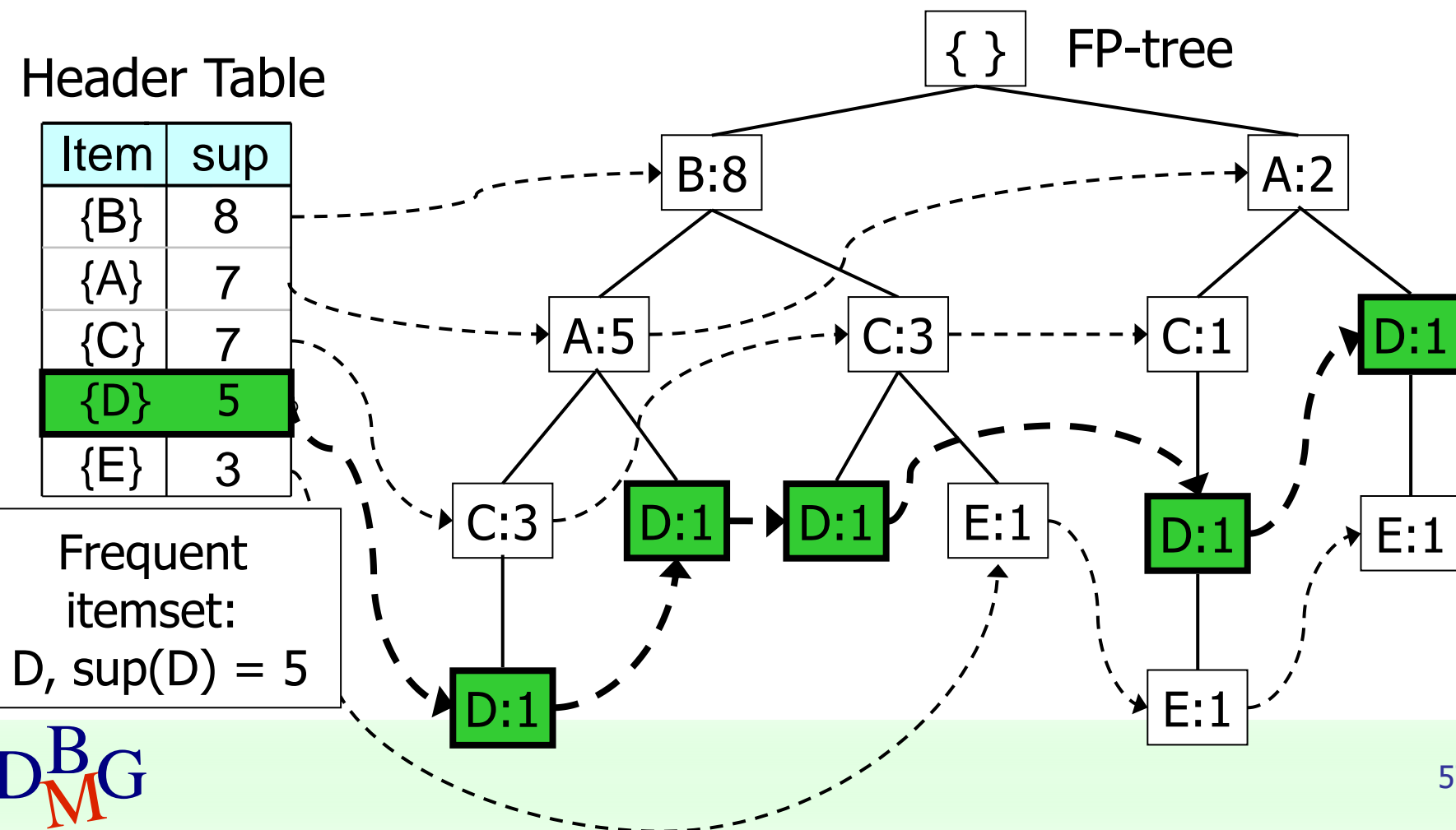
- (1) Build D-CPB
  - Select prefix-paths of item D from FP-tree

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
<b>{D}</b>	<b>5</b>
{E}	3

Frequent  
itemset:  
D,  $\text{sup}(D) = 5$

{ } FP-tree





# Conditional Pattern Base of D

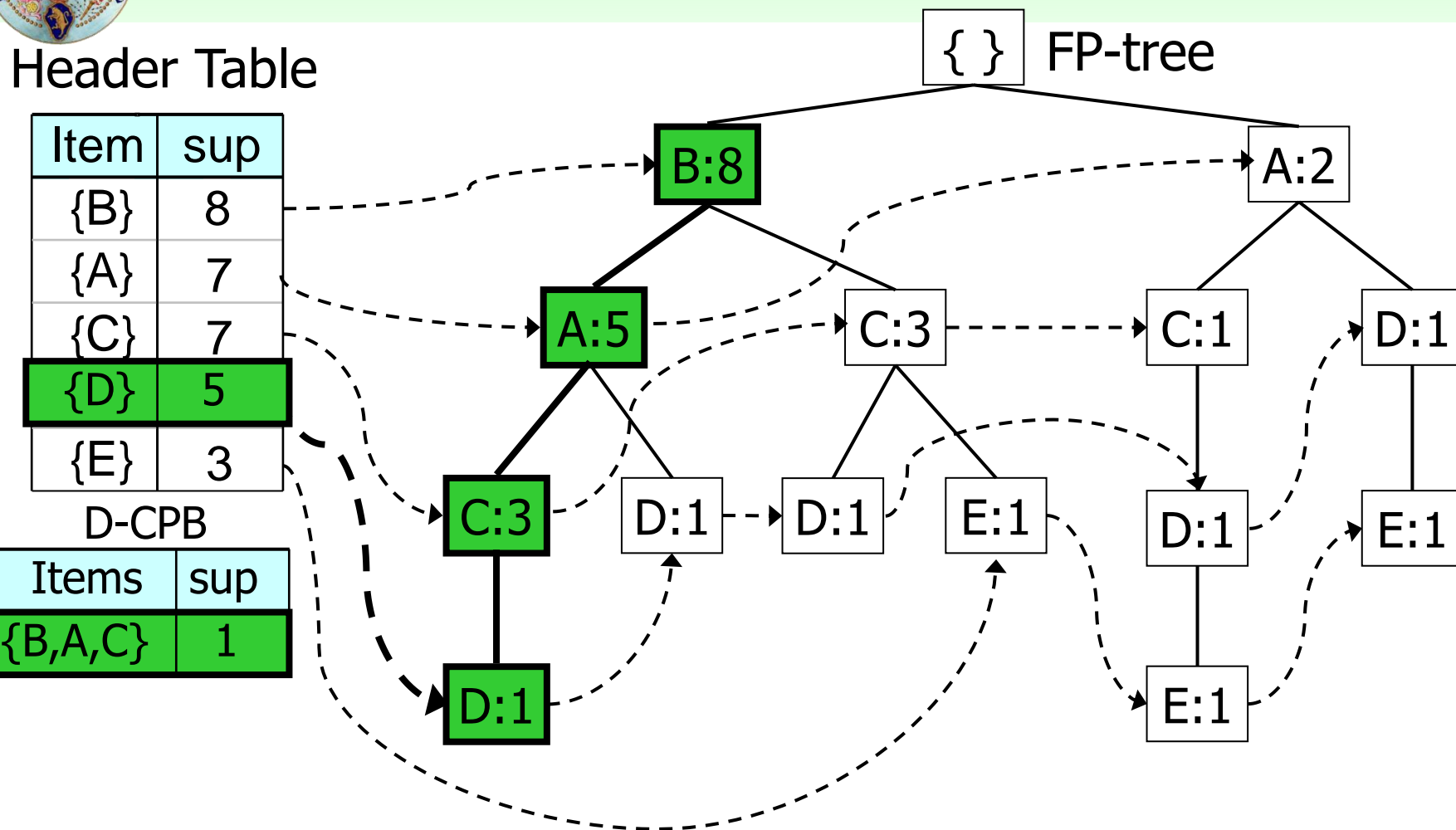
## Header Table

Item	sup
{B}	8
{A}	7
{C}	7
<b>{D}</b>	<b>5</b>
{E}	3

D-CPB

Items	sup
<b>{B,A,C}</b>	<b>1</b>

{ } FP-tree





# Conditional Pattern Base of D

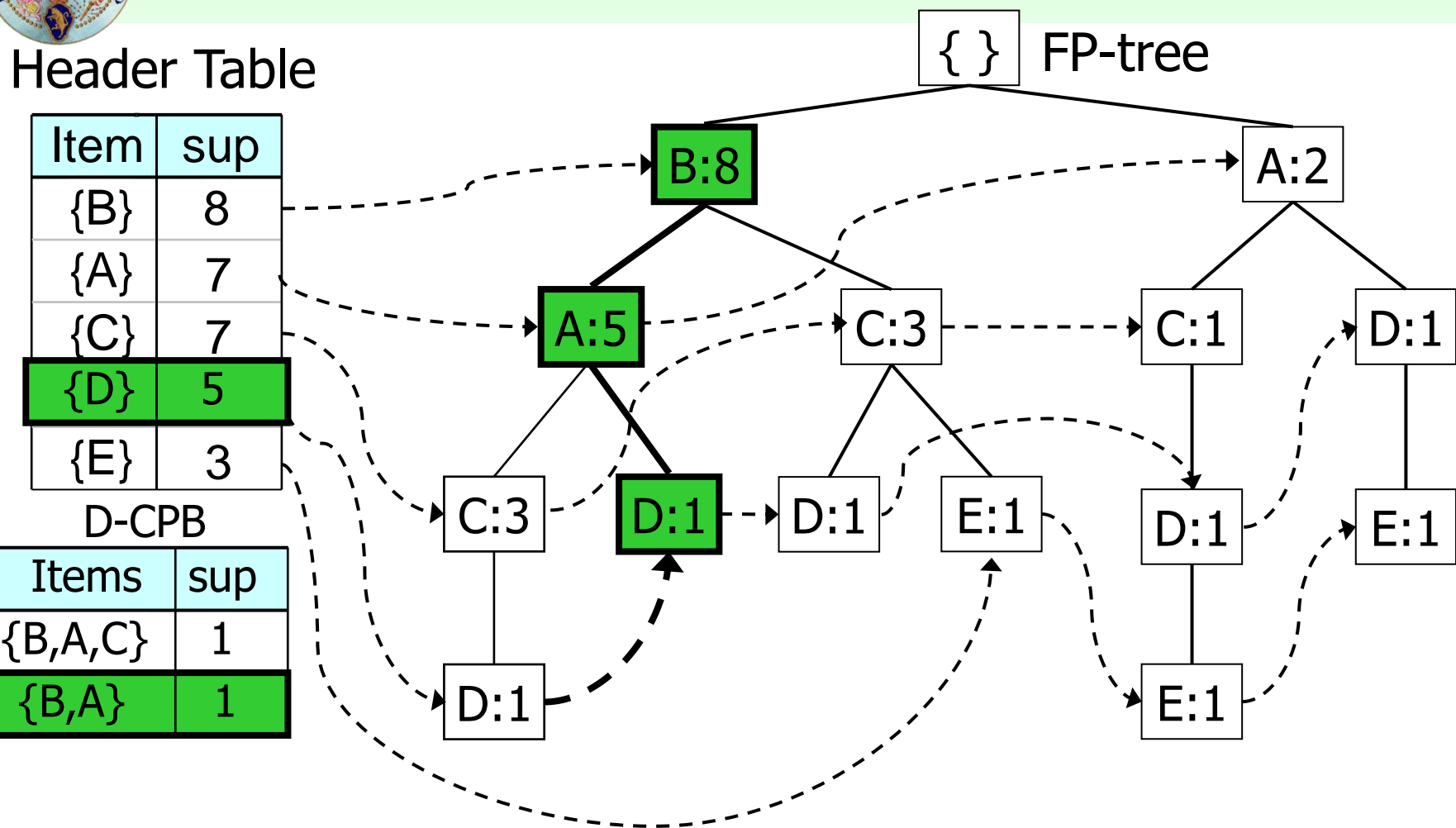
Header Table

Item	sup
{B}	8
{A}	7
{C}	7
<b>{D}</b>	<b>5</b>
{E}	3

D-CPB

Items	sup
{B,A,C}	1
<b>{B,A}</b>	<b>1</b>

{ } FP-tree





# Conditional Pattern Base of D

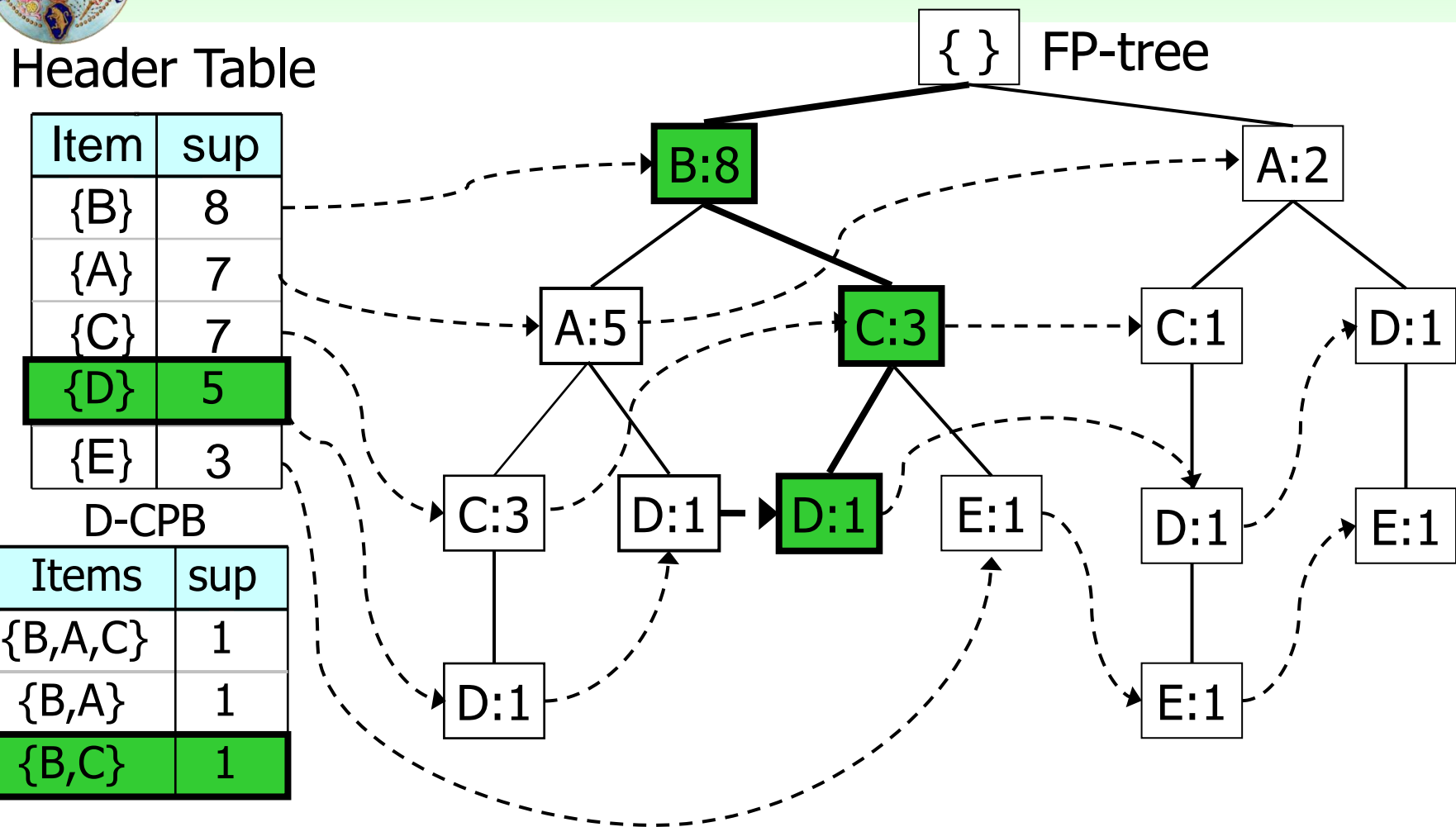
Header Table

Item	sup
{B}	8
{A}	7
{C}	7
<b>{D}</b>	<b>5</b>
{E}	3

D-CPB

Items	sup
{B,A,C}	1
{B,A}	1
<b>{B,C}</b>	<b>1</b>

{ } FP-tree





# Conditional Pattern Base of D

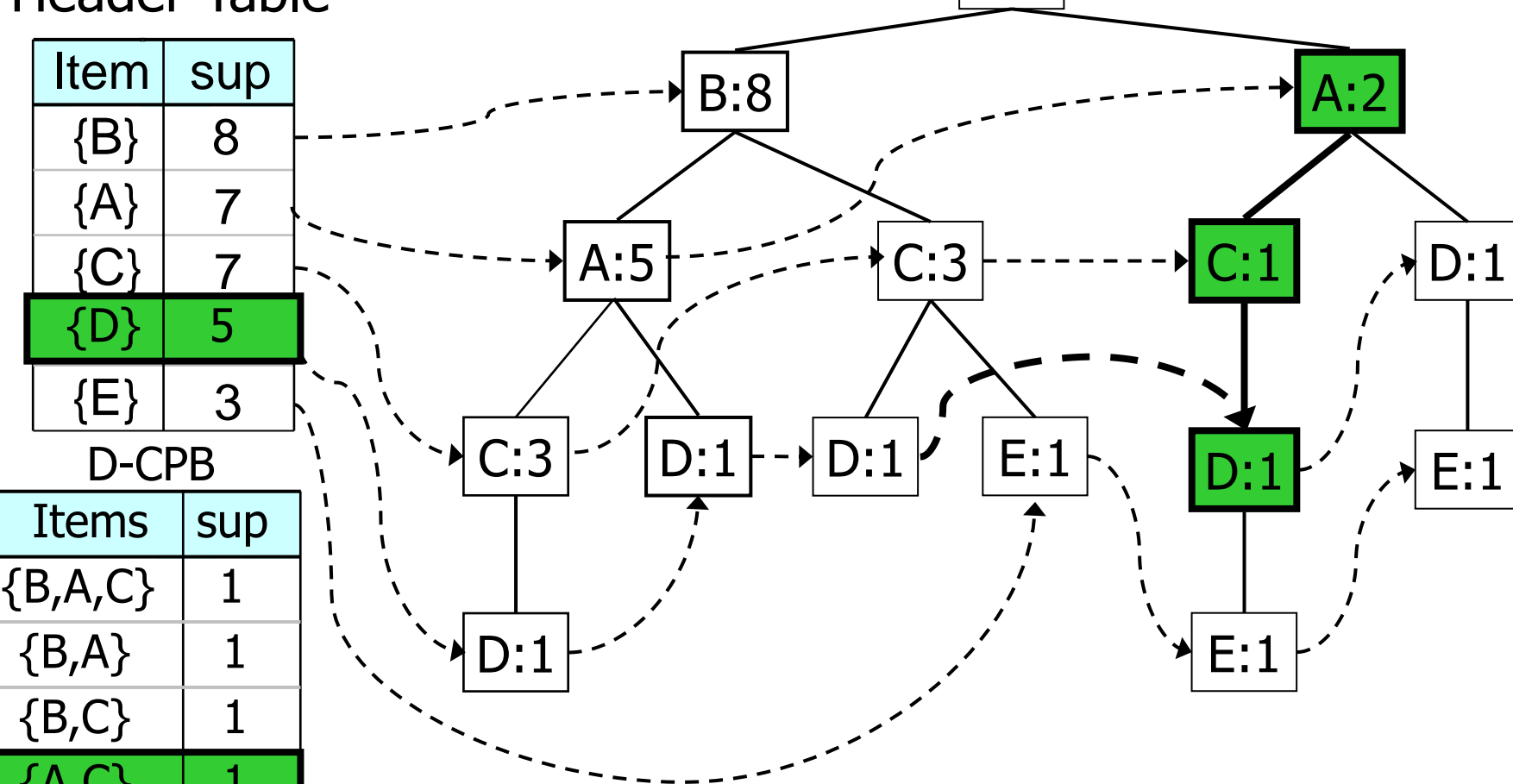
Header Table

Item	sup
{B}	8
{A}	7
{C}	7
<b>{D}</b>	<b>5</b>
{E}	3

D-CPB

Items	sup
{B,A,C}	1
{B,A}	1
{B,C}	1
<b>{A,C}</b>	<b>1</b>

{ } FP-tree





# Conditional Pattern Base of D

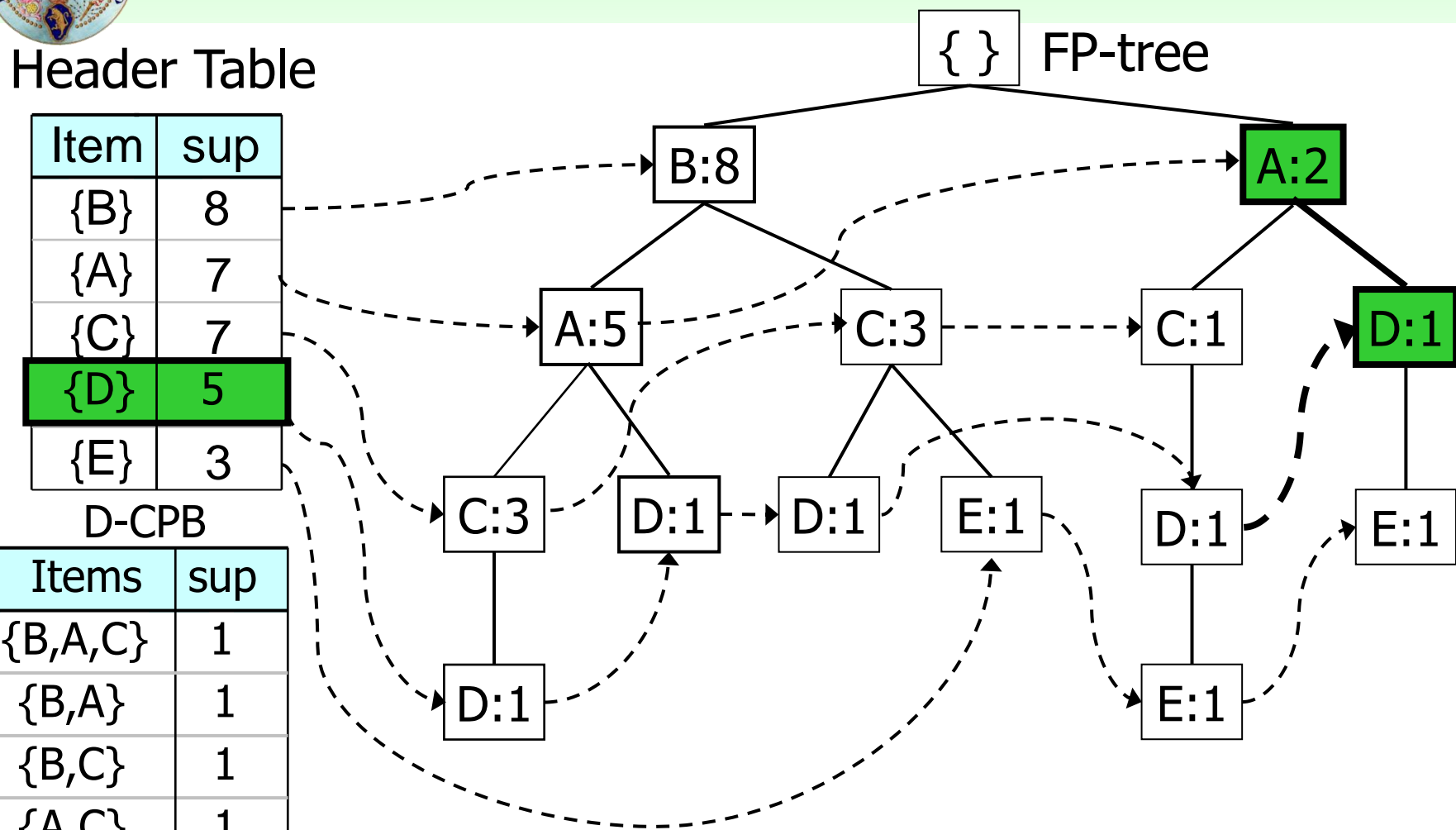
Header Table

Item	sup
{B}	8
{A}	7
{C}	7
<b>{D}</b>	<b>5</b>
{E}	3

D-CPB

Items	sup
{B,A,C}	1
{B,A}	1
{B,C}	1
{A,C}	1
<b>{A}</b>	<b>1</b>

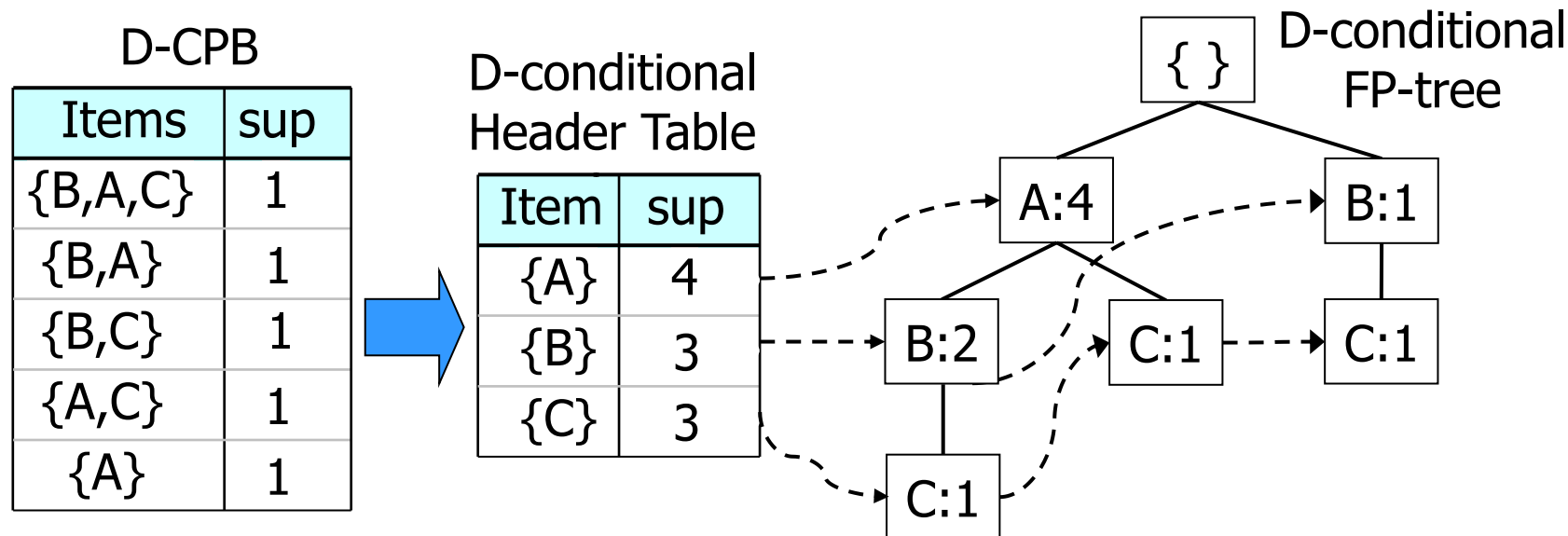
{ } FP-tree





# Conditional Pattern Base of D

- (1) Build D-CPB
  - Select prefix-paths of item D from FP-tree



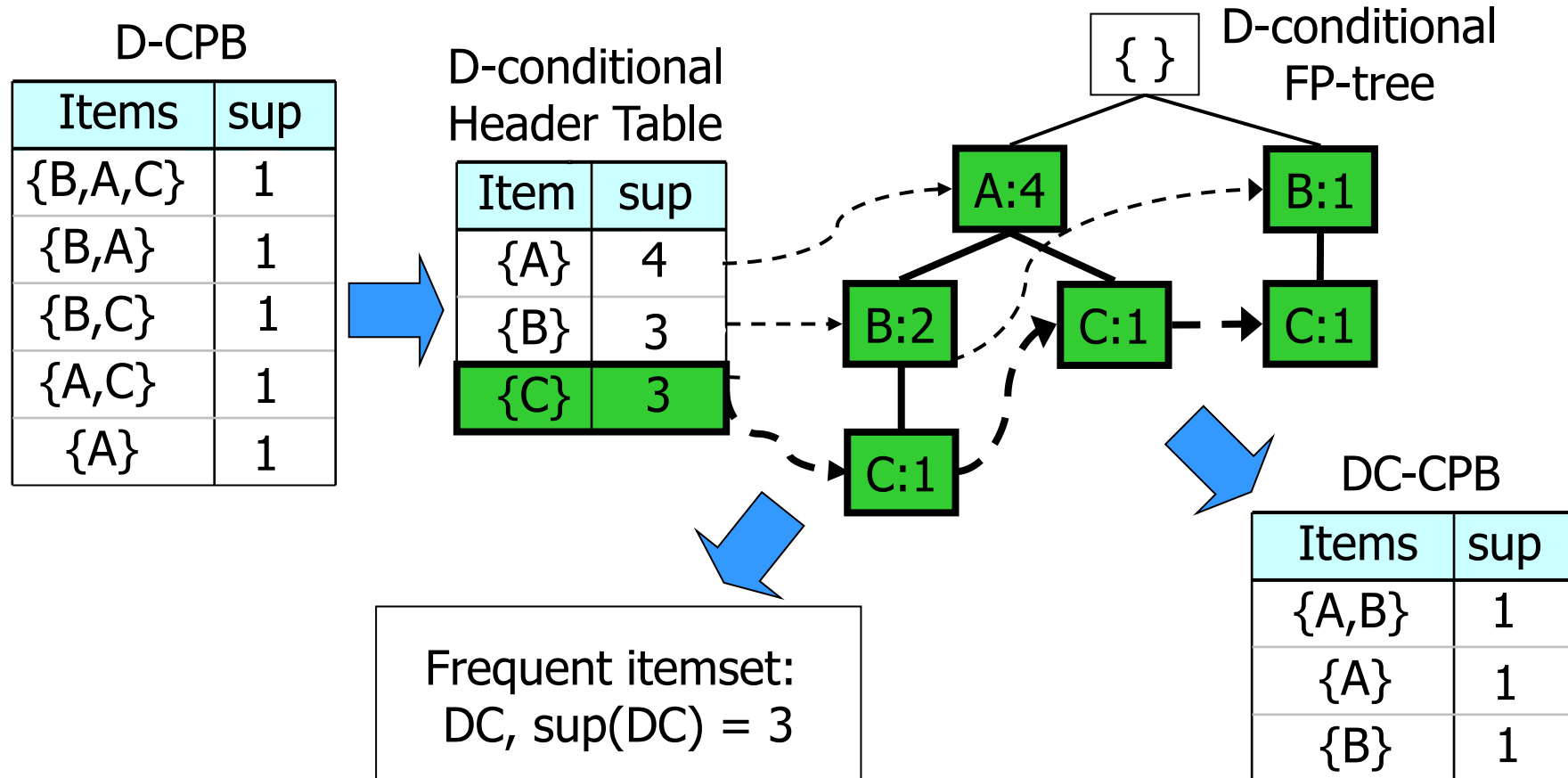
- (2) Recursive invocation of FP-growth on D-CPB



# Conditional Pattern Base of DC

## ■ (1) Build DC-CPB

- Select prefix-paths of item C from D-conditional FP-tree

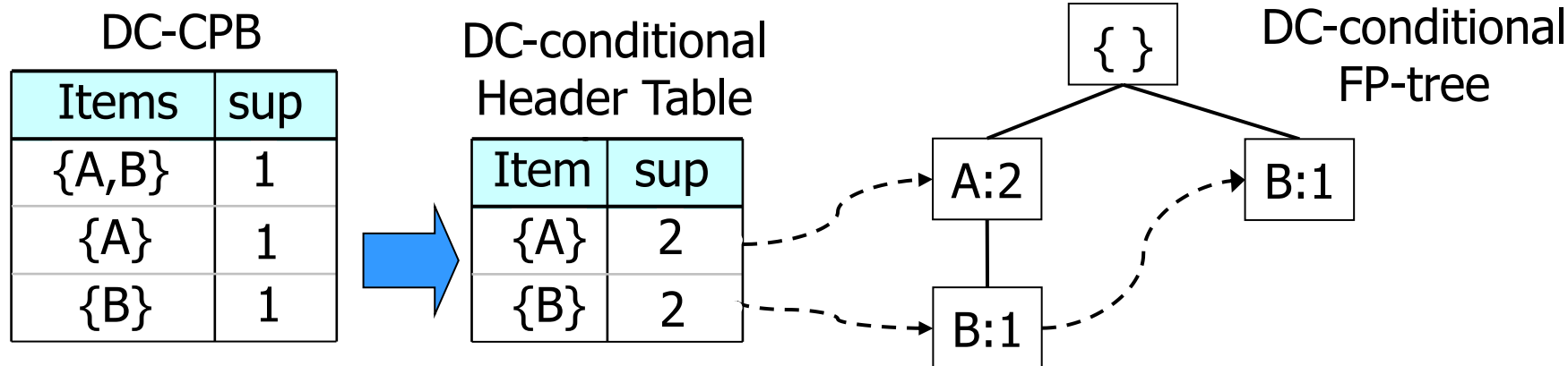






# Conditional Pattern Base of DC

- (1) Build DC-CPB
  - Select prefix-paths of item C from D-conditional FP-tree



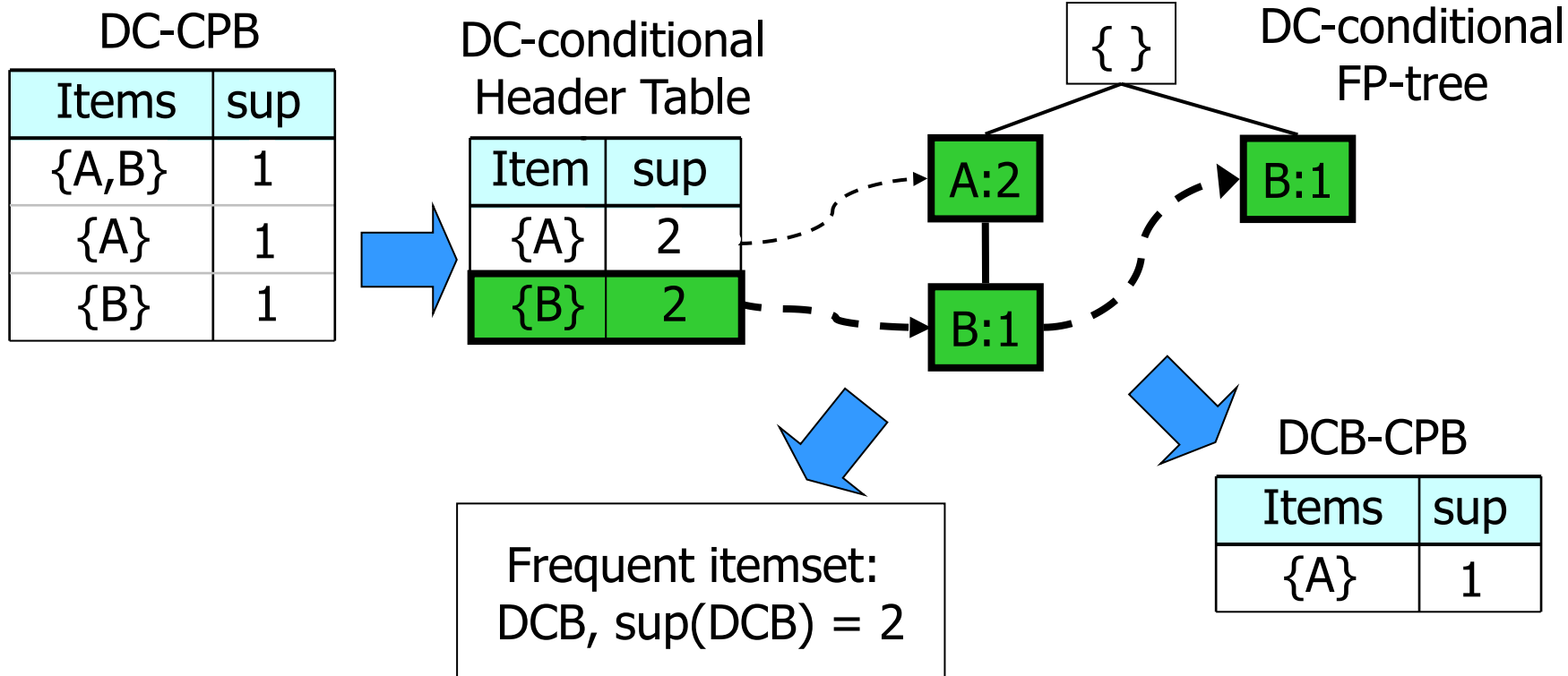
- (2) Recursive invocation of FP-growth on DC-CPB



# Conditional Pattern Base of DCB

## ■ (1) Build DCB-CPB

- Select prefix-paths of item B from DC-conditional FP-tree



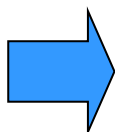


# Conditional Pattern Base of DCB

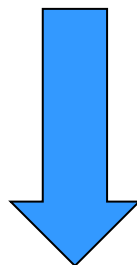
- (1) Build DCB-CPB
  - Select prefix-paths of item B from DC-conditional FP-tree

DCB-CPB

Items	sup
{A}	1



- Item A is infrequent in DCB-CPB
  - A is removed from DCB-CPB
  - DCB-CPB is empty



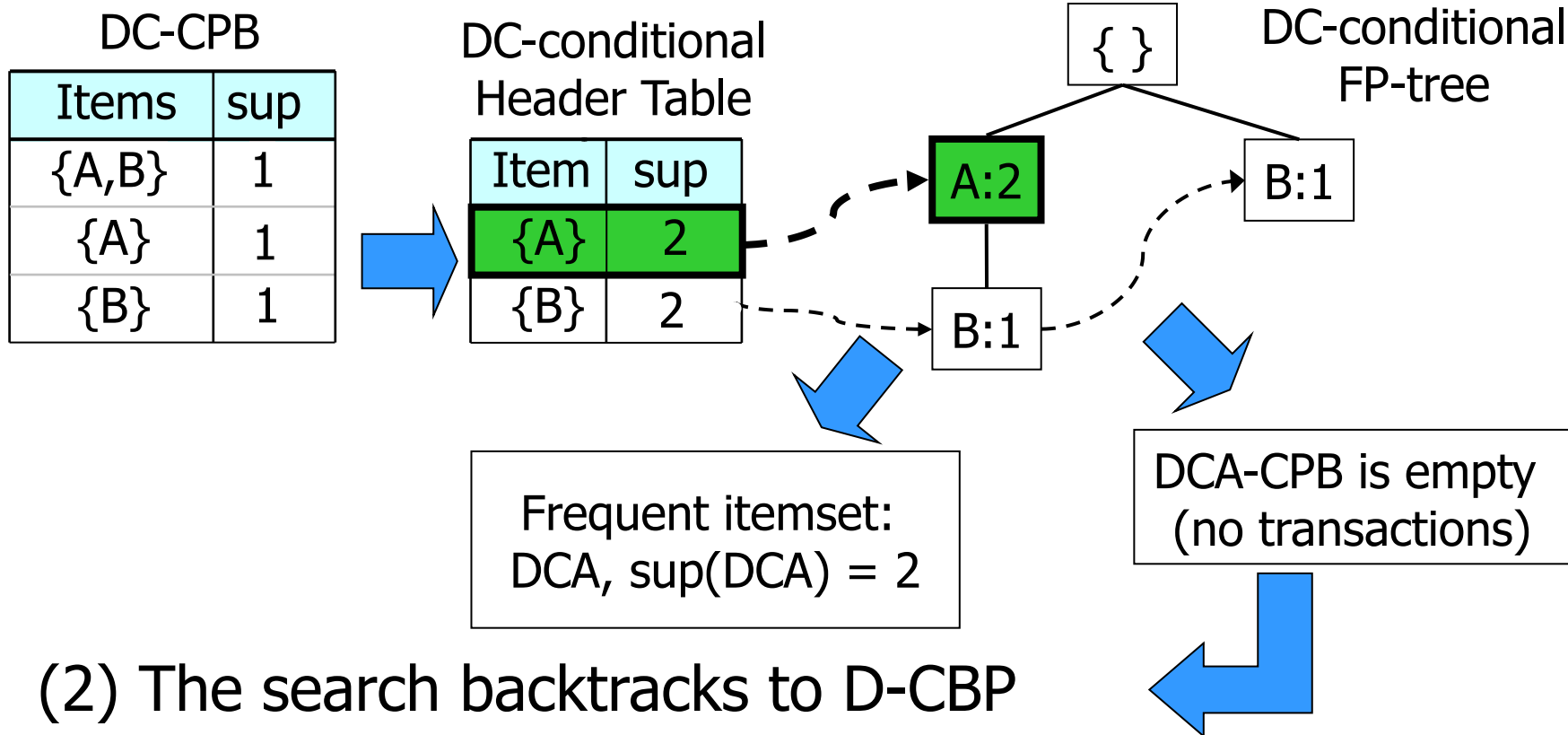
- (2) The search backtracks to DC-CPB



# Conditional Pattern Base of DCA

## ■ (1) Build DCA-CPB

- Select prefix-paths of item A from DC-conditional FP-tree



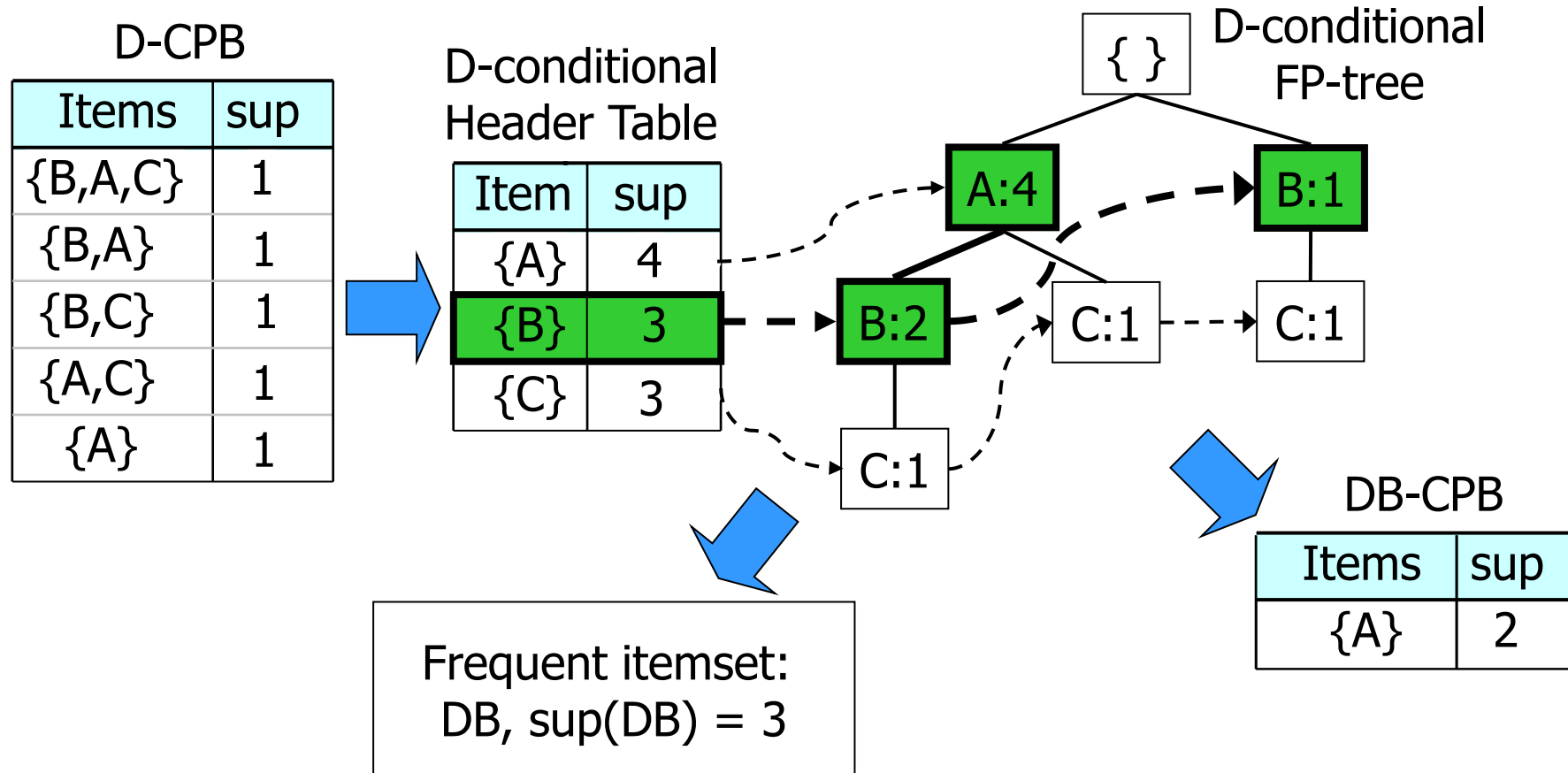
## ■ (2) The search backtracks to D-CBP



# Conditional Pattern Base of DB

## ■ (1) Build DB-CPB

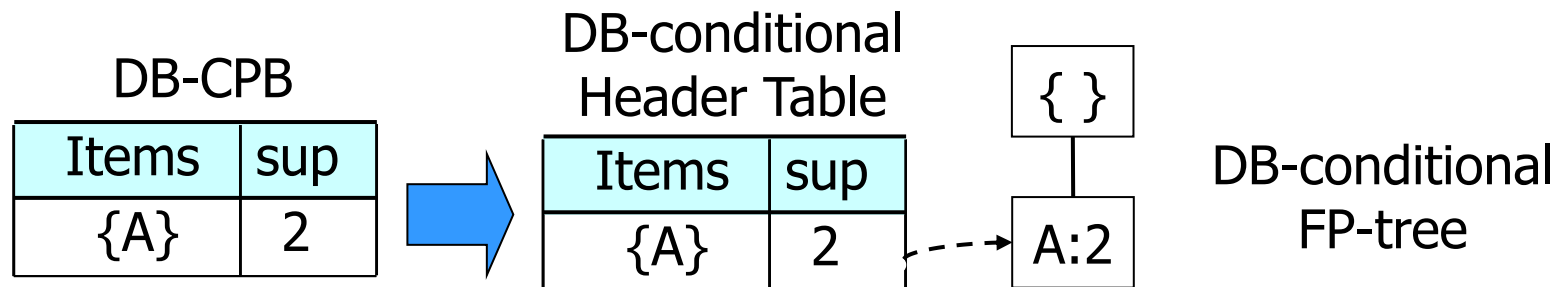
- Select prefix-paths of item B from D-conditional FP-tree





# Conditional Pattern Base of DB

- (1) Build DB-CPB
  - Select prefix-paths of item B from D-conditional FP-tree



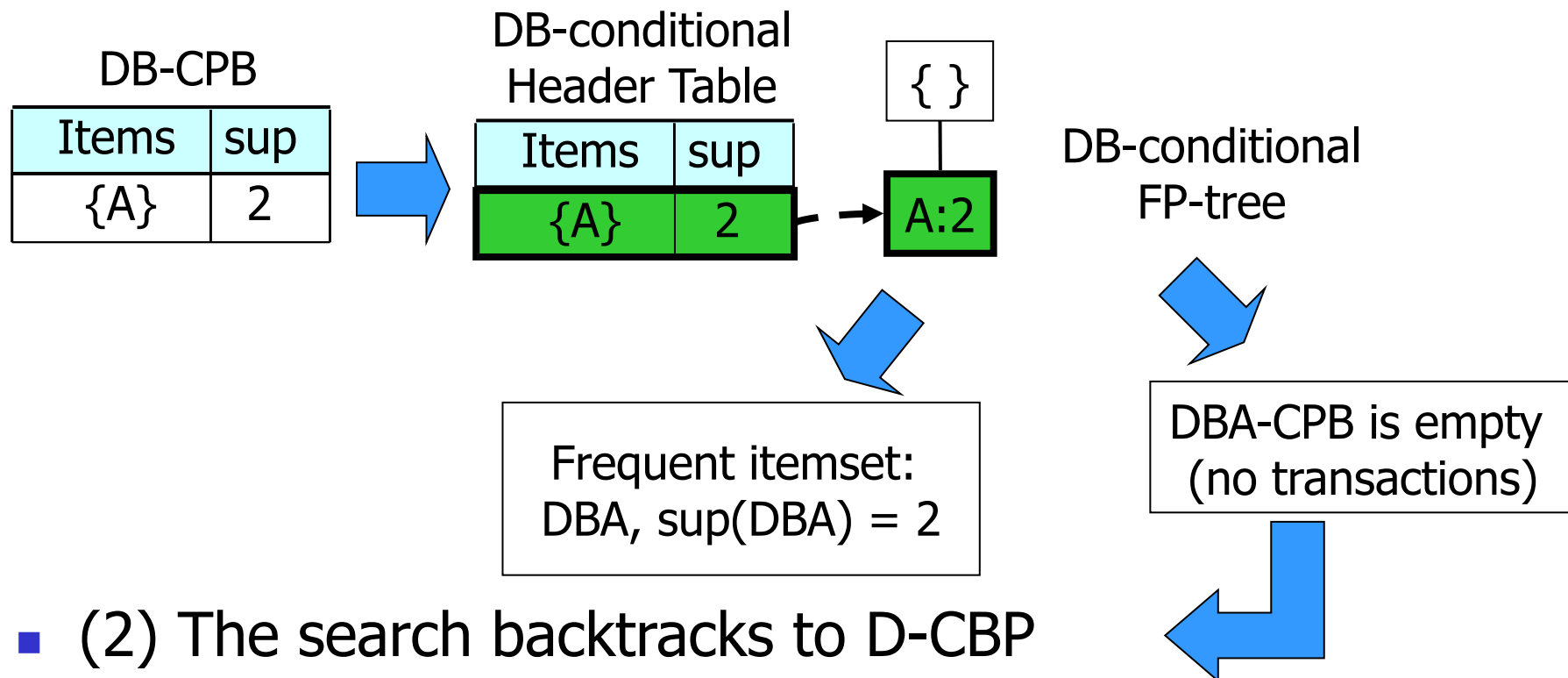
- (2) Recursive invocation of FP-growth on DB-CPB



# Conditional Pattern Base of DBA

- (1) Build DBA-CPB

- Select prefix-paths of item A from DB-conditional FP-tree

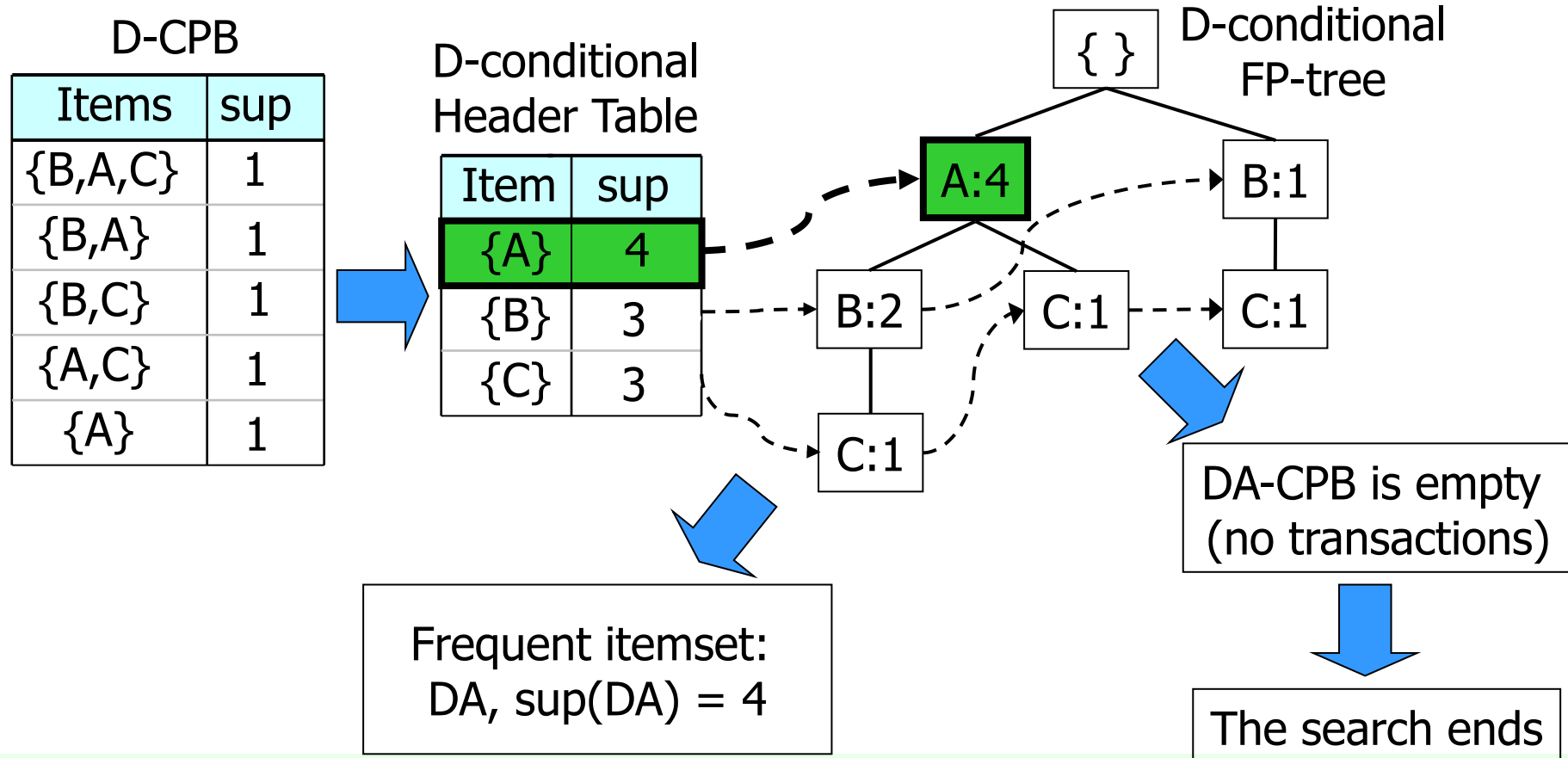




# Conditional Pattern Base of DA

## ■ (1) Build DA-CPB

- Select prefix-paths of item A from D-conditional FP-tree





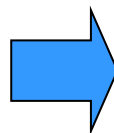


# Frequent itemsets with prefix D

- Frequent itemsets including D and supported combinations of items B,A,C

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}



itemsets	sup
{D}	5
{A,D}	4
{B,D}	3
{C,D}	3
{A,B,D}	2
{A,C,D}	2
{B,C,D}	2

$\text{minsup} > 1$



# Other approaches

- Many other approaches to frequent itemset extraction
- May exploit a different database representation
  - represent the tidset of each item [Zak00]

Horizontal  
Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				



# Compact Representations

- Some itemsets are redundant because they have identical support as their supersets

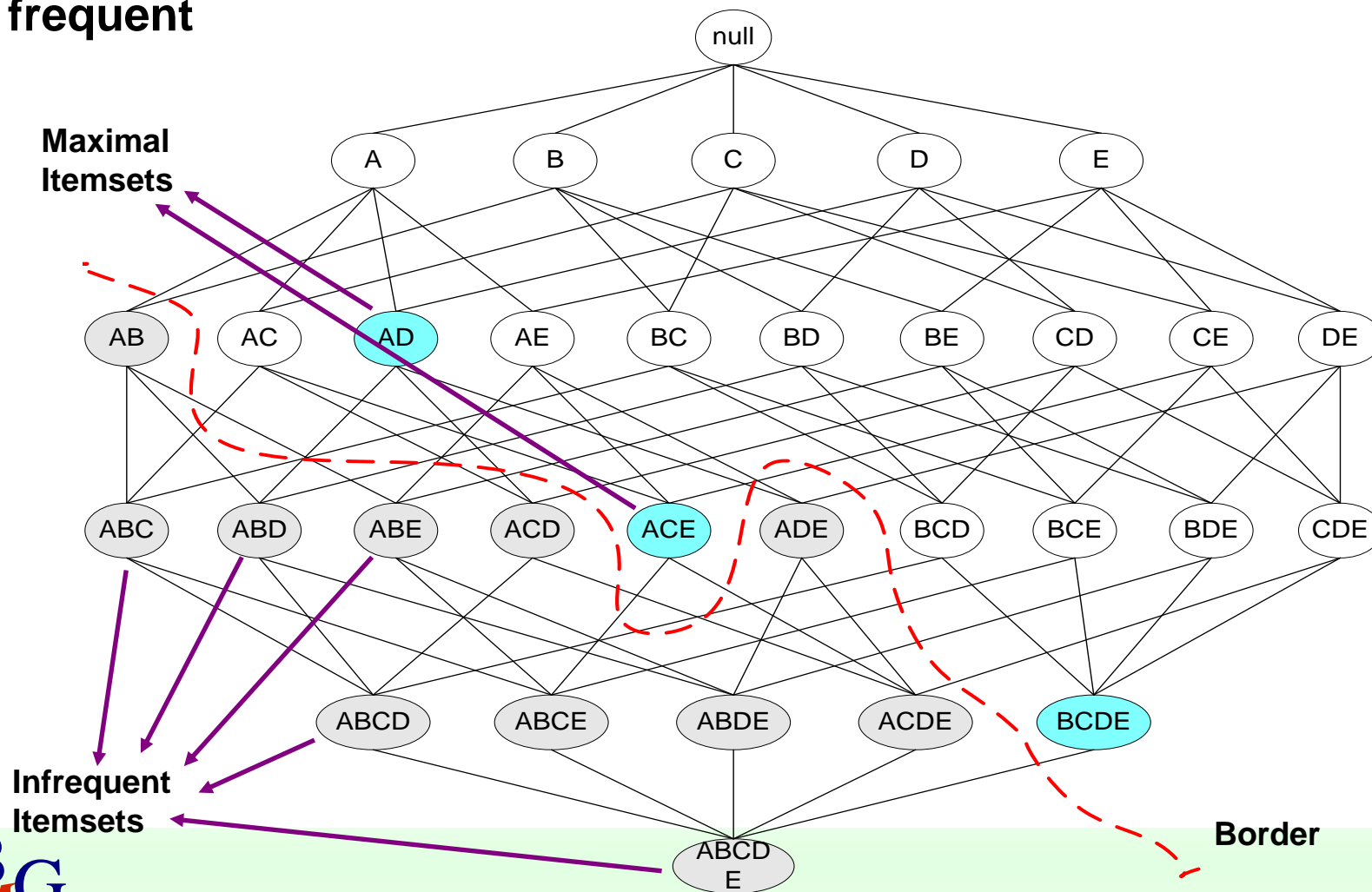
TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1

- Number of frequent itemsets  $= 3 \times \sum_{k=1}^{10} \binom{10}{k}$
- A compact representation is needed



# Maximal Frequent Itemset

An itemset is frequent maximal if none of its immediate supersets is frequent





# Closed Itemset

- An itemset is closed if none of its immediate supersets has the same support as the itemset

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

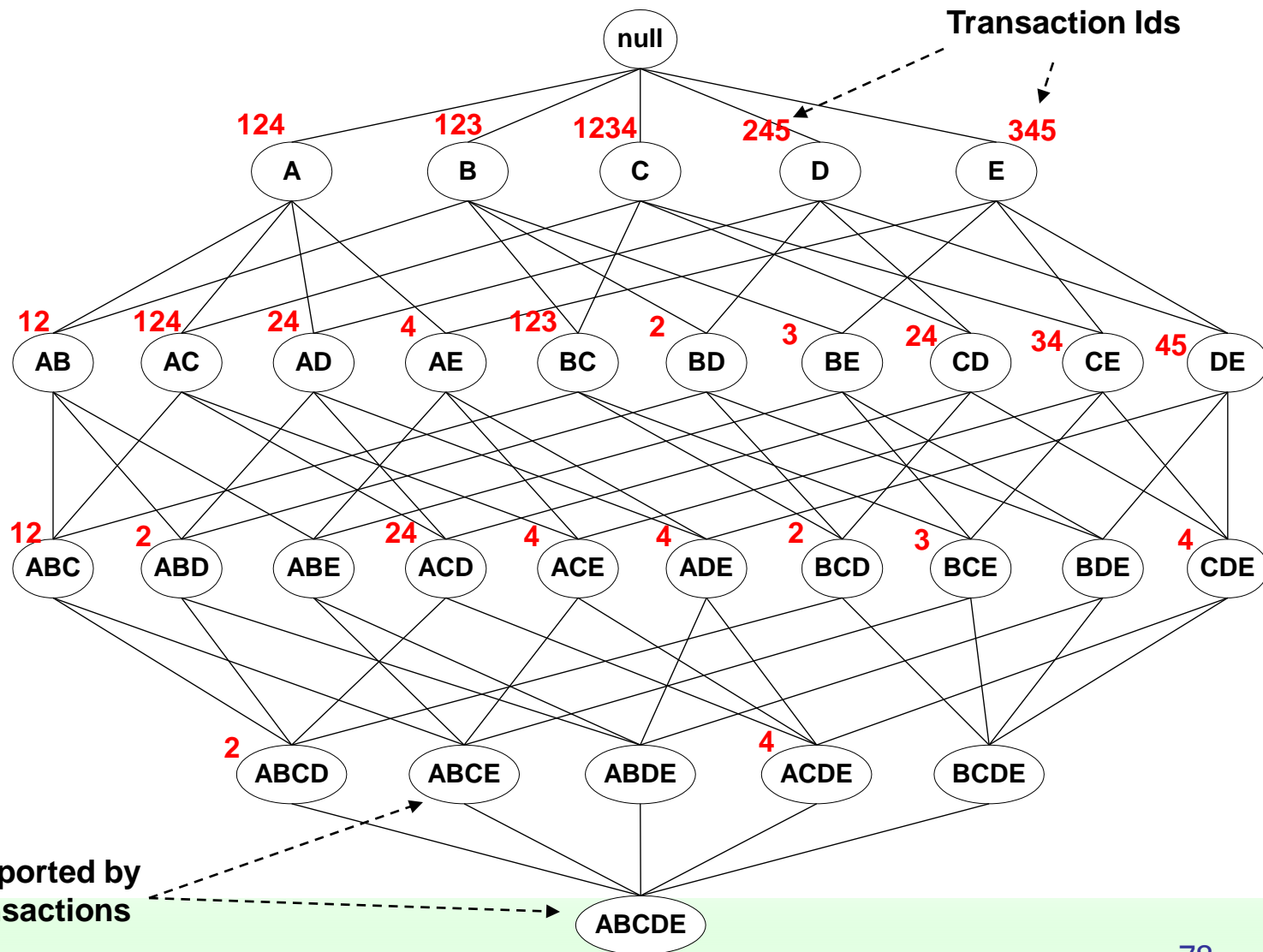
itemset	sup
{A}	4
<b>{B}</b>	<b>5</b>
{C}	3
{D}	4
<b>{A,B}</b>	<b>4</b>
{A,C}	2
{A,D}	3
{B,C}	3
<b>{B,D}</b>	<b>4</b>
{C,D}	3

itemset	sup
{A,B,C}	2
<b>{A,B,D}</b>	<b>3</b>
{A,C,D}	2
<b>{B,C,D}</b>	<b>3</b>
<b>{A,B,C,D}</b>	<b>2</b>



# Maximal vs Closed Itemsets

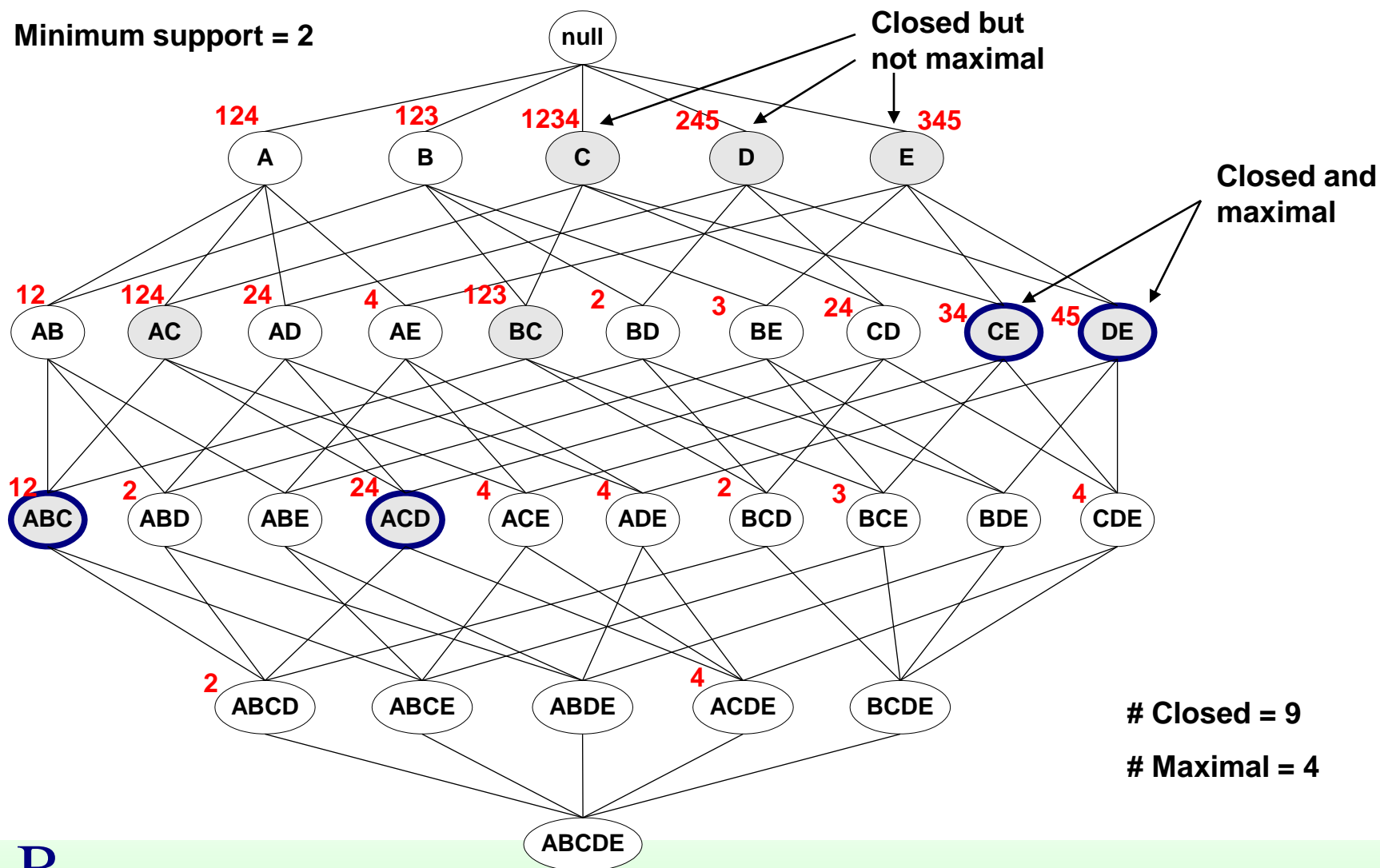
TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE





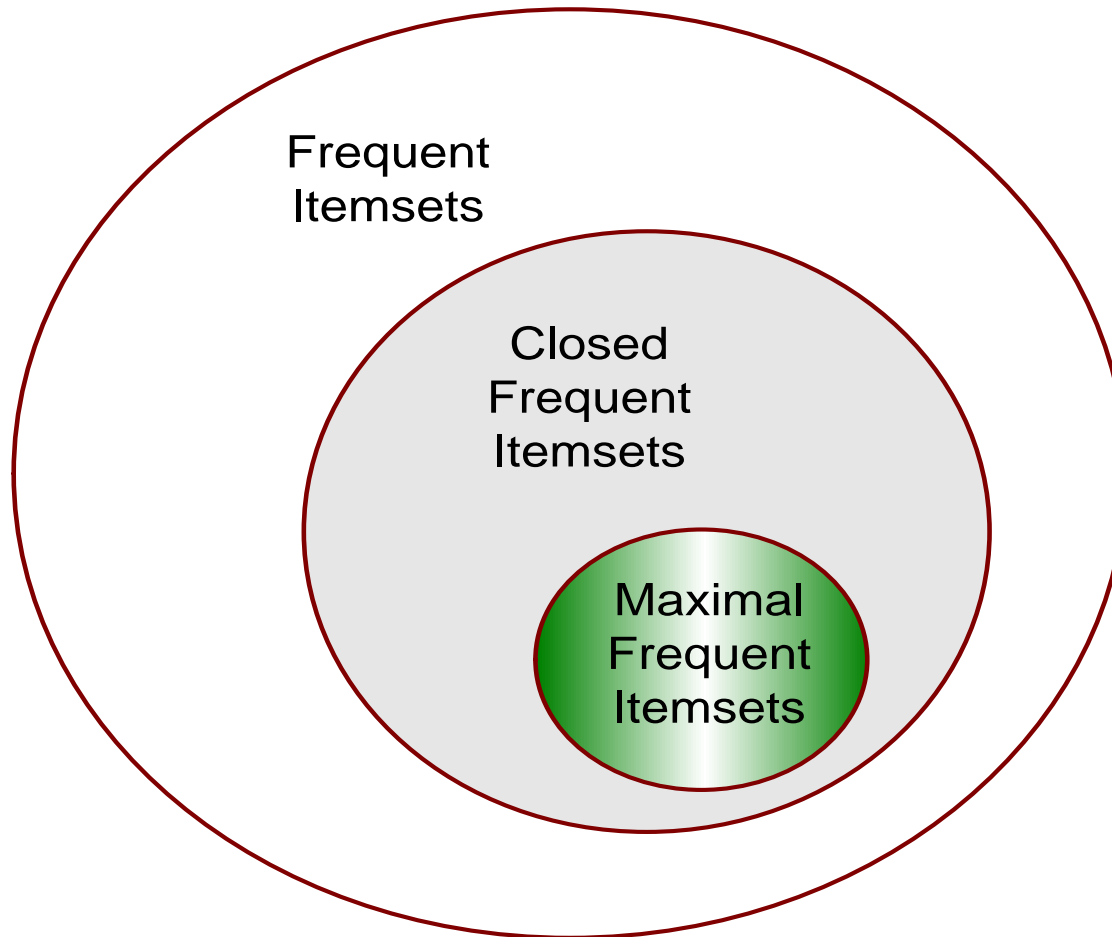
# Maximal vs Closed Frequent Itemsets

Minimum support = 2





# Maximal vs Closed Itemsets



From: Tan, Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006





# Effect of Support Threshold

- Selection of the appropriate *minsup* threshold is not obvious
  - If *minsup* is too high
    - itemsets including rare but interesting items may be lost
      - example: pieces of jewellery (or other expensive products)
  - If *minsup* is too low
    - it may become computationally *very expensive*
    - the number of frequent itemsets becomes *very large*



# Interestingness Measures

- A large number of pattern may be extracted
  - rank patterns by their interestingness
- Objective measures
  - rank patterns based on statistics computed from data
  - initial framework [Agr94] only considered support and confidence
    - other statistical measures available
- Subjective measures
  - rank patterns according to user interpretation [Silb98]
    - interesting if it contradicts the expectation of a user
    - interesting if it is actionable



# Confidence measure: always reliable?

- 5000 high school students are given
  - 3750 eat cereals
  - 3000 play basket
  - 2000 eat cereals and play basket

- Rule

play basket  $\Rightarrow$  eat cereals

sup = 40%, conf = 66,7%

is misleading because eat cereals has sup 75% ( $>66,7\%$ )

- Problem caused by high frequency of rule head

- negative correlation

	basket	not basket	total
cereals	2000	1750	3750
not cereals	1000	250	1250
total	3000	2000	5000



# Correlation or lift

$r: A \Rightarrow B$

$$\text{Correlation} = \frac{P(A, B)}{P(A)P(B)} = \frac{\text{conf}(r)}{\text{sup}(B)}$$

- Statistical independence
  - Correlation = 1
- Positive correlation
  - Correlation > 1
- Negative correlation
  - Correlation < 1



# Example

- Association rule

play basket  $\Rightarrow$  eat cereals

has corr = 0.89

- negative correlation

- but rule

play basket  $\Rightarrow$  not (eat cereals)

has corr = 1,34



#	Measure	Formula
1	$\phi$ -coefficient	$\frac{P(A,B) - P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's ( $\lambda$ )	$\frac{\sum_j \max_k P(A_j, B_k) + \sum_k \max_j P(A_j, B_k) - \max_j P(A_j) - \max_k P(B_k)}{2 - \max_j P(A_j) - \max_k P(B_k)}$
3	Odds ratio ( $\alpha$ )	$\frac{P(A,B)P(\bar{A},\bar{B})}{P(A,\bar{B})P(\bar{A},B)}$
4	Yule's $Q$	$\frac{P(A,B)P(\bar{A}\bar{B}) - P(A,\bar{B})P(\bar{A},B)}{P(A,B)P(\bar{A}\bar{B}) + P(A,\bar{B})P(\bar{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's $Y$	$\frac{\sqrt{P(A,B)P(\bar{A}\bar{B})} - \sqrt{P(A,\bar{B})P(\bar{A},B)}}{\sqrt{P(A,B)P(\bar{A}\bar{B})} + \sqrt{P(A,\bar{B})P(\bar{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa ( $\kappa$ )	$\frac{P(A,B) + P(\bar{A},\bar{B}) - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}$
7	Mutual Information ( $M$ )	$\frac{\sum_i \sum_j P(A_i, B_j) \log \frac{P(A_i, B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i) \log P(A_i), -\sum_j P(B_j) \log P(B_j))}$
8	J-Measure ( $J$ )	$\max \left( P(A, B) \log \left( \frac{P(B A)}{P(B)} \right) + P(\bar{A}\bar{B}) \log \left( \frac{P(\bar{B} \bar{A})}{P(\bar{B})} \right), \right. \\ \left. P(A, B) \log \left( \frac{P(A B)}{P(A)} \right) + P(\bar{A}\bar{B}) \log \left( \frac{P(\bar{A} \bar{B})}{P(\bar{A})} \right) \right)$
9	Gini index ( $G$ )	$\max \left( P(A)[P(B A)^2 + P(\bar{B} A)^2] + P(\bar{A})[P(B \bar{A})^2 + P(\bar{B} \bar{A})^2] \right. \\ \left. - P(B)^2 - P(\bar{B})^2, \right. \\ \left. P(B)[P(A B)^2 + P(\bar{A} B)^2] + P(\bar{B})[P(A \bar{B})^2 + P(\bar{A} \bar{B})^2] \right. \\ \left. - P(A)^2 - P(\bar{A})^2 \right)$
10	Support ( $s$ )	$P(A, B)$
11	Confidence ( $c$ )	$\max(P(B A), P(A B))$
12	Laplace ( $L$ )	$\max \left( \frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2} \right)$
13	Conviction ( $V$ )	$\max \left( \frac{P(A)P(\bar{B})}{P(\bar{A}B)}, \frac{P(B)P(\bar{A})}{P(\bar{B}A)} \right)$
14	Interest ( $I$ )	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine ( $IS$ )	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's ( $PS$ )	$P(A, B) - P(A)P(B)$
17	Certainty factor ( $F$ )	$\max \left( \frac{P(B A) - P(B)}{1 - P(B)}, \frac{P(A B) - P(A)}{1 - P(A)} \right)$
18	Added Value ( $AV$ )	$\max(P(B A) - P(B), P(A B) - P(A))$
19	Collective strength ( $S$ )	$\frac{P(A,B) + P(\bar{A}\bar{B})}{P(A)P(B) + P(\bar{A})P(\bar{B})} \times \frac{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A,B) - P(\bar{A}\bar{B})}$
20	Jaccard ( $\zeta$ )	$\frac{P(A,B)}{P(A) + P(B) - P(A,B)}$
21	Klosgen ( $K$ )	$\sqrt{P(A, B)} \max(P(B A) - P(B), P(A B) - P(A))$



# Considering weight

- Items may be characterized by different importance within a transaction
  - Example: product quantity or price in transactions
- Transactions may be weighted
  - Example: discount on entire market basket
- Weighted dataset
  - Each item is assigned a weight measuring its relevance in the corresponding transaction



# Weighted association rules

- Consider item/transaction weights during association rule extraction
- Extend rule quality measures
  - E.g., weighted support, weighted confidence
- Apply ad-hoc weight aggregation functions
  - E.g., min, max, avg





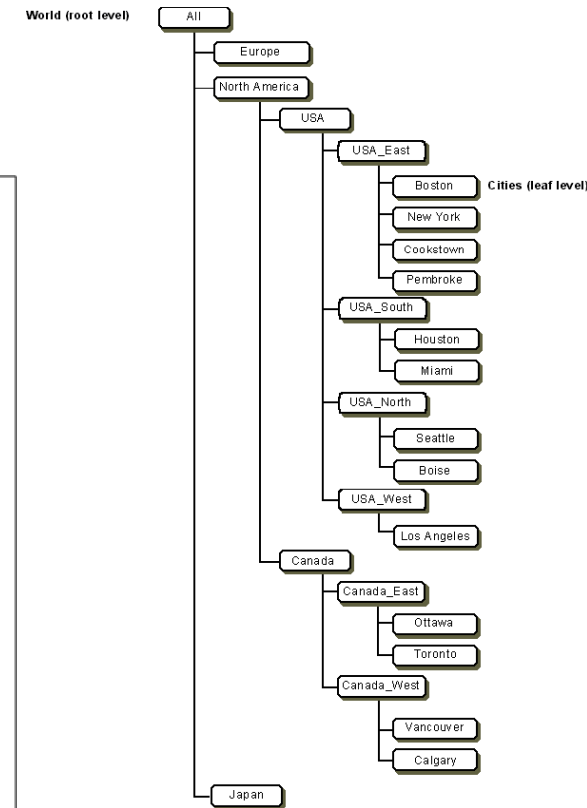
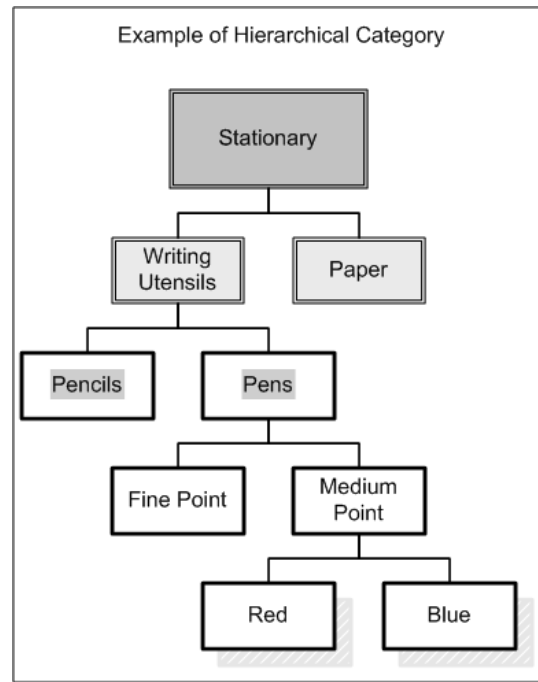
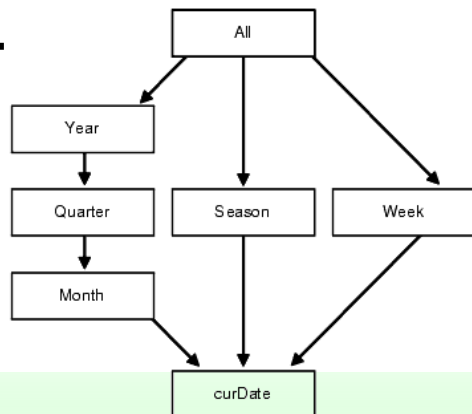
# Considering hierarchies

## ■ Generalization hierarchies

- Aggregation over attributes in a dataset
- Typically user provided

## ■ Examples

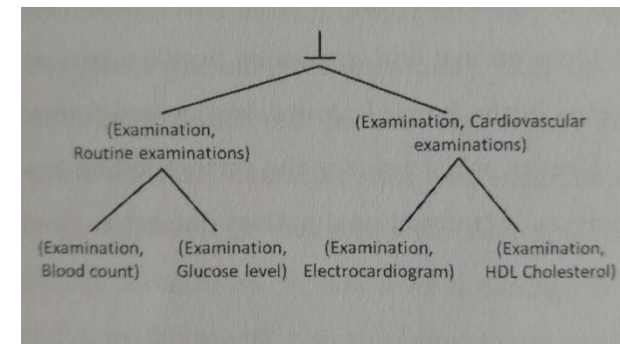
- Time hierarchy
- Product category
- Location hierarchy
- ...





# Taxonomy

- A taxonomy is a set of is-a hierarchies that aggregate data items into higher-level concepts
- Data item
  - Instance in the (transactional) dataset
  - Represents detailed concepts
- Generalized item
  - Aggregation in higher-level concepts
  - Represents abstractions on instances





# Generalized itemsets

- Sets of items at different generalization levels
  - May contain data items together with generalized items defined in the taxonomy
  - Summarize knowledge represented by a set of lower-level descendants
    - Both frequent and infrequent
- A generalized itemset covers a transaction when all
  - its generalized items are ancestors of items included in the transaction
  - its data items are included in the transaction
- Generalized itemset support
  - ratio between number of covered transactions and dataset cardinality



# Context-aware data analysis

- Context data provided by different, possibly heterogeneous, sources
  - Mobile devices provide information on
    - the user context (e.g., GPS coordinates)
    - the supplied services
      - temporal information
      - service description
      - duration
  - Additional information available
    - demographics of the user requesting the service



# Generalized itemset example

***user: John, time: 6.05 p.m., service: Weather***  
**(s = 0.005%)**

- A very low support
  - The itemset may be discarded
- By generalizing
  - the time attribute on a time period
  - the user on a user category

***user: employee, time: 6 p.m. to 7 p.m., service: Weather***  
**(s = 0.2%)**

- May discover interesting properties generalizing *infrequent* items



# Generalized association rules

- Extension of “classical” association rules

$$X \rightarrow Y$$

- X and Y are either generalized or not generalized itemsets
  - Extract associations among data items at multiple abstraction levels
  - Support, confidence and lift are defined accordingly



# Patient data analysis

- Analysis of multiple level correlations on patient treatment historical data
  - Dataset collected by an Italian Local Health Center
    - Diabetes complications at various severity levels
    - 95K records, 3.5K patients
  - Features
    - Prescribed examinations (26 examinations, 7 categories)
    - Prescribed drugs (200 drugs, 14 categories)
    - Census patient data (gender, age discretized in age groups)
- Sparse dataset
  - Difficult setting of support threshold
    - Low: generates too many rules
    - High: interesting information at lower levels of abstraction may remain hidden



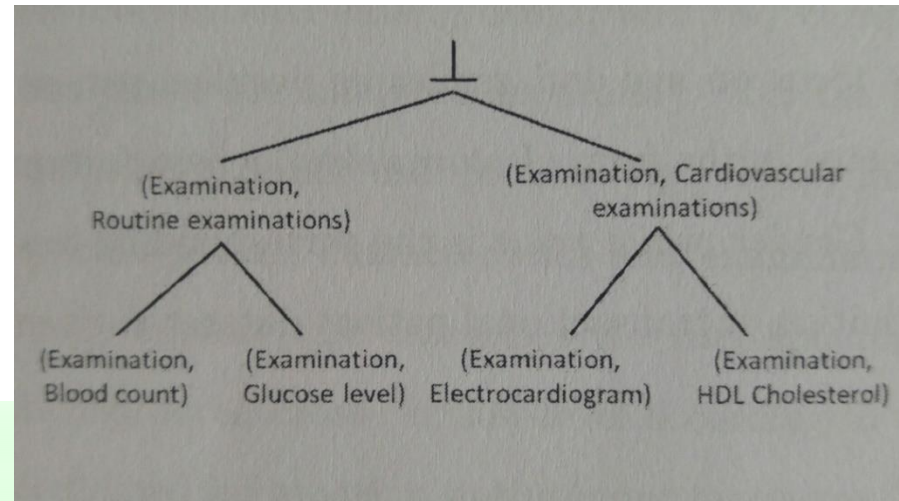
# High level rules

- Only generalized itemsets
  - Represent general knowledge
    - May be too high level to perform targeted analyses

- Example

(Examination, Liver) -> (Examination, Kidney)

- Frequently prescribed together
- May be used for examination scheduling







# Cross level rules

- Different abstraction levels (generalized items and data items)
  - Combine detailed and general information
- Example
  - (Examination, Liver) -> (Examination, Uric acid)
  - Insight into specific kidney examinations correlated with liver examinations
    - Confidence: 74.8%



# Low-level rules

- Only not generalized itemsets (only data items)
  - Very detailed knowledge
    - Covered by high and cross-level rules
  - Large rule set
    - Challenging exploration task
  - Drill down exploration based on formerly extracted high and cross-level rules