

Visual perception

Data Management and Visualization



SoftEng
<http://softeng.polito.it>

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VISUAL INTEGRITY

- **Proportionality**

- ◆ Representation as physical quantities should be proportional to the represented numbers

- **Utility**

- ◆ Graphical element should convey useful information

- **Clarity**

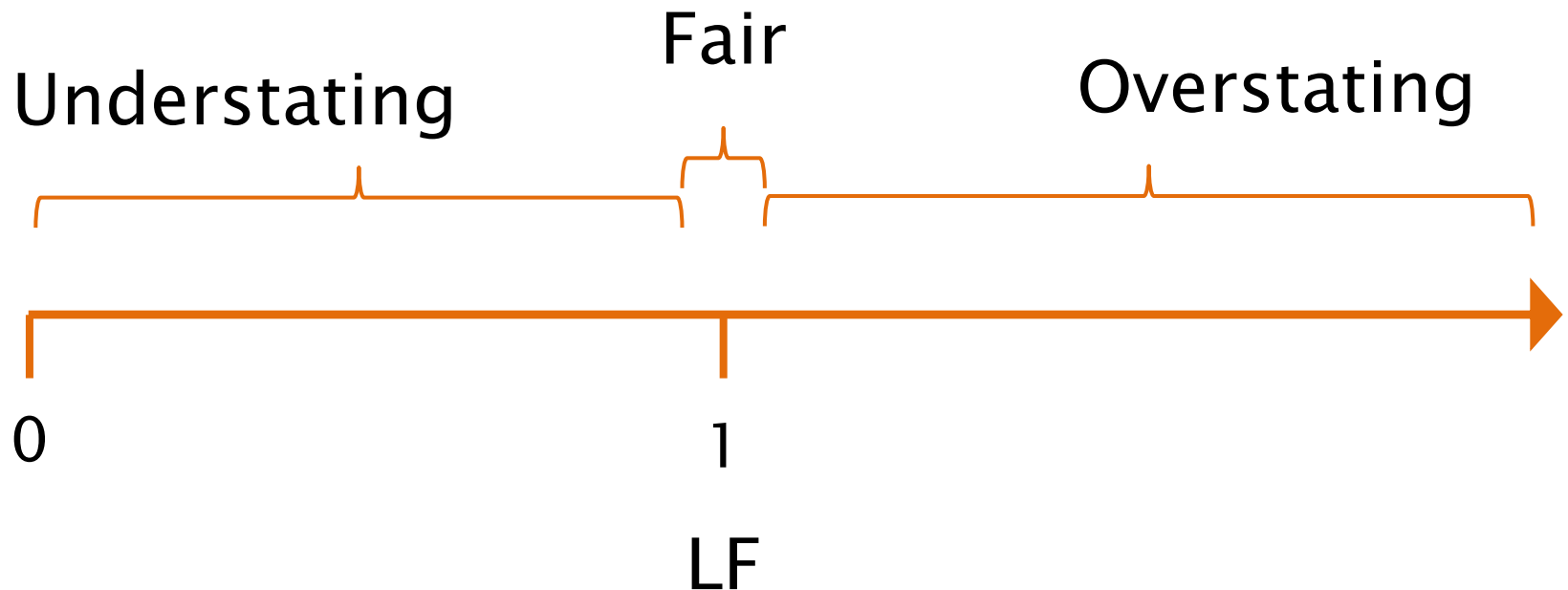
- ◆ Labeling should counter graphical distortion and ambiguity

- The magnitude of visual attributes should represent faithfully the magnitude of measures
- They should allow
 - ◆ Discrimination: are they different?
 - ◆ Comparison: which is larger?
 - ◆ Magnitude Assessment: how much larger?

$$LF = \frac{\text{size of effect shown in graphic}}{\text{size of effect in data}}$$

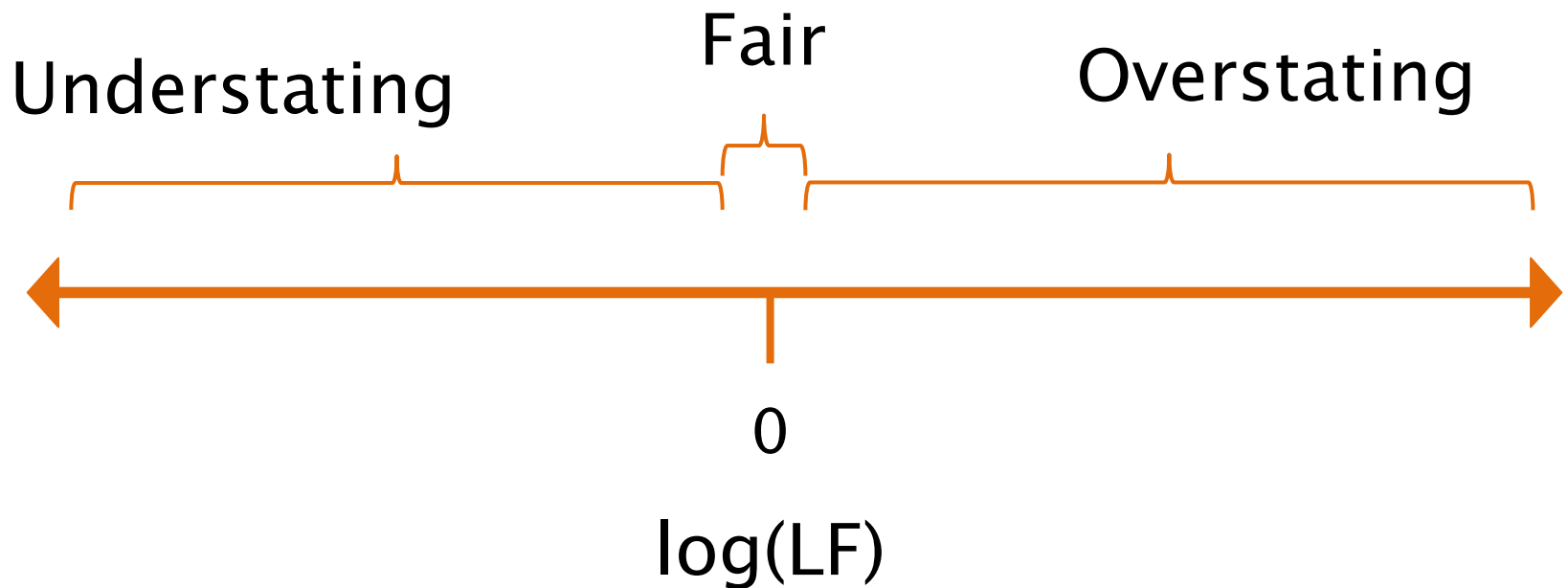
- Overstating
 - ◆ $LF > 1 \Leftrightarrow \text{Log}(LF) > 0$
- Understating
 - ◆ $LF < 1 \Leftrightarrow \text{Log}(LF) < 0$
- Fair
 - $LF = 1 \Leftrightarrow \text{Log}(LF) = 0$

$$LF = \frac{\text{size of effect shown in graphic}}{\text{size of effect in data}}$$

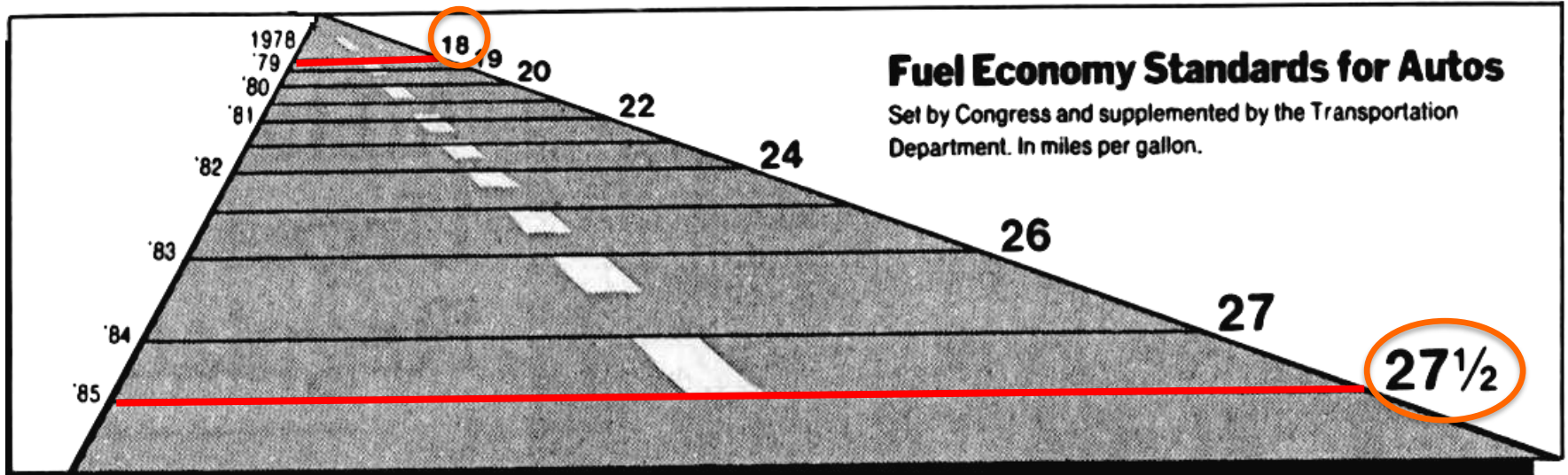


Lie Factor

$$LF = \frac{\text{size of effect shown in graphic}}{\text{size of effect in data}}$$



Lie Factor



$$\frac{18.7}{2.2} = 8.5 \text{ on graphic}$$

$$\frac{27.5}{18} = 1.52 \text{ in data}$$

$$LF = 8.5 / 1.52 = 5.59$$

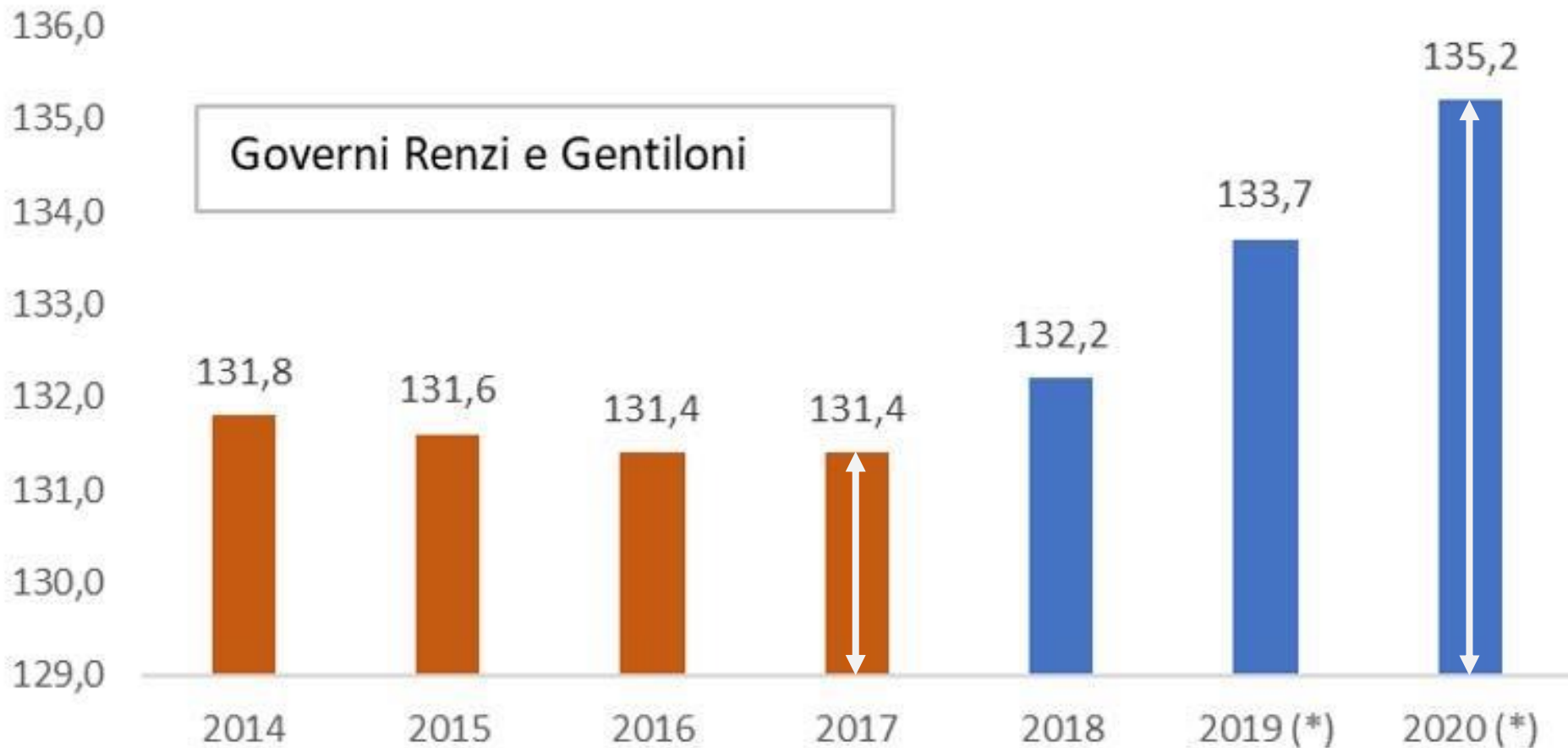
Example

Debito pubblico (% PIL)

(*) previsioni Commissione UE

Governo Conte

Governi Renzi e Gentiloni

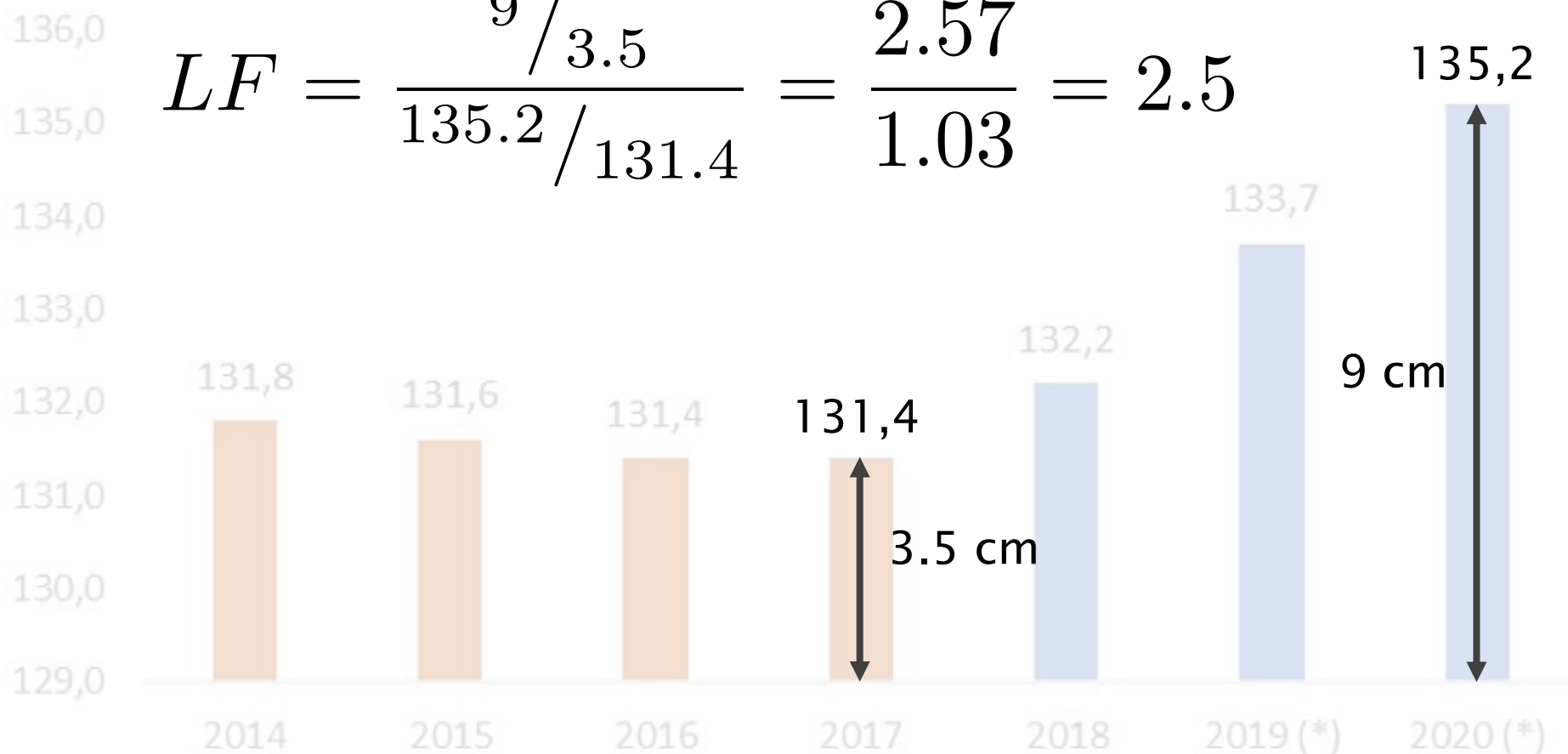


Example – Lie Factor

Debito pubblico (% PIL)

(*) previsioni Commissione UE

$$LF = \frac{9 / 3.5}{135.2 / 131.4} = \frac{2.57}{1.03} = 2.5$$

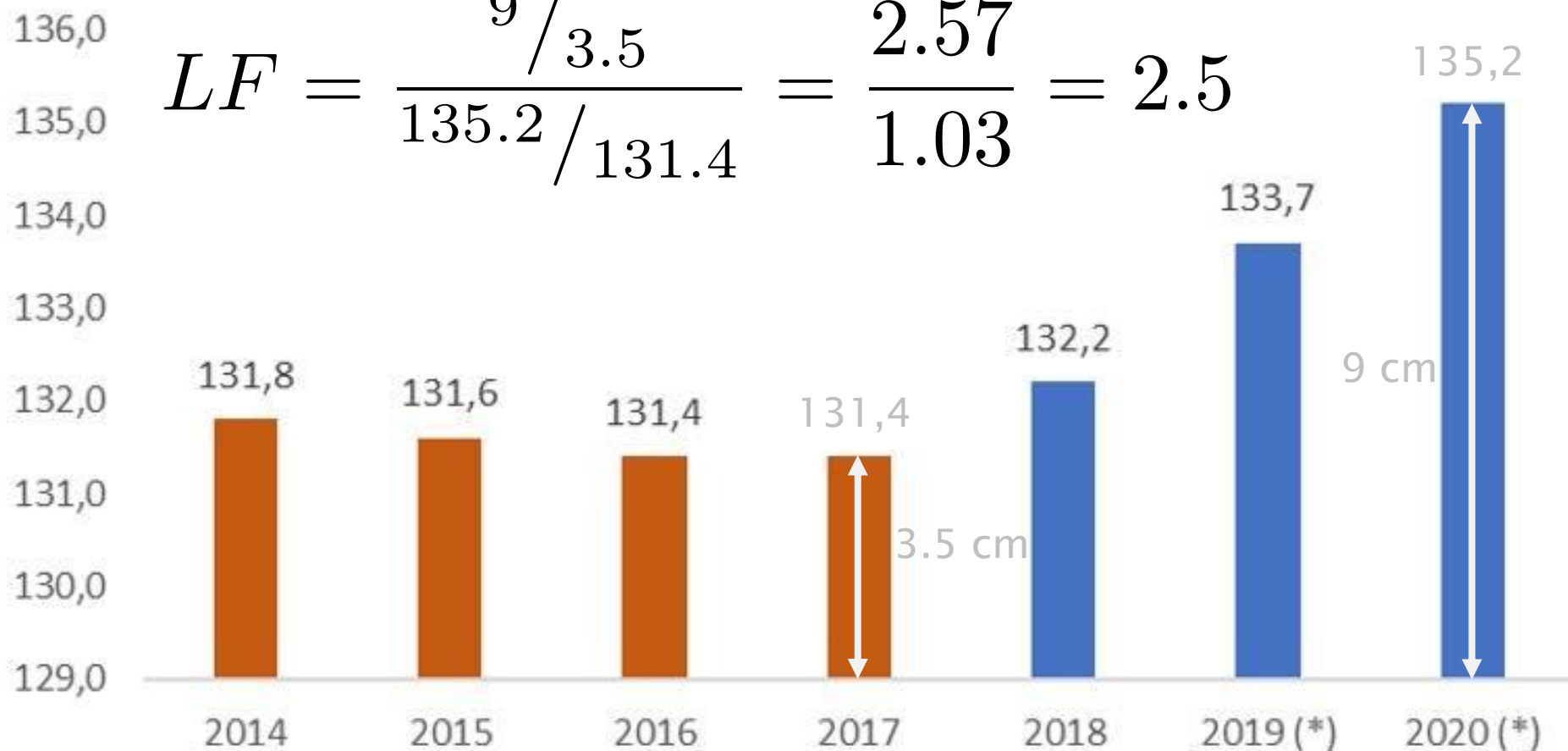


Example – Lie Factor

Debito pubblico (% PIL)

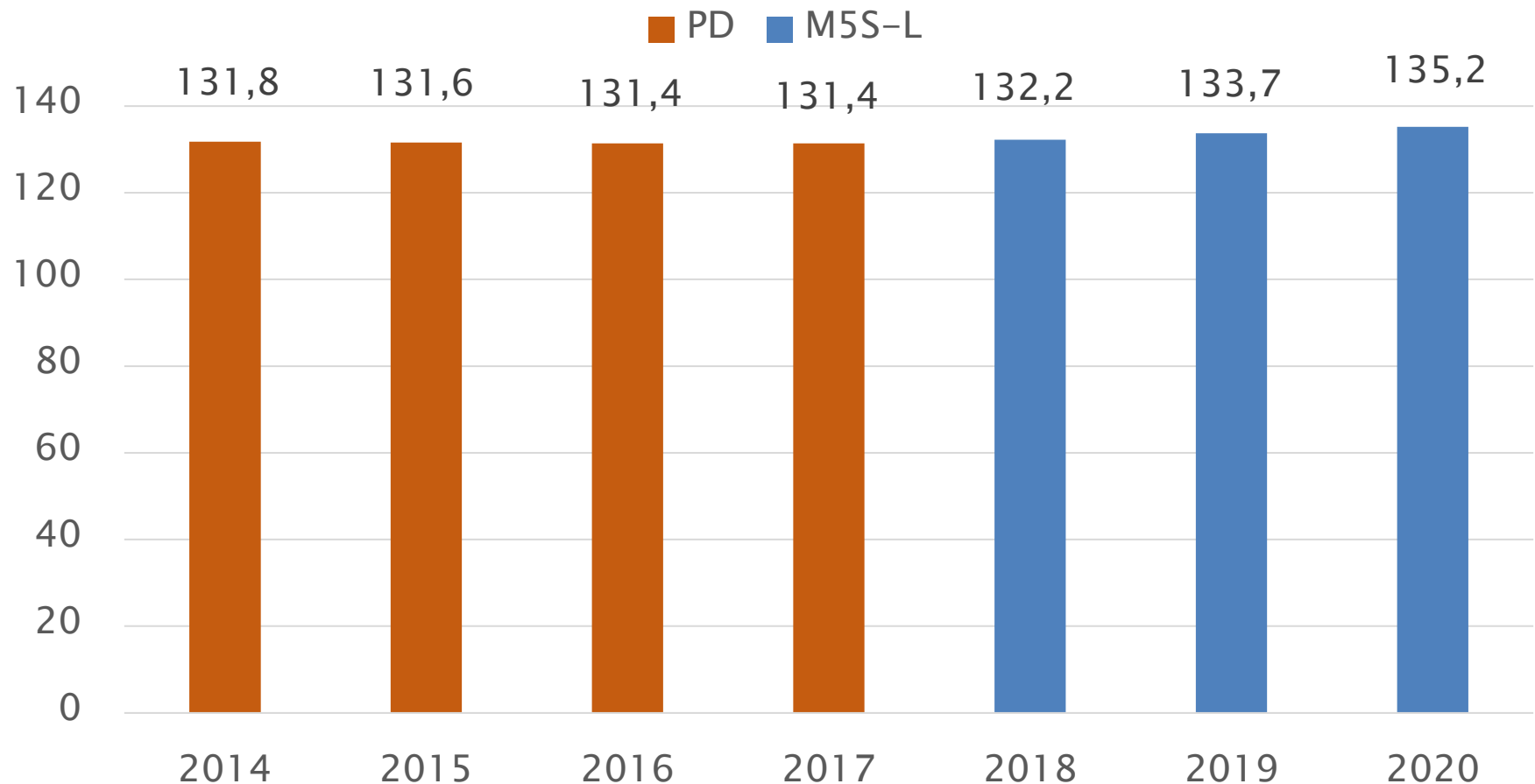
(*) previsioni Commissione UE

$$LF = \frac{9 / 3.5}{135.2 / 131.4} = \frac{2.57}{1.03} = 2.5$$



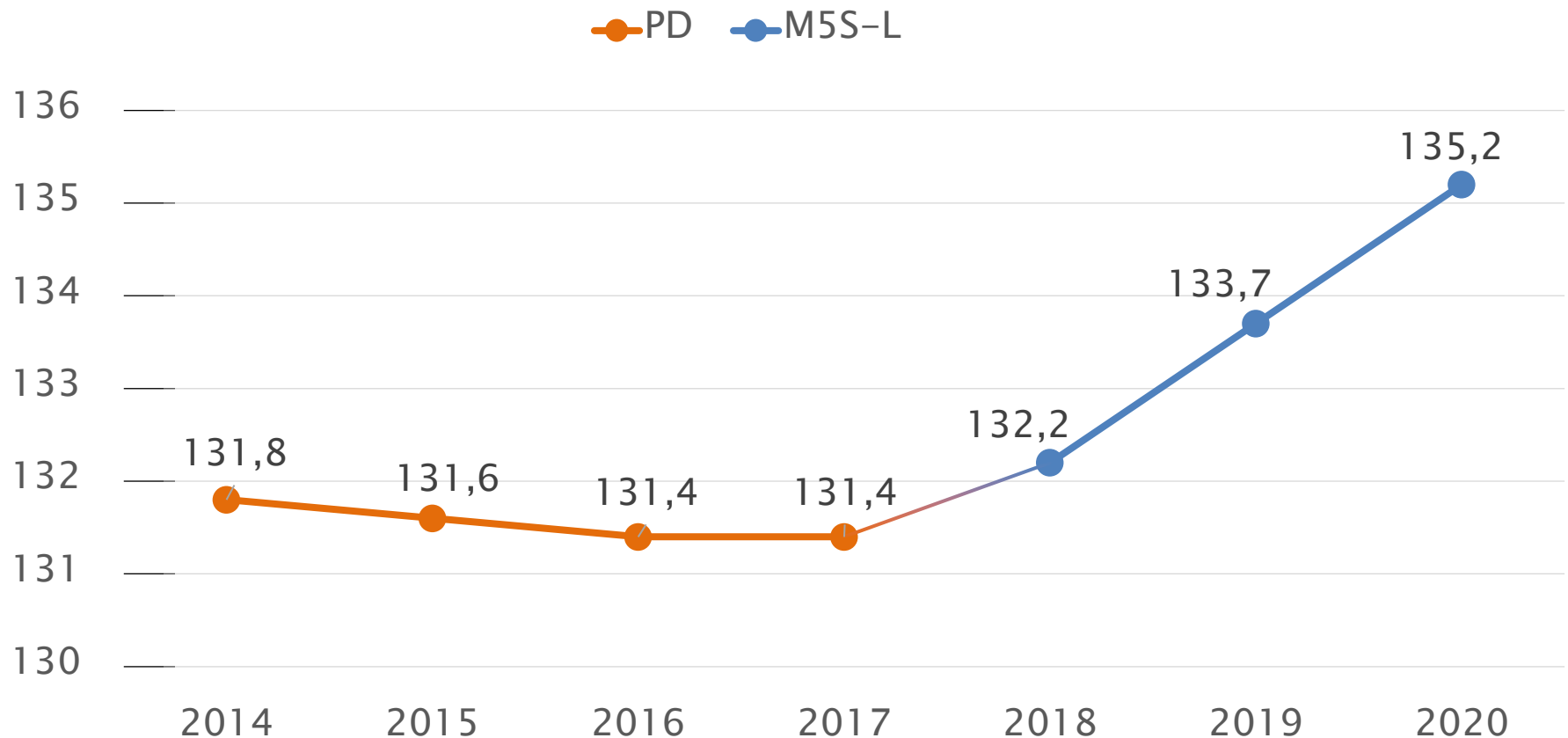
Example – Redesign

Debito Pubblico (% PIL)



Example – Redesign

Debito Pubblico (% PIL)



Guidelines for design

- Keep the physical Lie Factor = 1
- Limit the perceptual Lie Factor as much as possible

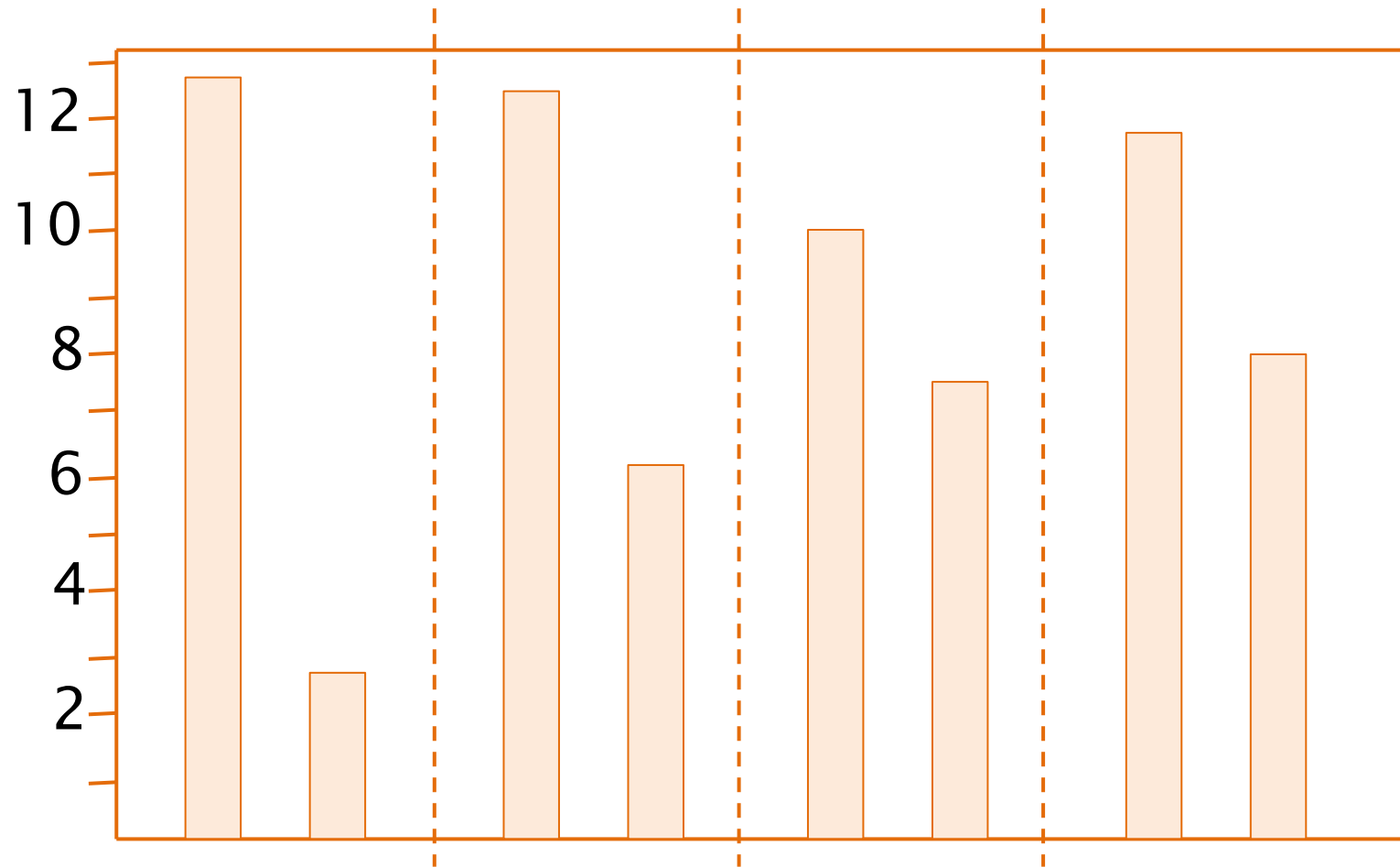
- Every element should convey useful information
- Unnecessary visual objects or attributes distract from the message
 - ◆ Different attributes trigger a search for a rationale (e.g. random colors)

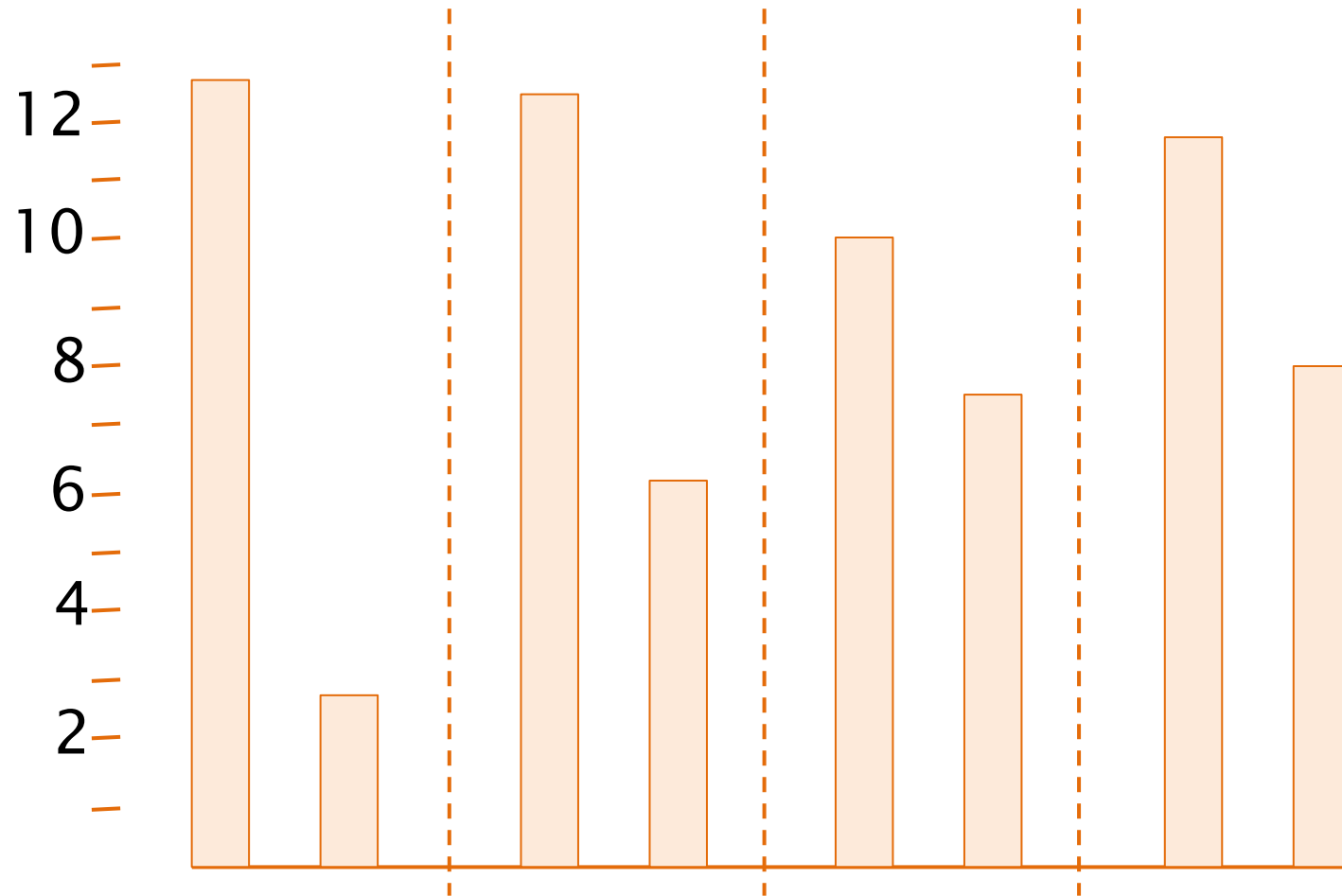
$$\text{Data-ink ratio} = \frac{\text{data ink}}{\text{total ink used to print the graphic}}$$

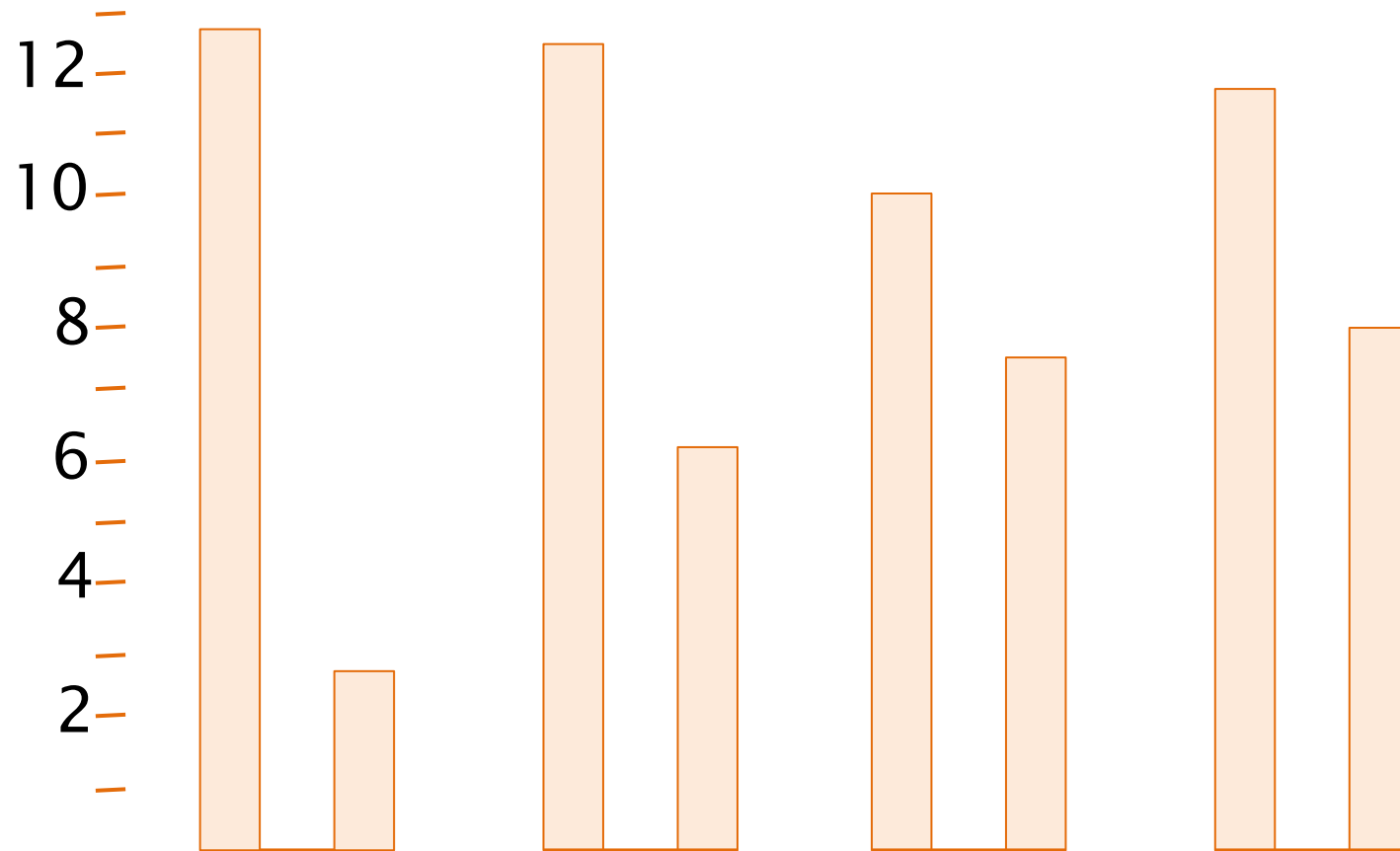
- Proportion of a graphic's ink devoted to the non-redundant display of data information

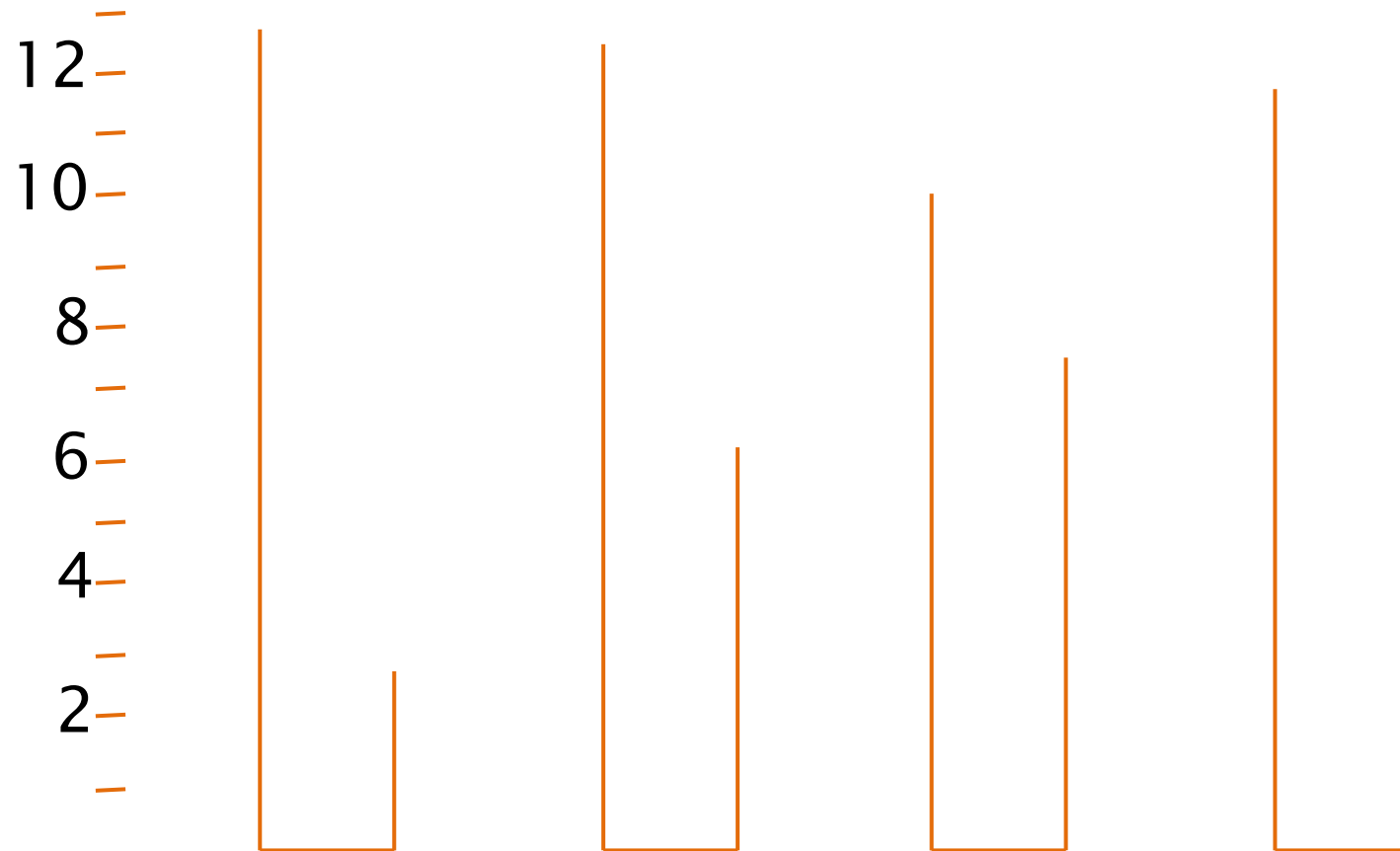
◆ Or:

$$1 - \frac{\text{ink that can be erased without loss of information}}{\text{total ink used to print the graphic}}$$









Tufte's proposed redesign

- Maximize data–ink ratio
 - ◆ Erase non–data–ink
 - ◆ Erase redundant data–ink

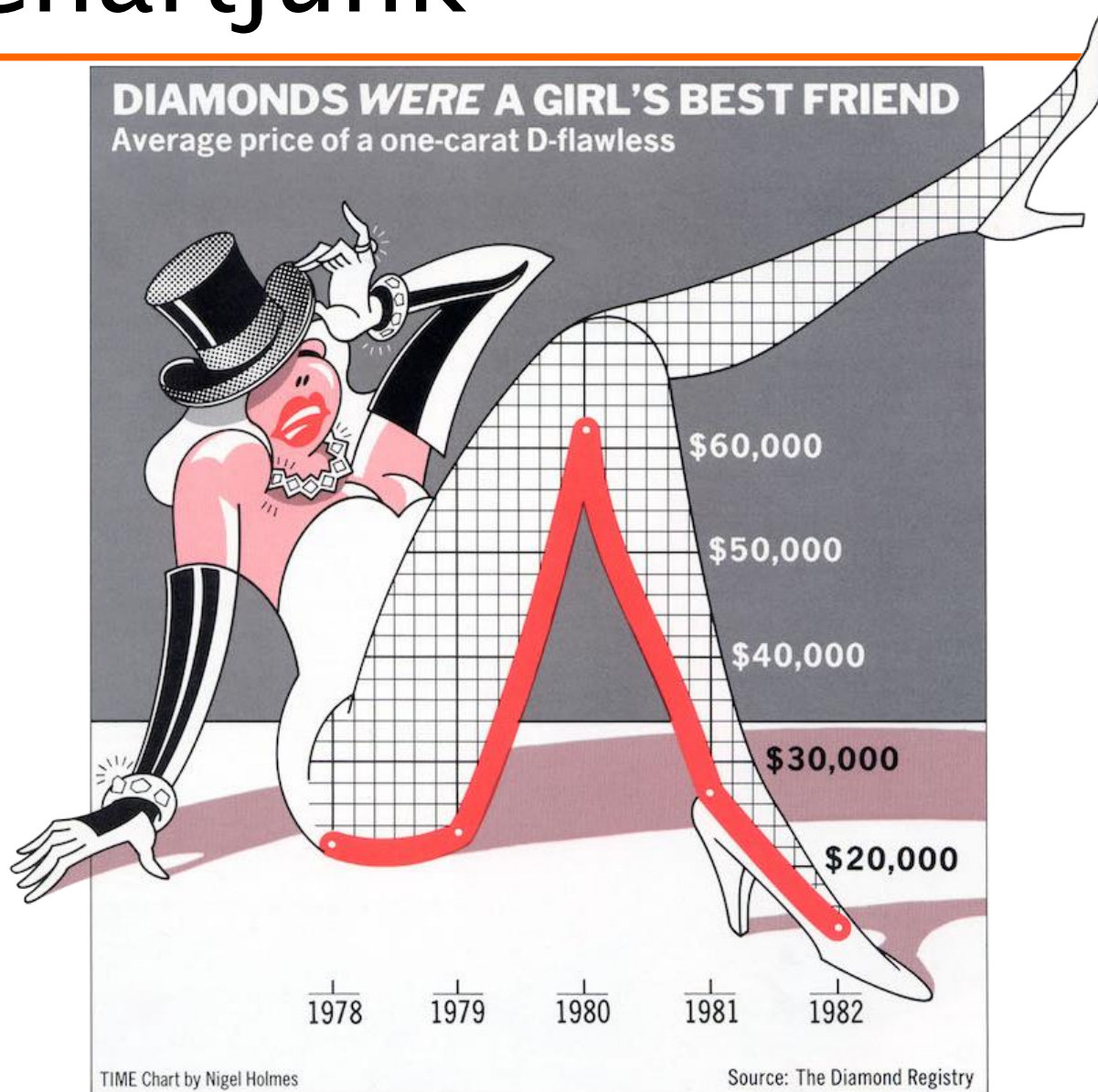
- “Within reason”

Above all else show the data
E. Tufte

- Include differences corresponding to actual differences
- Effective when one item is different in a context of other items that are the same
 - ◆ Bright saturated color among mid colors

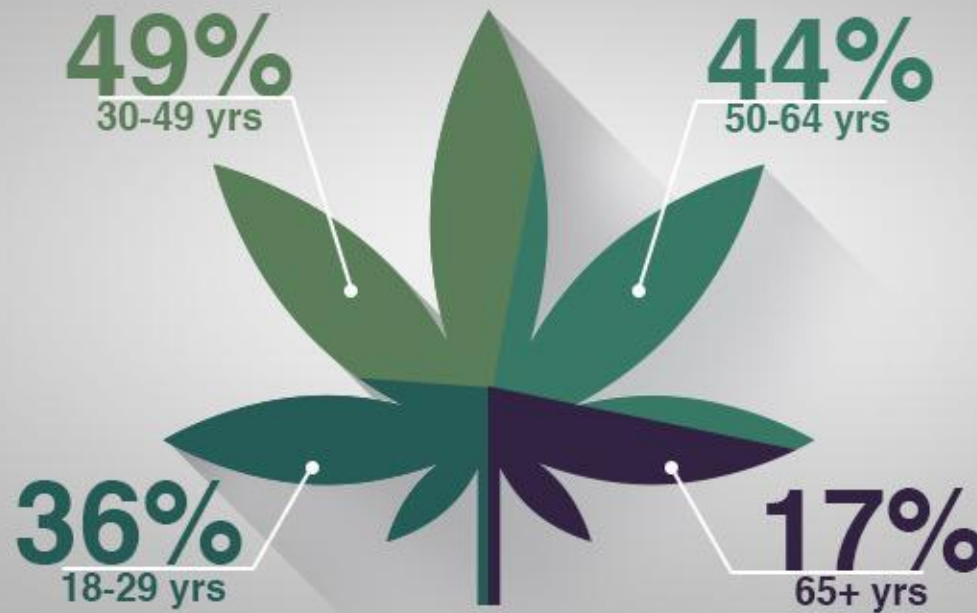
- The presence of unnecessary elements that distract or hide the message conveyed by the diagram

Chartjunk



Nigel Holmes:
<http://nigelholmes.com>

AMERICANS WHO HAVE TRIED WEED



Source: Gallup



SUNDAYS
10^P
ET/PT

#highprofits

- Visual encoding and layout should make perception tasks easy and effortless
- Textual and support elements should provide effective support to understanding the information
- Any variation in the graph should represent useful information otherwise it is noise obfuscating the message

- **Textual** elements should provide effective support to understanding
 - ◆ Hierarchical
 - Size and position reflects importance
 - ◆ Readable
 - Large enough
 - ◆ Horizontal
 - ◆ Close to data (avoid legends)
- Always label the axes

- Get it right in black and white
- Use medium hues or pastels
 - ◆ Bright colors distract and tire out
- Use color only when needed to serve a particular communication goal

Cognitive Dissonance



RED



BLUE



GREEN



YELLOW

Efficiency and efficacy of perception tasks is affected by:

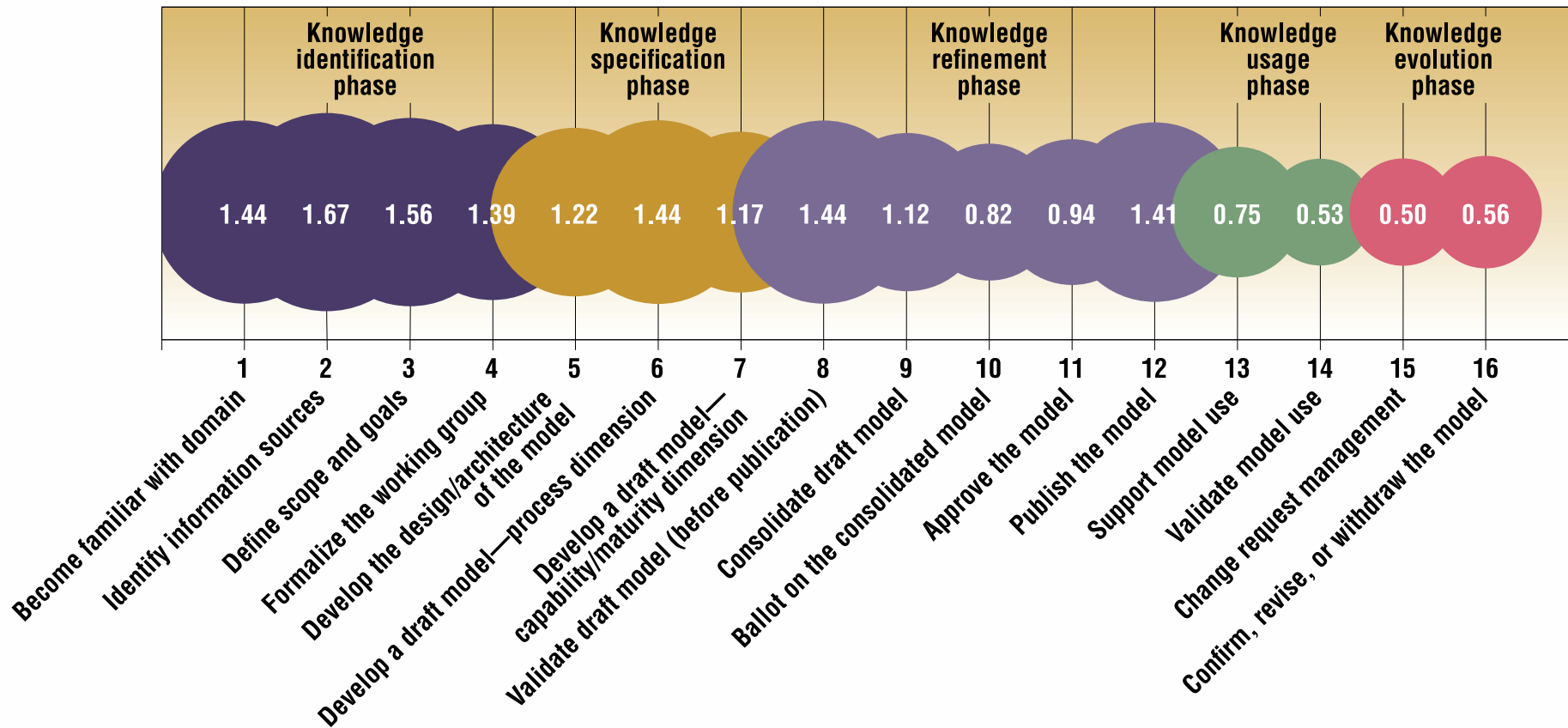
- **Detection**

The capability to visually identify the objects that represent the data to be compared

- **Separation**

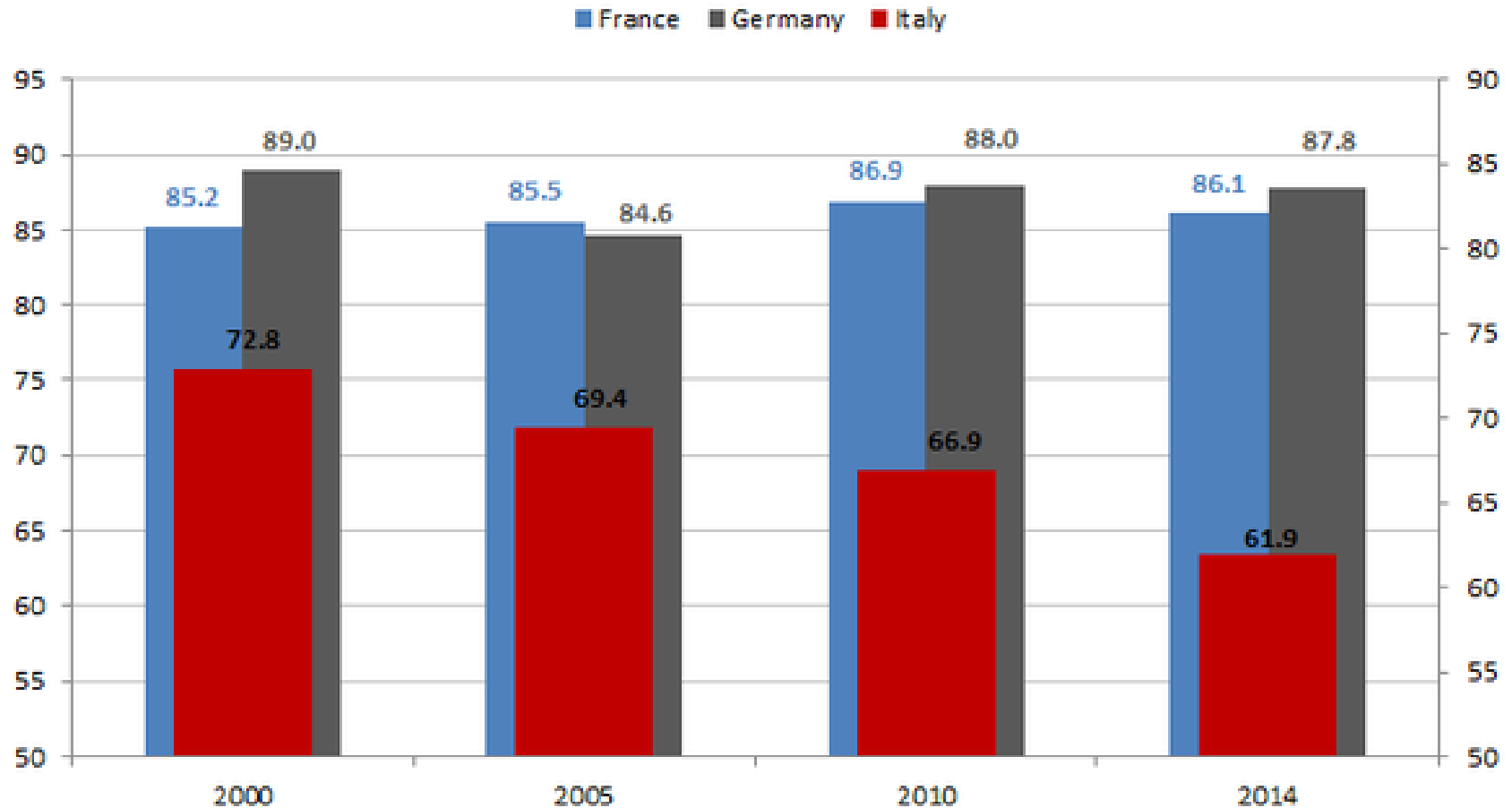
The distance between the objects to be compared

- affects negatively the accuracy



Example

Trends in employment rates of 25-34 with a tertiary degree



Analysis

- Proportionality

- ◆ Due to non-zero base bars, it has a large lie factor (2.2):
 - ratio of real values: 87.8 : 61.9
 - ratio on graph: 37.8 : 11.9

- Utility

- ◆ Most elements appear useful
- ◆ X-axis ticks can be removed
- ◆ Y grid could be made less prominent

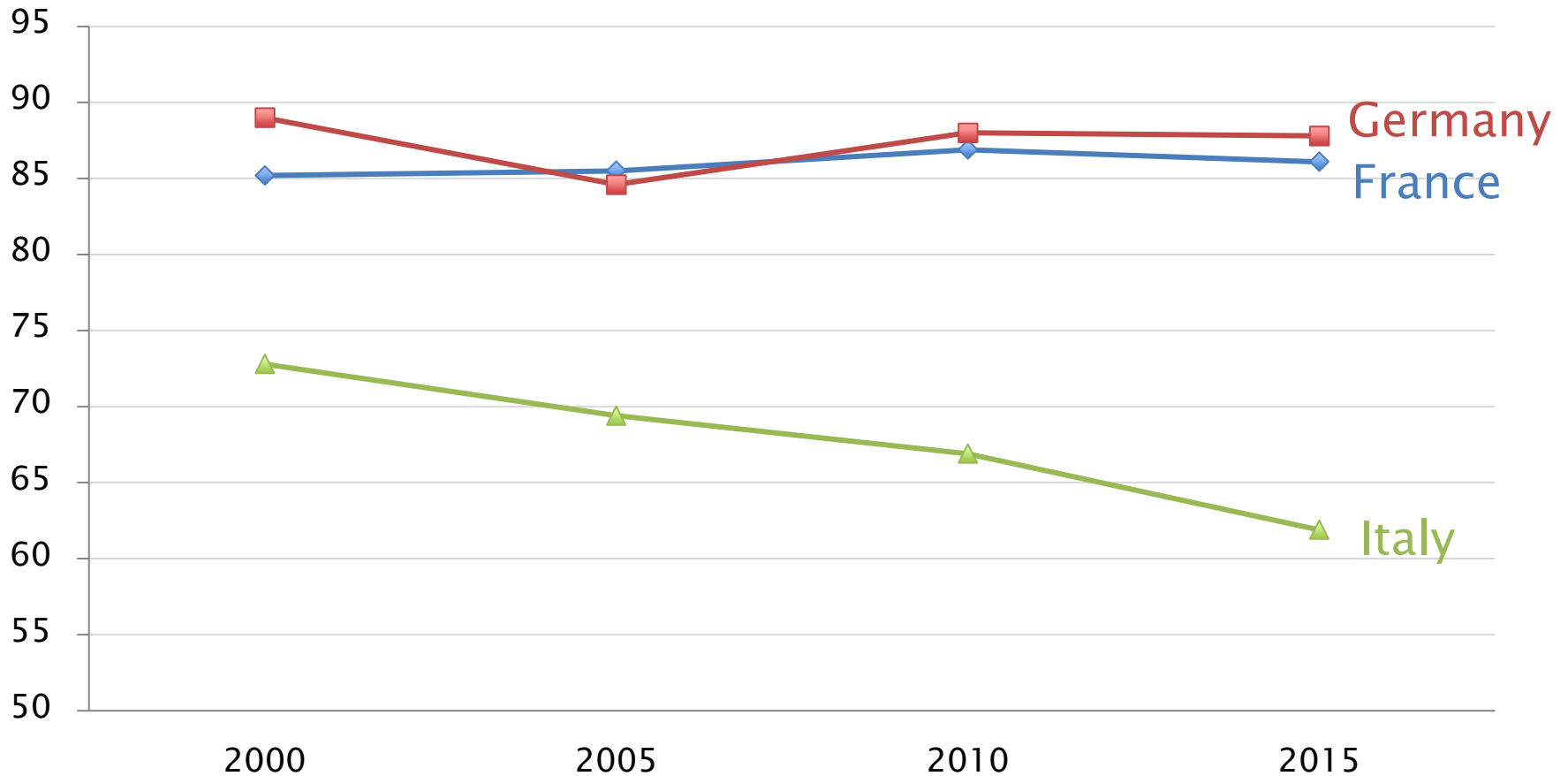
Analysis

- Clarity

- ◆ It uses a **dual scale** that confuses and makes very hard a visual comparison of the values and further distorting the compared values.
- ◆ The dual scale is not mentioned anywhere and it is not clear which values refer to which scale.
- ◆ In general the usage of bars is not the most appropriate visual representation if the goal is to show a trend or evolution in time.

Redesign

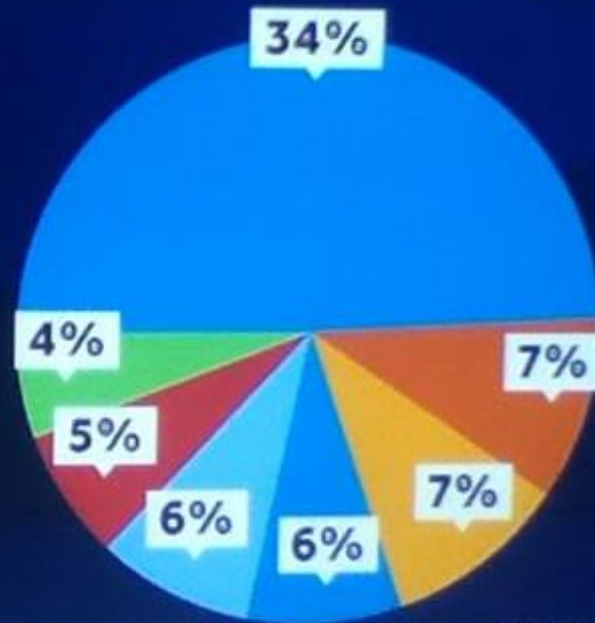
Trends in employment rates of 25–34 with a tertiary degree



Case study

WHICH NFL TEAM IS YOUR FAVORITE?

- PANTHERS
- COWBOYS
- PACKERS
- PATRIOTS
- STEELERS
- REDSKINS
- BRONCOS



SOURCE: PUBLIC
POLICY POLLING

WXII
12

Assessment

- Question:
 - ◆ Is there one (or more) question addressed by the visualization?
- Data:
 - ◆ Is the data quality appropriate?
- Visual Integrity:
 - ◆ Are the visual features appropriate?

Visual Integrity

- **Proportionality:**
 - ◆ Are the values encoded in a uniformly proportional way?
- **Utility:**
 - ◆ All the elements in the graph convey useful information?
- **Clarity:**
 - ◆ Are the data in the graph identifiable and understandable (properly described)?

Question

- What are the most popular/favorite NFL teams in our audience?
- ...

Data

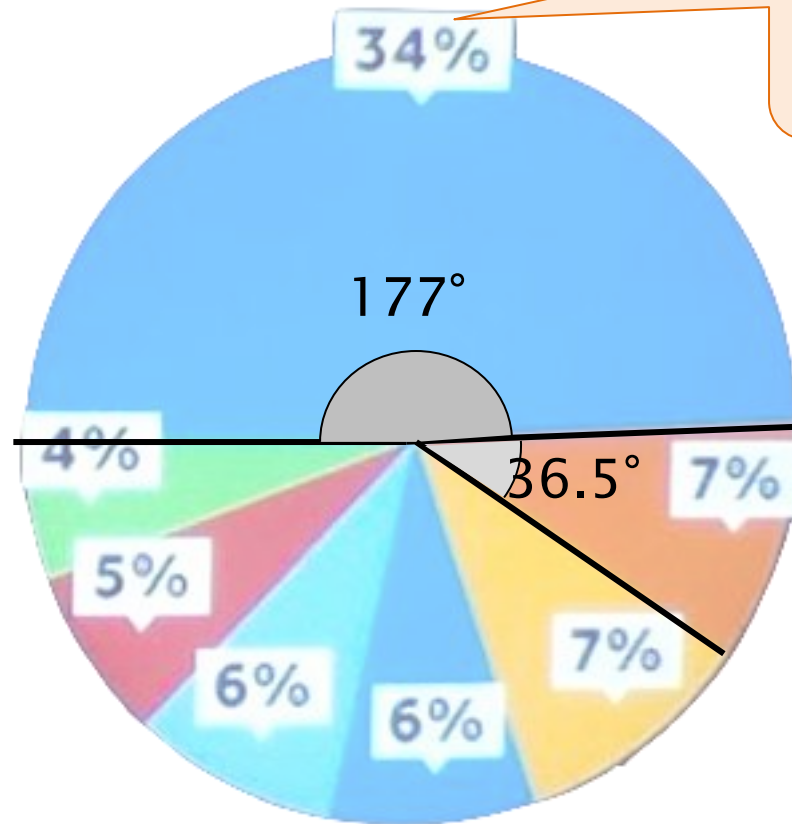
WXII-TV is an NBC-affiliated television station serving North Carolina: home of Panthers

Team	Preferences
Panthers	34%
Cowboys	7%
Packers	7%
Patriots	6%
Steelers	6%
Redskins	5%
Broncos	4%
Total:	69%

Full data

Team	Preferences
Panthers	34%
Cowboys	7%
Packers	7%
Patriots	6%
Steelers	6%
Redskins	5%
Broncos	4%
<i>Other</i>	31%
Total:	100%

Integrity – Proportionality



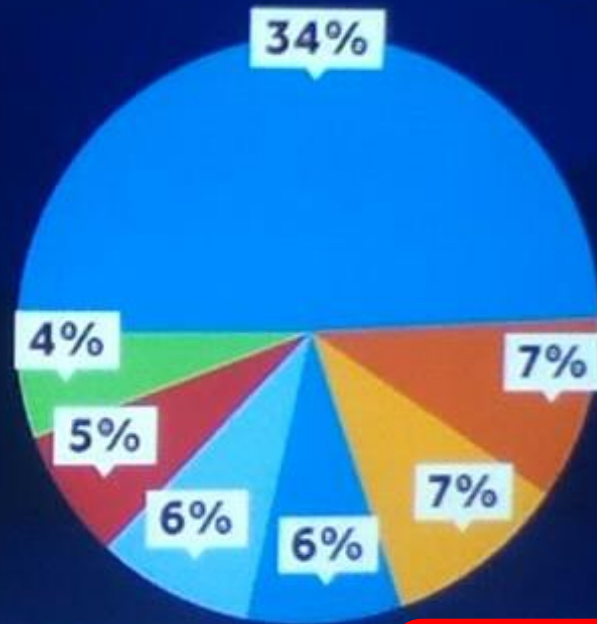
34% corresponds to 50% of the pie!

$$\begin{aligned} 177 / 36.5 &= 4.8 \\ 34 / 7 &= 4.8 \end{aligned}$$

Utility

WHICH NFL TEAM IS YOUR FAVORITE?

- PANTHERS
- COWBOYS
- PACKERS
- PATRIOTS
- STEELERS
- REDSKINS
- BRONCOS

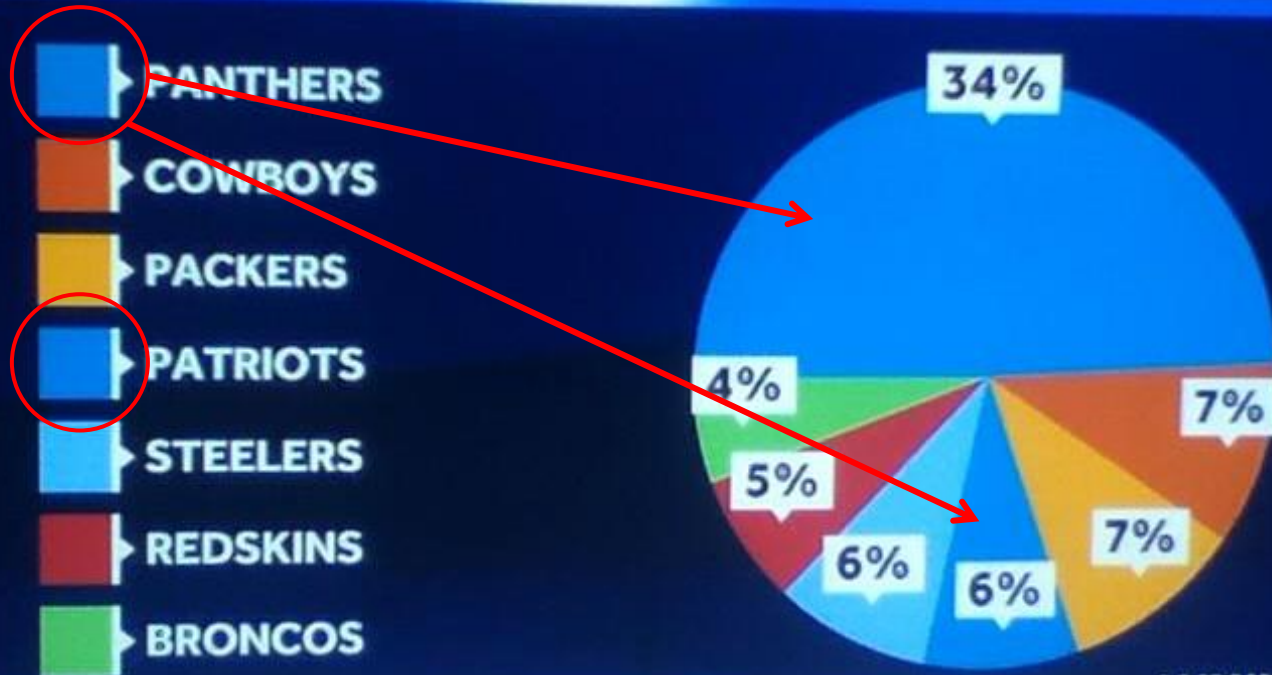


SOURCE: PUBLIC
POLICY POLLING

WXII
12

Clarity

WHICH NFL TEAM IS YOUR FAVORITE?

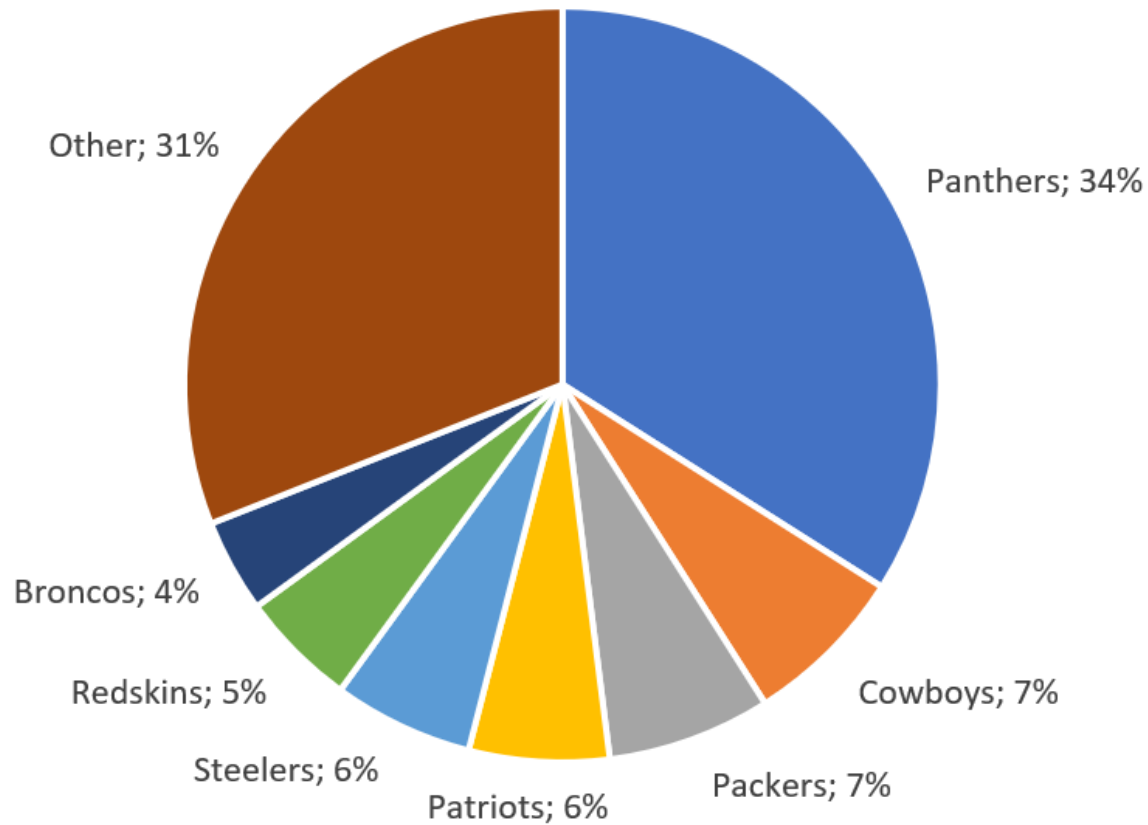


SOURCE: PUBLIC
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WXII
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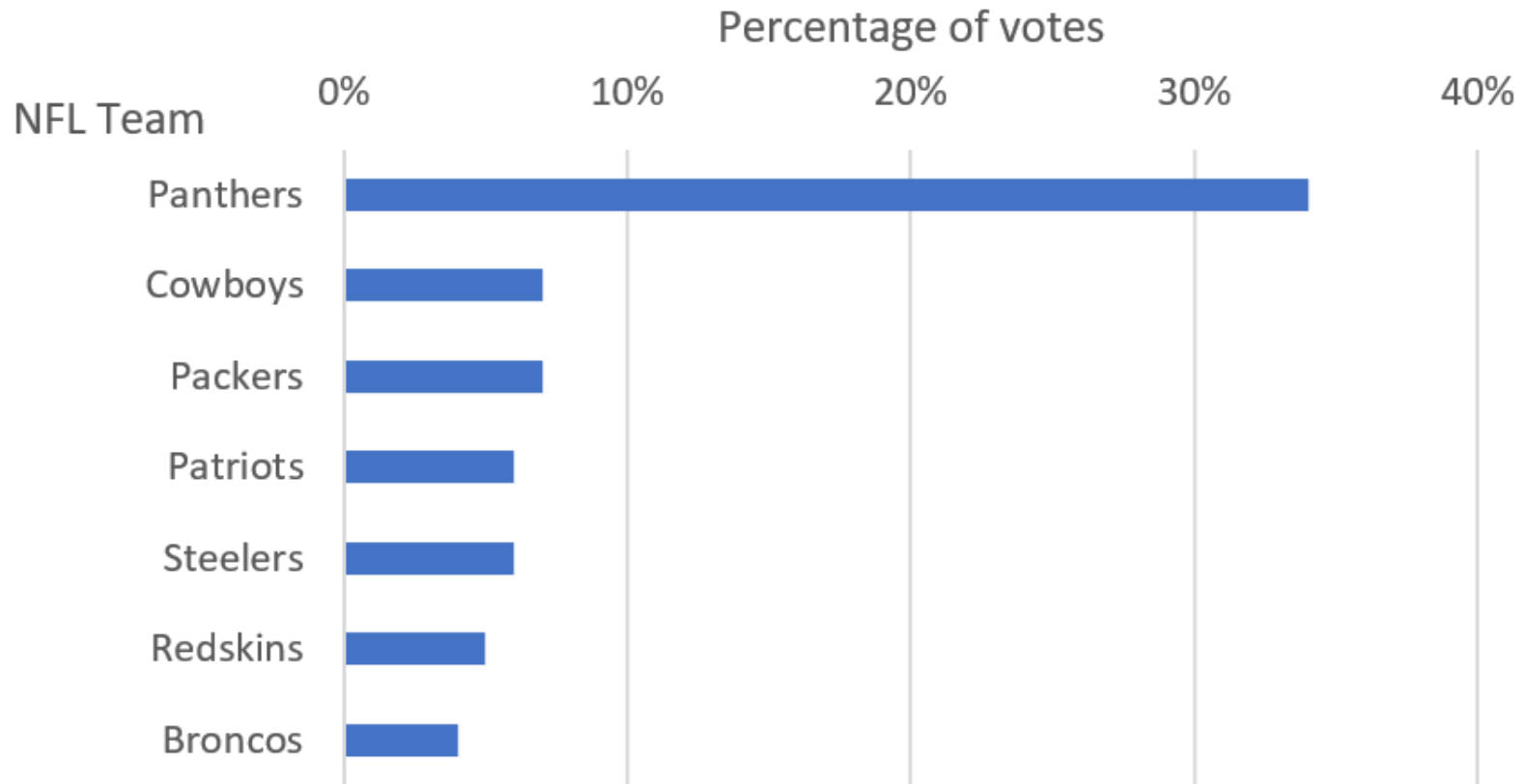
Redesign #1

Preferences



Redesign #2

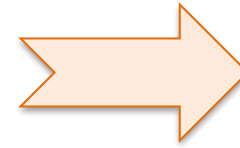
Favorite NFL teams in our audience



VISUALIZATION PIPELINE

Visualization Pipeline

Knowledge



Decisions

Information Understanding

Visual Patterns, Trends, Exceptions

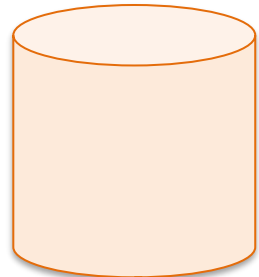
Quantitative Reasoning

Quantitative Relationship & Comparison

Visual Perception

Visual Properties & Objects

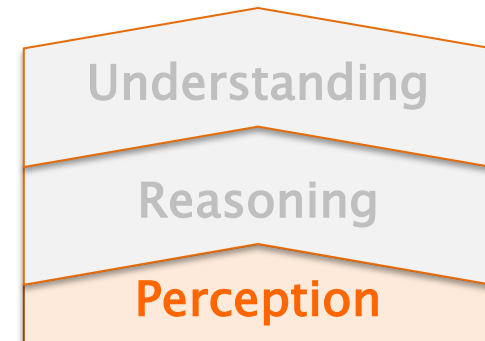
Data



Representation/Encoding

Visual Perception

- Any variable (measure) must be **visually encoded**, i.e. we need to identify:
 - ◆ Visual object to represent entity
 - ◆ Visual attribute to represent the measure



Example

Votes received by four candidates in recent elections

Candidate	Votes	Proportion
Sergio	1 97800	50.09%
Alberto	1 40545	35.59%
Giorgio	53748	13.61%
Valter	2759	0.70%

<http://www.comune.torino.it/elezioni/2019/regionali/presidente/citta/>

Encoding

- Visual object: line
- Visual attribute: length

Alberto

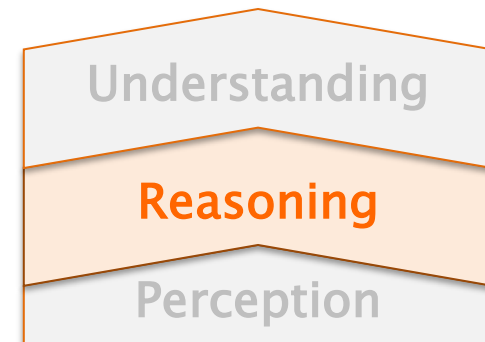
- Giorgio
Valter

Sergio

Visual Reasoning

Layout and visual attributes allow:

- **Discrimination**
 - ◆ Distinguish visual objects or group of –
- **Comparison**
 - ◆ Place visual objects in order
- **Magnitude assessment**
 - ◆ Evaluate the (relative) magnitude of visual objects



Reasoning

Alberto

- Giorgio
Valter

Sergio

Reasoning

- Discrimination

Alberto

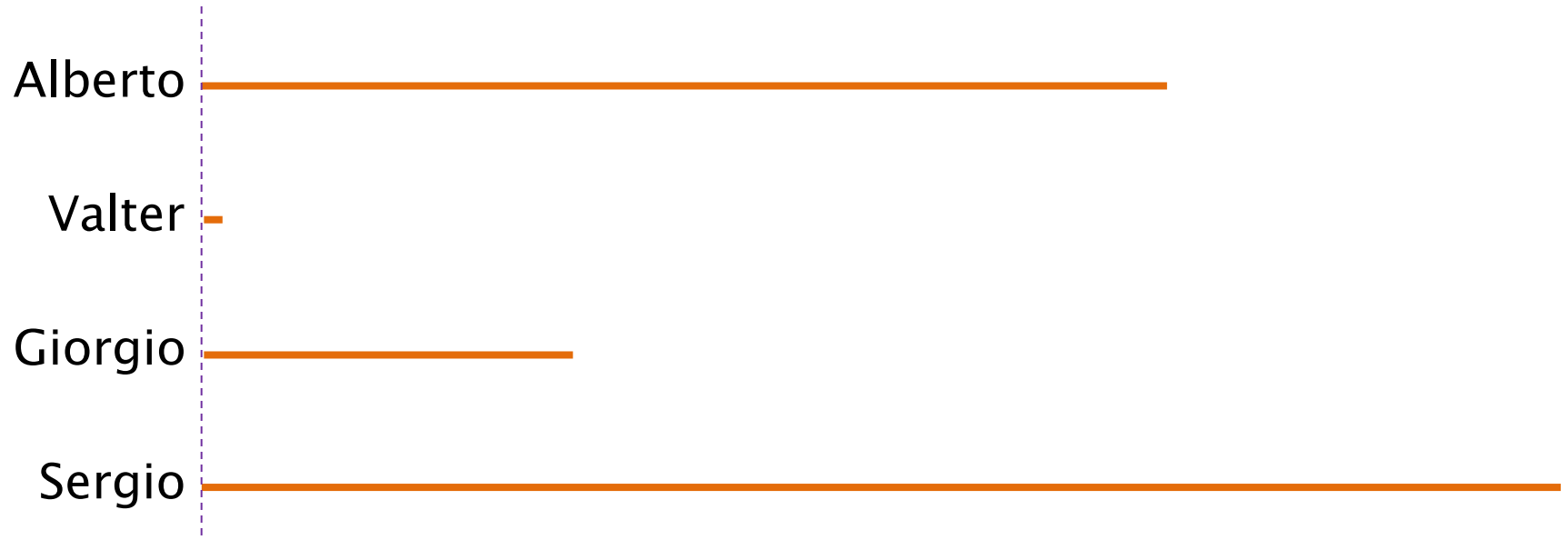
Valter -

Giorgio

Sergio

Reasoning

- Comparison



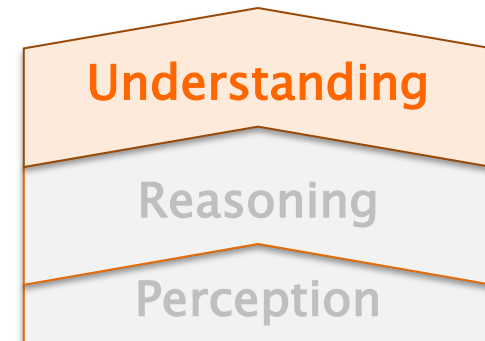
Reasoning

- Assessment



Understanding

- Variation within quantitative measures
 - ◆ Distribution
 - ◆ Deviation
 - ◆ Correlation
- Variation within category
 - ◆ Ranking
 - ◆ Part-to-whole
 - ◆ Time
 - ◆ Space
- Multivariate



Understanding



Understanding

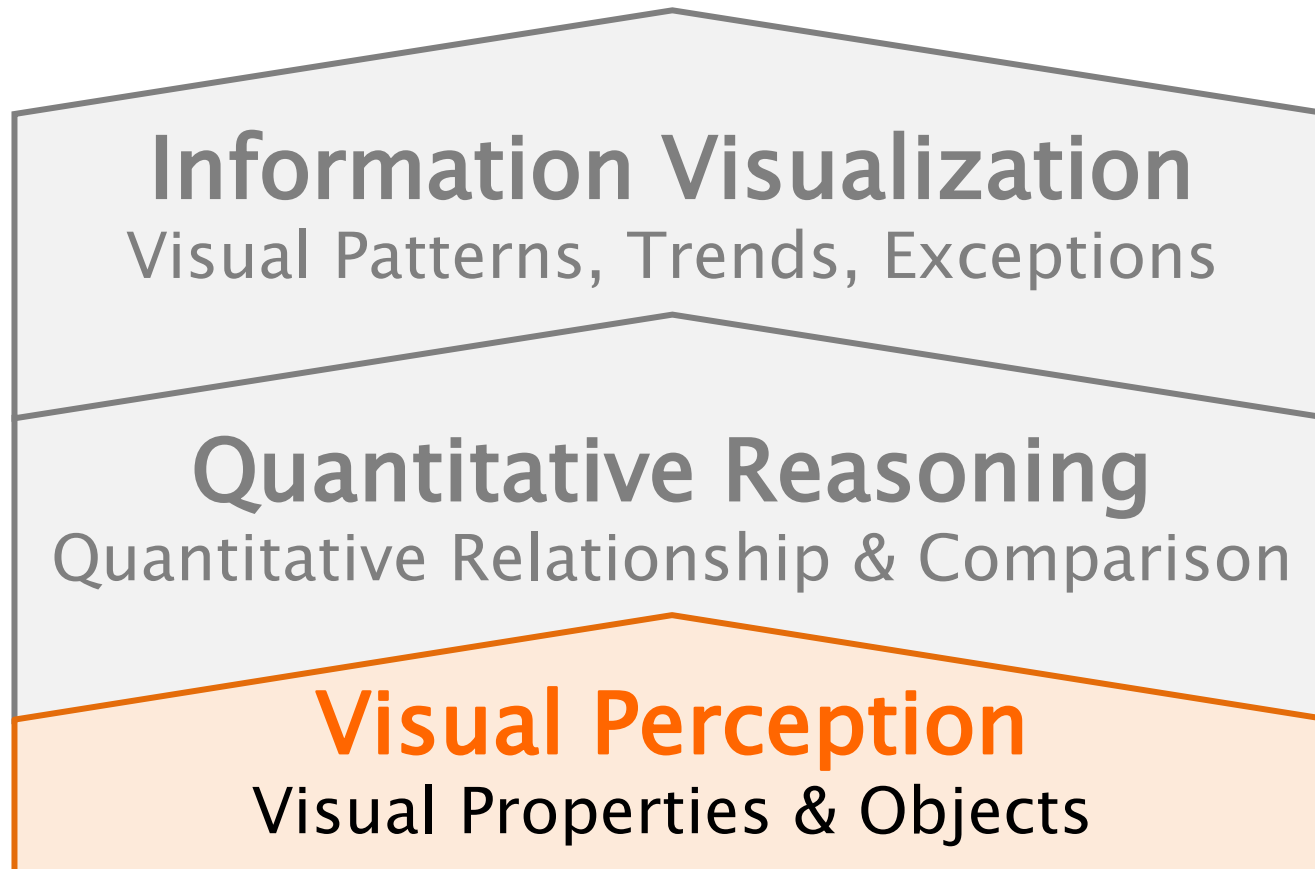
- Ranking



VISUAL PERCEPTION

Data Visualization

Understanding

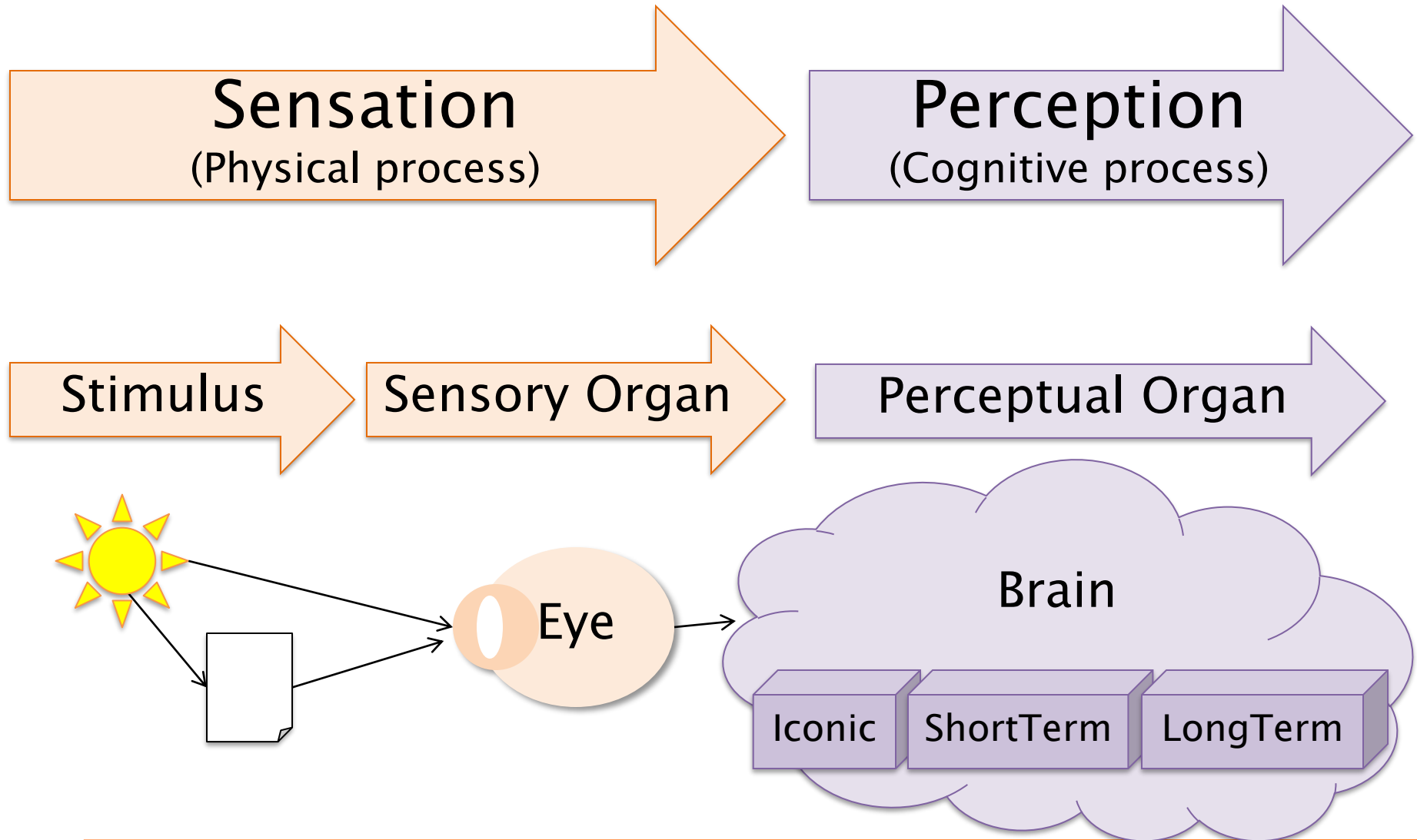


Data



Representation/Encoding

Visual perception



Memory Hierarchy

- Iconic memory (visual sensory register)
 - ◆ Pre-attentive processing
 - ◆ Detects a **limited number of attributes**
- Short-term memory (working memory)
 - ◆ Store visual chunks
 - ◆ Limited number
- Long-term memory
 - ◆ Store high-level knowledge

Simplified Model

- The three levels of memory represent a simplified model
 - ◆ does not correspond to “real” physical brain structure
- Useful to explain a few phenomena
 - ◆ The 7 ± 2 rule
 - ◆ Change blindness

Change blindness



Change blindness



Pre-Attentive Attributes

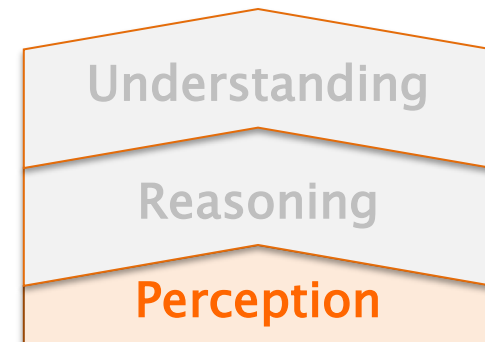
5	7	8	4	9	8	3	1	1	0	6	8	8	2	1	1	5	2	6	6	5
9	5	1	8	4	6	8	4	9	3	0	4	5	3	4	9	2	5	8	5	8
5	0	5	4	6	2	6	5	7	3	7	8	6	5	3	7	2	6	3	1	5
5	8	6	6	8	3	7	6	5	0	9	6	3	4	6	1	9	5	6	6	4
1	6	7	3	9	9	2	8	3	4	0	3	5	1	6	3	5	3	9	3	4
8	6	9	7	5	4	2	4	7	4	9	5	8	5	3	0	7	6	0	6	7
0	3	1	5	3	2	3	5	6	7	2	8	9	8	5	3	7	8	8	2	4
5	5	3	4	8	1	5	6	2	3	5	5	1	2	1	0	8	7	2	6	3
7	4	3	8	4	8	2	6	7	9	5	6	2	3	6	7	8	0	8	3	6
4	9	5	6	7	2	2	2	8	3	1	1	0	1	8	6	2	6	2	1	4

Pre-Attentive Attributes

5	7	8	4	9	8	3	1	1	0	6	8	8	2	1	1	5	2	6	6	5
9	5	1	8	4	6	8	4	9	3	0	4	5	3	4	9	2	5	8	5	8
5	0	5	4	6	2	6	5	7	3	7	8	6	5	3	7	2	6	3	1	5
5	8	6	6	8	3	7	6	5	0	9	6	3	4	6	1	9	5	6	6	4
1	6	7	3	9	9	2	8	3	4	0	3	5	1	6	3	5	3	9	3	4
8	6	9	7	5	4	2	4	7	4	9	5	8	5	3	0	7	6	0	6	7
0	3	1	5	3	2	3	5	6	7	2	8	9	8	5	3	7	8	8	2	4
5	5	3	4	8	1	5	6	2	3	5	5	1	2	1	0	8	7	2	6	3
7	4	3	8	4	8	2	6	7	9	5	6	2	3	6	7	8	0	8	3	6
4	9	5	6	7	2	2	2	8	3	1	1	0	1	8	6	2	6	2	1	4

Encoding

- Encoding is the key to enable visual perception
 - ◆ Visual object to represent entity
 - ◆ Visual attribute to represent the measure
- Two main types
 - ◆ Quantitative (different properties)
 - ◆ Categorical (ordinal or not)



Pre-Attentive attributes

Category	Attribute
Form	Orientation Length/distance Line width Size Shape Curvature Added marks Enclosure
Color	Hue Intensity
Spatial position	2-D position
Motion	Flicker Direction Speed

Perception task

Visual attributes allow:

- Discrimination
 - ◆ Distinguish visual objects
- Comparison
 - ◆ Place visual objects in order
- Magnitude assessment
 - ◆ Evaluate the (relative) magnitude of visual objects

Just noticeable difference

- Given a physical dimension (length, brightness, etc.) x
- d is the **just noticeable difference** if:
 - ♦ difference between x and $x+d$ is perceivable
 - ♦ but not smaller differences
- d depends on many factors:
 - ♦ Subject
 - ♦ Environment
 - ♦ Physical dimension

Weber's law

- Just noticeable difference d is:

$$d_p(x) = k_p \cdot x$$

- Where
 - ♦ x : dimension
 - ♦ $d_p(x)$: just noticeable difference
 - ♦ k_p : constant
 - Subjective
 - Environmental

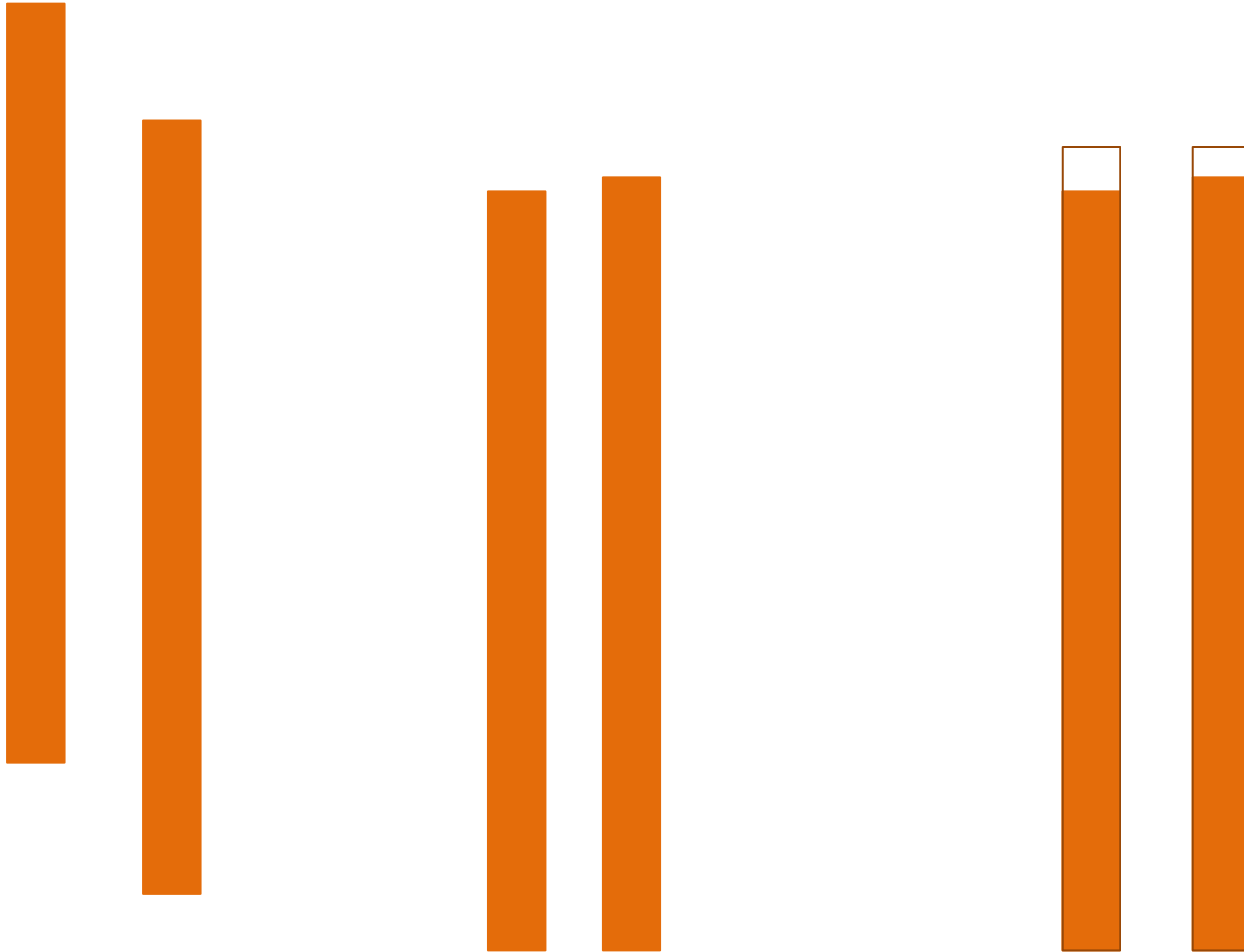
Consequences of Weber's law

- It is easier to compare lengths that differ by a large percentage
- The same difference is easier to notice between smaller measures
 - ◆ More likely to be larger than just noticeable difference

$$x < y \implies d_p(x) < d_p(y)$$

- Length of non-aligned objects is harder to compare
 - ◆ Double comparison

Non-aligned objects lengths



Non-aligned objects lengths

- Additional references may help comparison
 - ◆ They provide alternative possible comparisons
- If lengths range between 0 and a maximum (L), e.g. percentages
- Comparing l_1 and l_2 (close to L) that differ by a small amount d
 - ◆ Difference $L-l_1$ vs. $L-l_2$ easier to notice than l_1 vs. l_2

Stevens's law

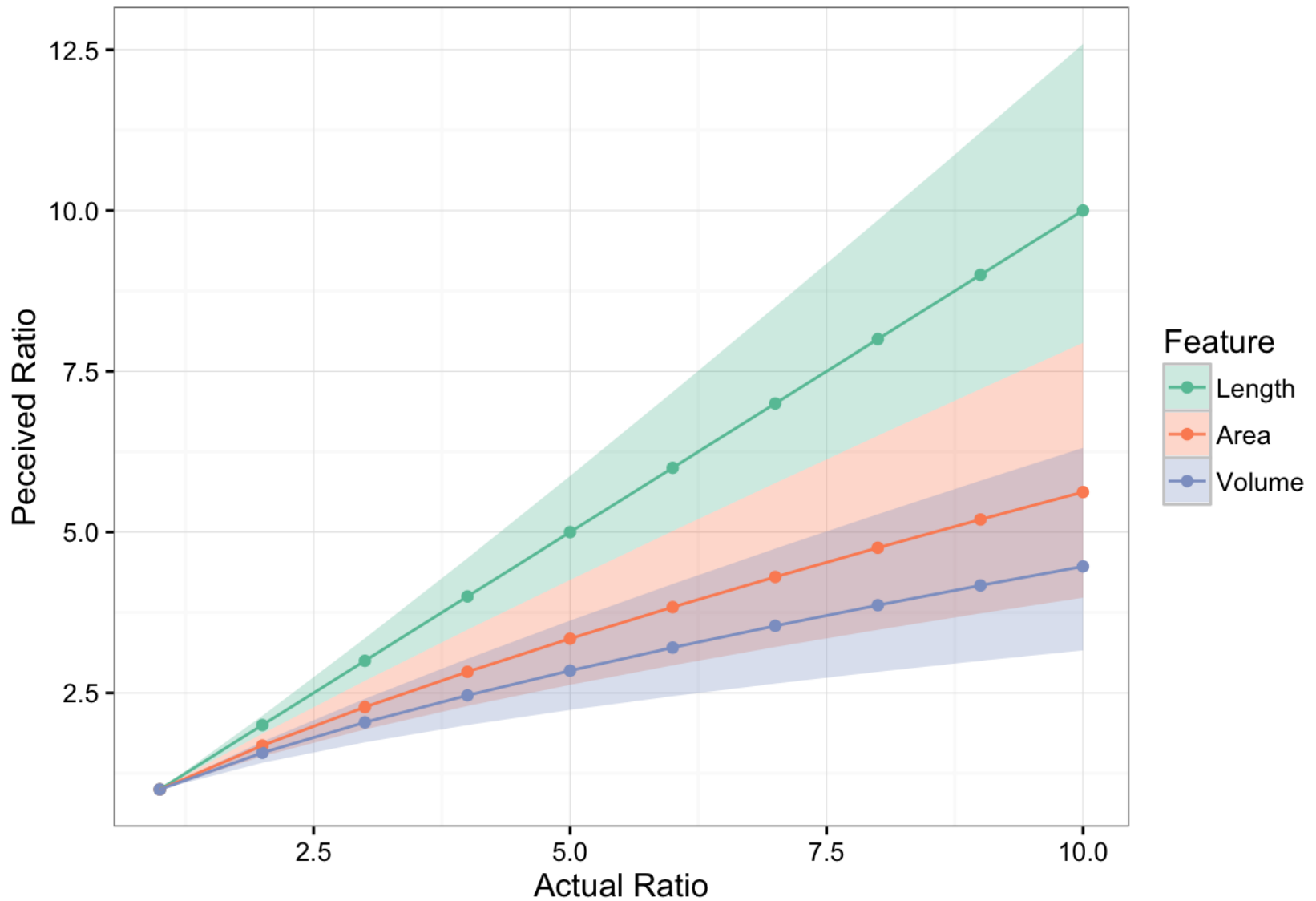
- Perceive scale (magnitude ratio)

$$p(x) = c \cdot x^{\beta}$$

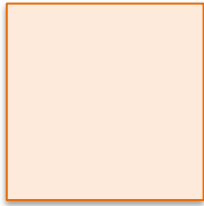
- Where β depends on spatial dimension
 - ♦ 1D: Length $\rightarrow \beta$ in $[0.9, 1.1]$
 - ♦ 2D: Area $\rightarrow \beta$ in $[0.6, 0.9]$
 - ♦ 3D: Volume $\rightarrow \beta$ in $[0.5, 0.8]$

Stevens S. S. (1975). Psychophysics, John Wiley & Sons.

Stevens's law



Stevens's law

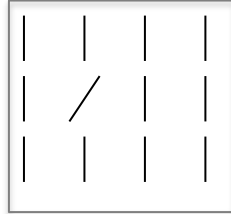


Consequences

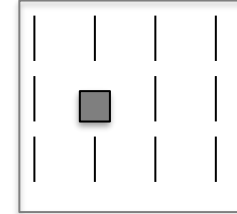
- Prefer comparing lengths
- Avoid comparison between areas
 - ◆ Except for ordinal measures
- Never–ever make volume comparisons

Attributes of form

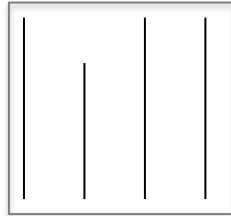
Orientation



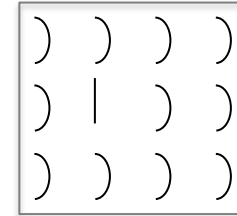
Shape



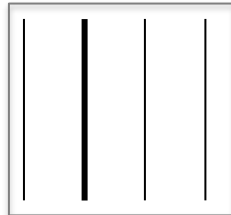
Line Length



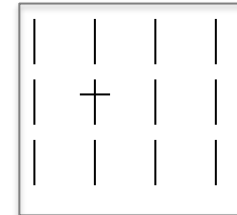
Curvature



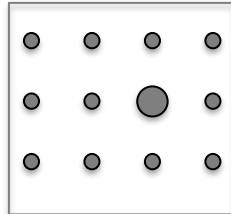
Line Width



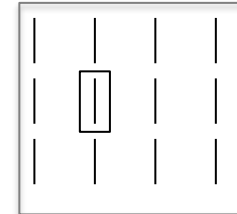
Added mark



Size

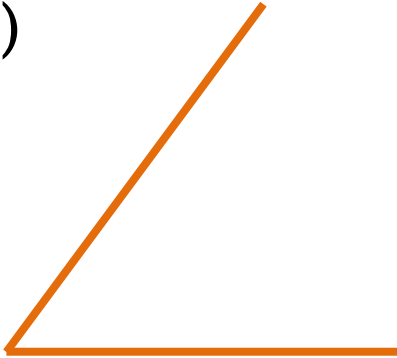


Enclosure

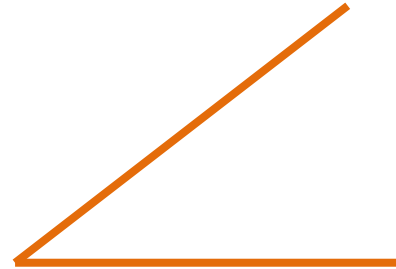


Orientation (angle or slope)

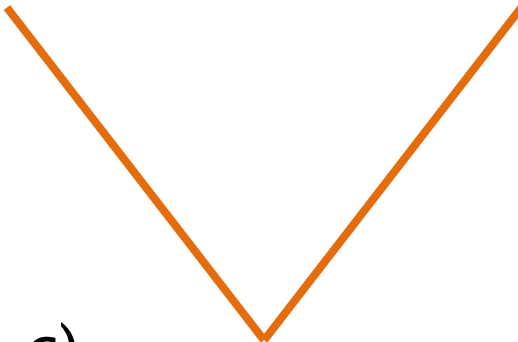
a)



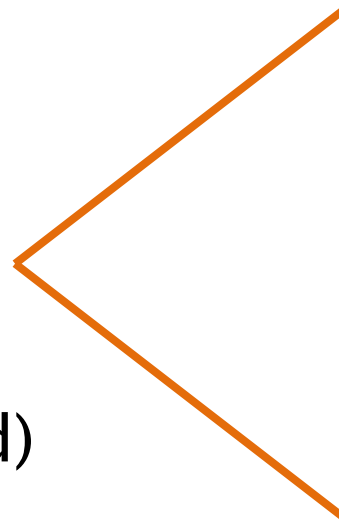
b)



c)

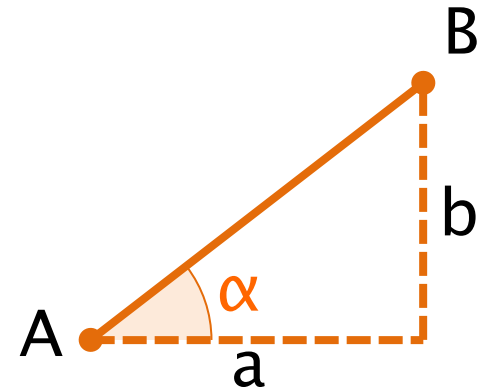


d)



Angle vs. Slope

- Slope of A–B is b/a
 - ♦ $\tan(\alpha)$
- Slope judgment typically falls back to an angle judgment
 - ♦ Given an error ϵ in the angle judgment
 - ♦ It is reflected in a slope error

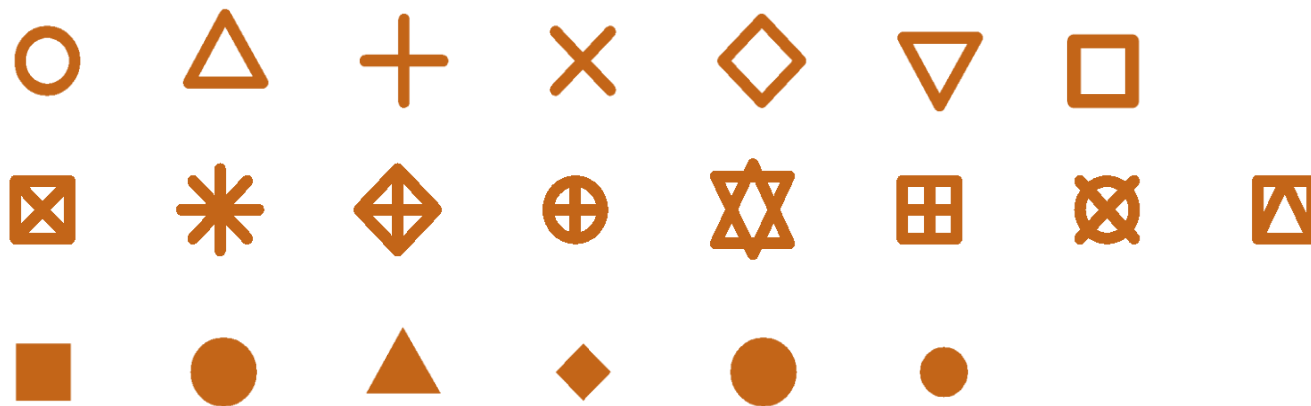


$$\tan(\alpha + \epsilon) - \tan(\alpha) = \epsilon \cdot \tan'(\alpha) = \frac{\epsilon}{\cos^2(\alpha)}$$

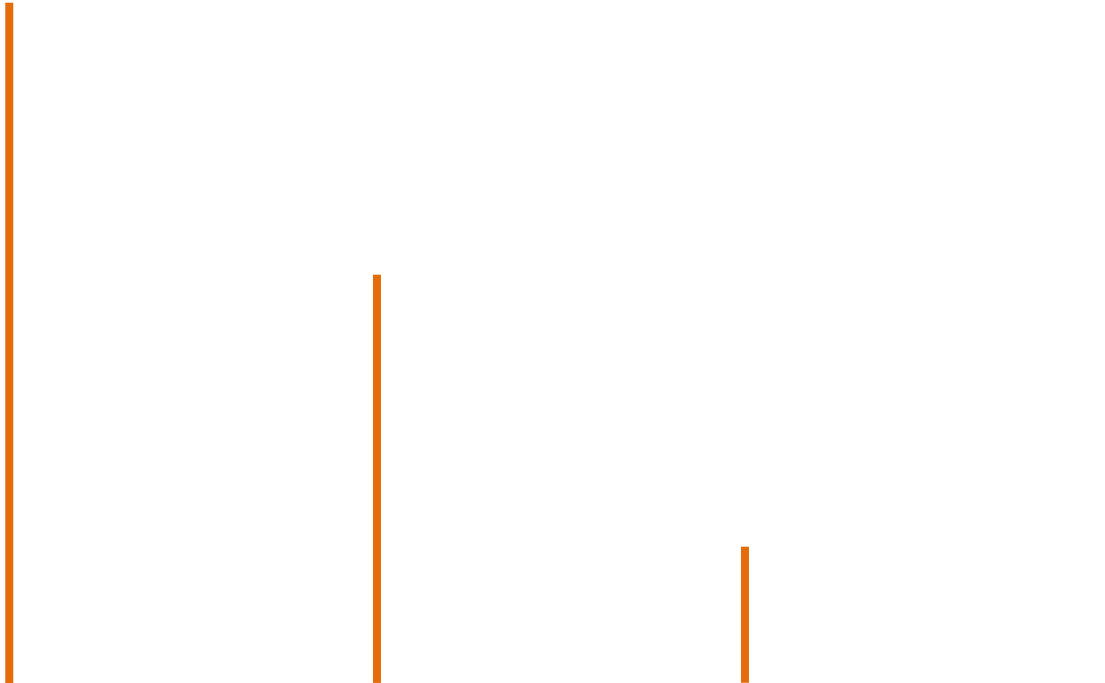
– Getting infinite as α approaches to $\pi/2$

Shape

- There is no common quantitative semantics for the shapes
 - Unless they are characters...
- ◆ Fill textures are shapes too



Length



Effect of context



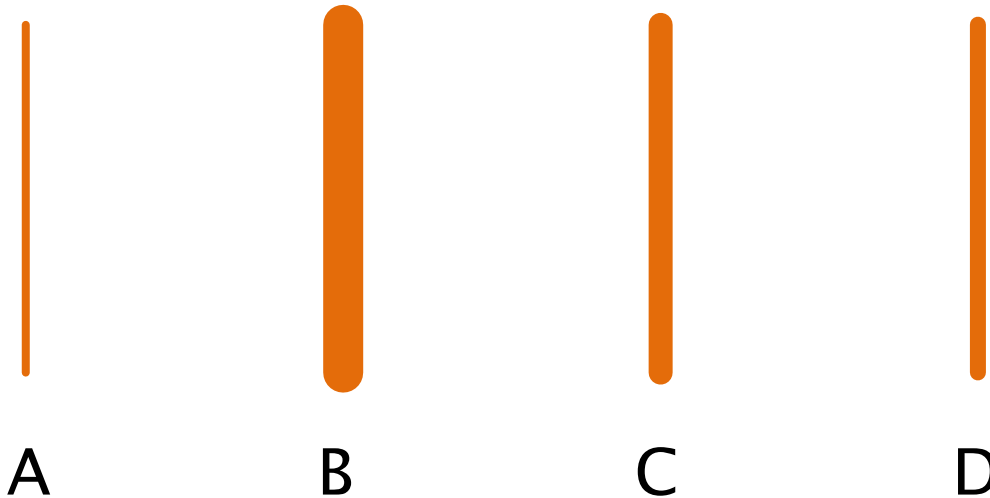
Curvature

- There is no common magnitude assessment for the curvature



Width

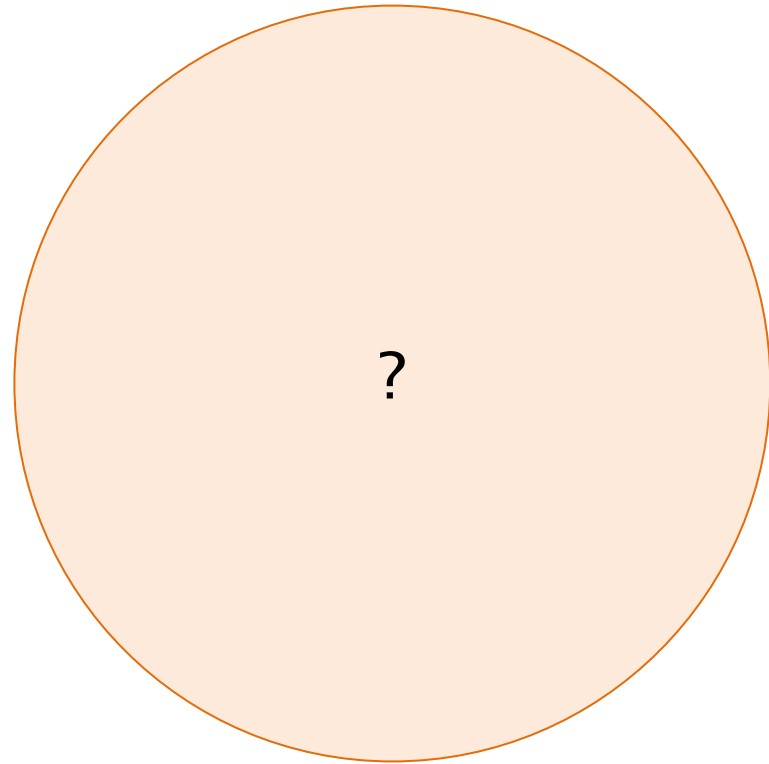
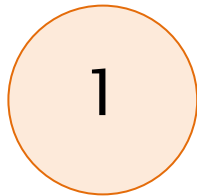
- Order can be identified
 - ◆ Difficult to appreciate actual magnitude



Mark

- No common quantitative semantics of marks
- Number of marks could encode a natural number
 - ◆ Harder to read than a cipher

Size / Area

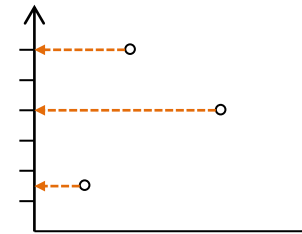


Enclosure

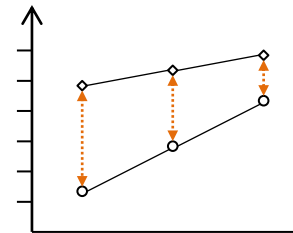
- No common quantitative semantics for enclosure
 - ◆ Except counting items enclosed

Spatial Position

- Position along axis
 - ◆ Common scale
 - ◆ Distinct identical scales
 - Possibly un-aligned

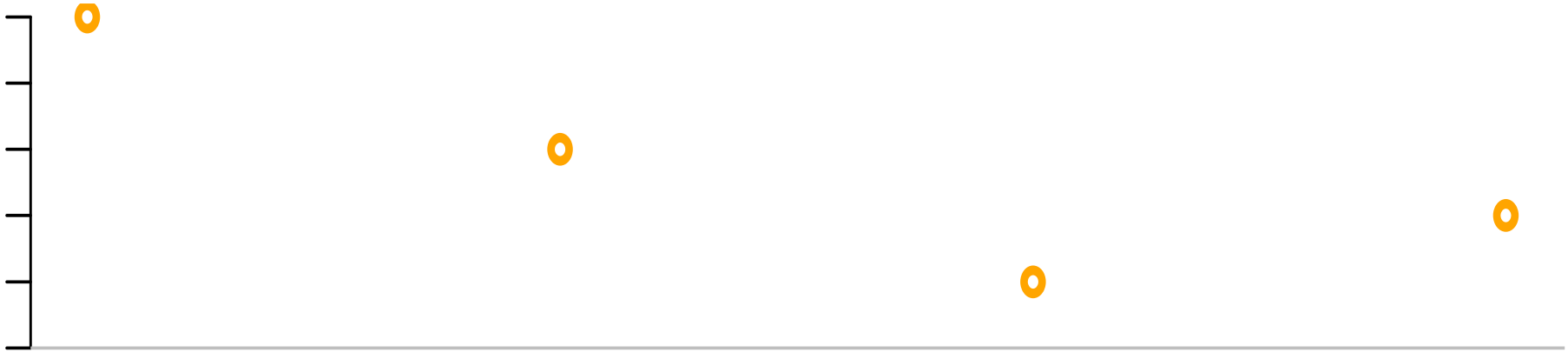


- Distance



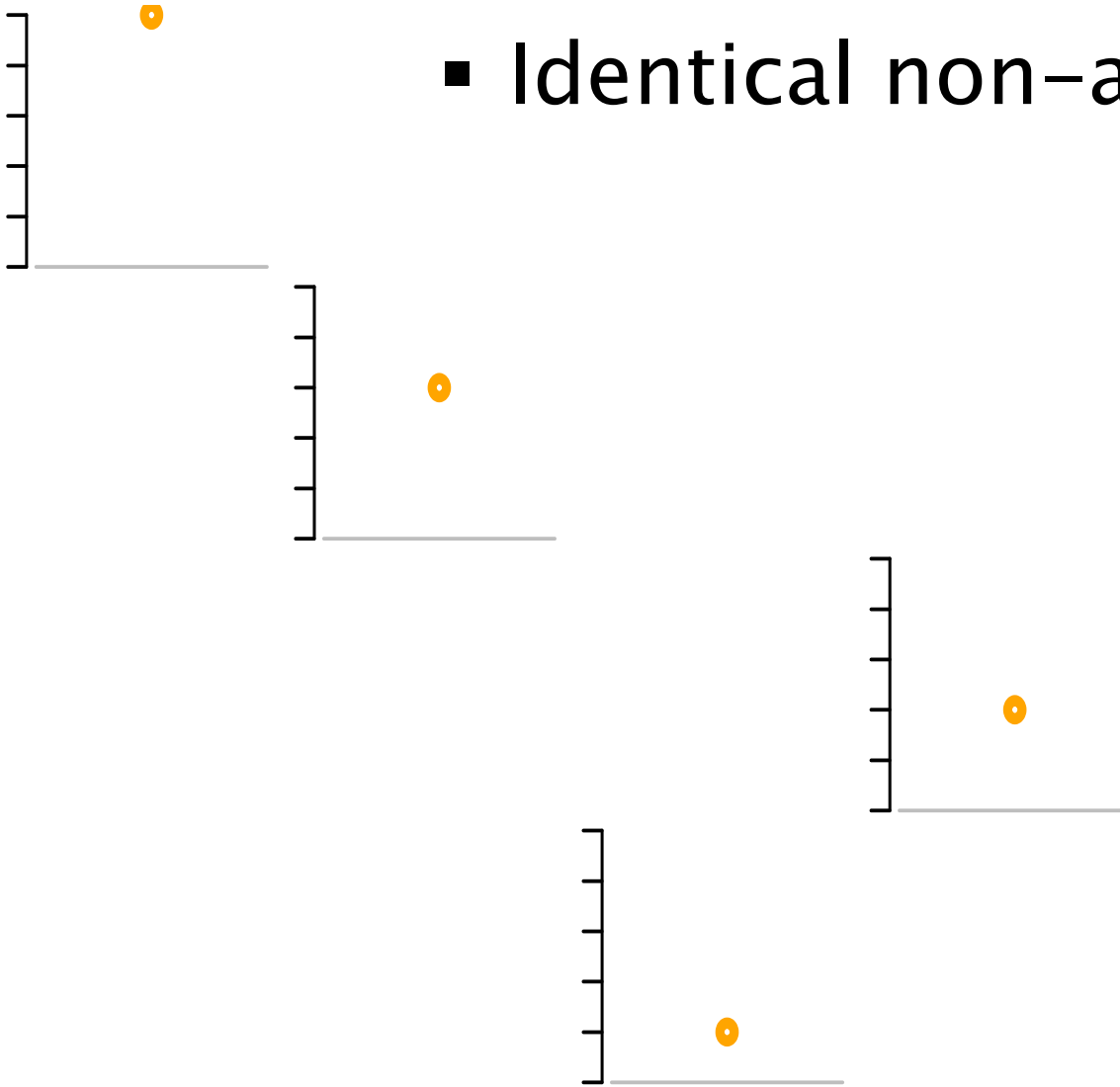
Position

- A common scale



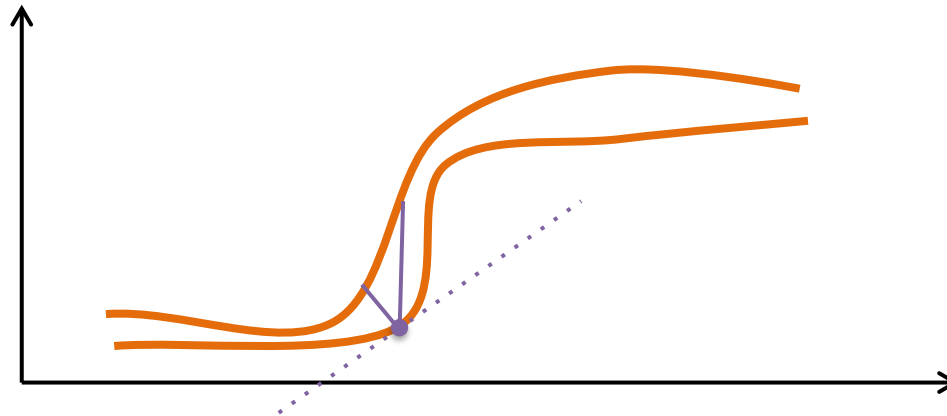
Position

- Identical non-aligned scales



Distance

- Points
 - ◆ Use length of imaginary connecting lines
- Lines
 - ◆ Distance orthogonal to tangent
 - Not what is meant in xy plots



Detection and Separation

Comparison is affected by:

- Detection

- ◆ The capability to visually identify the objects that represent the data to be compared

- Separation

- ◆ The distance between the objects to be compared
 - affects negatively the accuracy

Attributes of color

- Hue



- Saturation



- Intensity



- ♦ Luminance

- ♦ Value

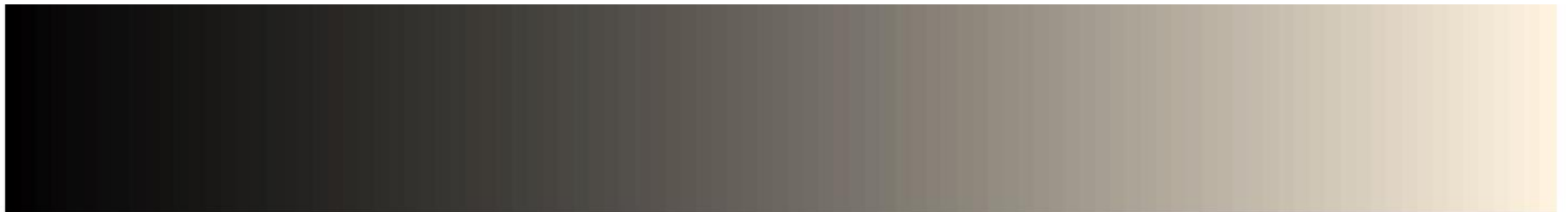
Hue

- There is no common ordering semantics for hues
 - ◆ High spatial frequencies are perceived through intensity changes
 - ◆ Often perceived as separated into bands of almost constant hue, with sharp transitions between hues
- Nominal values can be represented by suitably spaced values



Intensity

- ♦ a.k.a. Luminance, Value
- Provides a perceptually unambiguous ordering
 - ♦ Context can affect accuracy

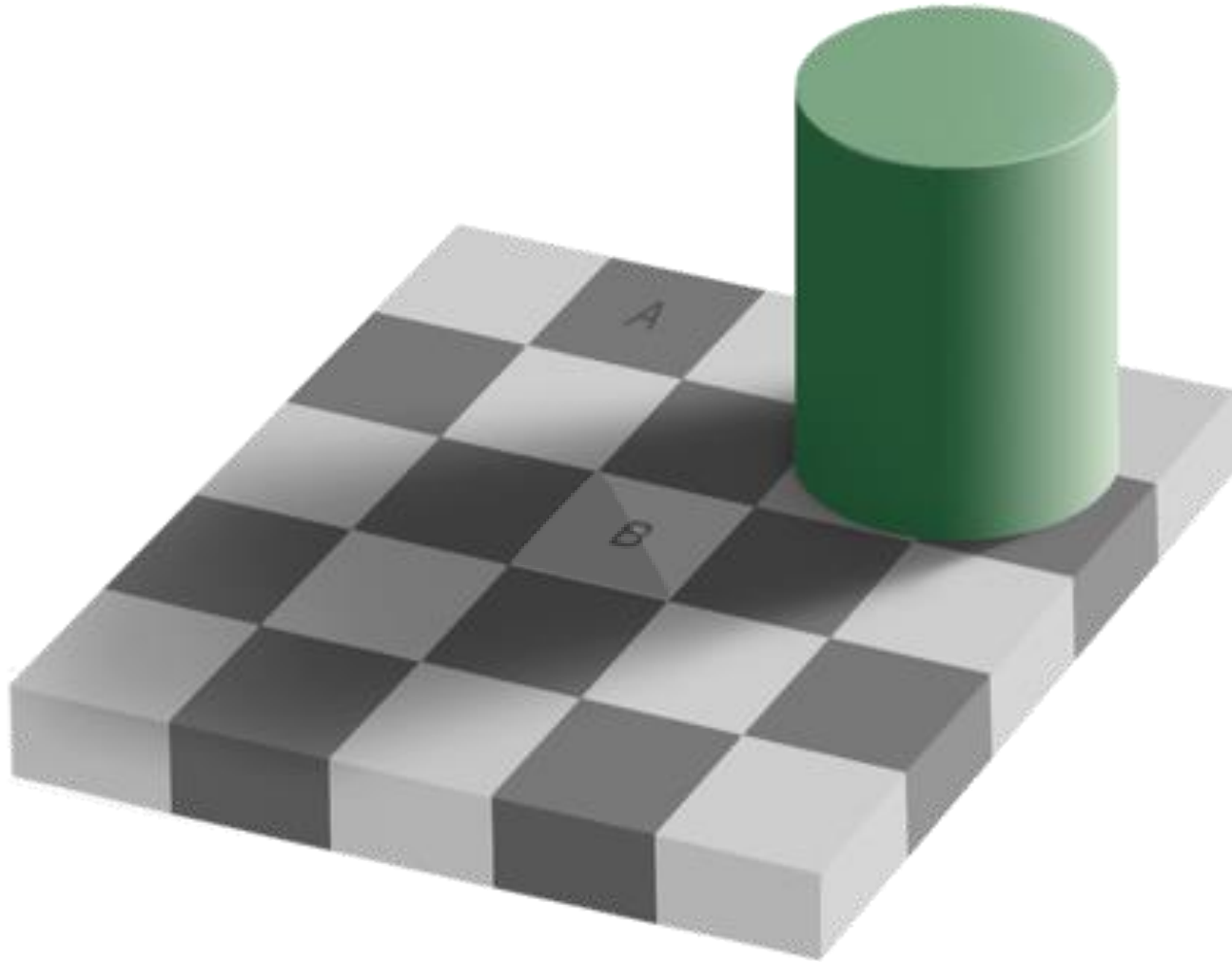


Saturation

- Perceptually difficult to associate an ordered semantics
 - ◆ Can be combined with hue to increase discrimination



Effect of Context

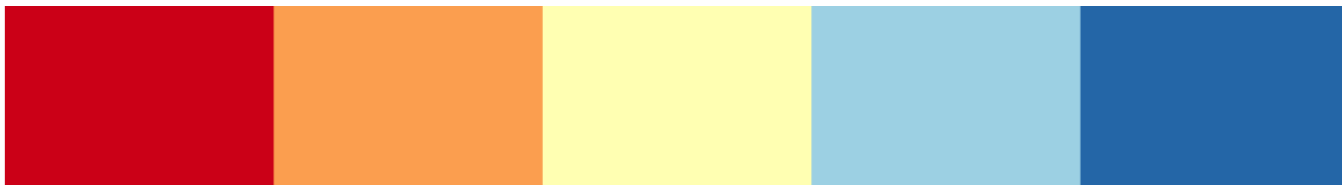


Effect of Context

- Use uniform background
 - ◆ To make distinct visual objects for the same feature look the same
- Use a background color that is contrasting enough with the visual objects' color
 - ◆ To make visual objects easily seen
- Avoid non-uniform background

Color usage

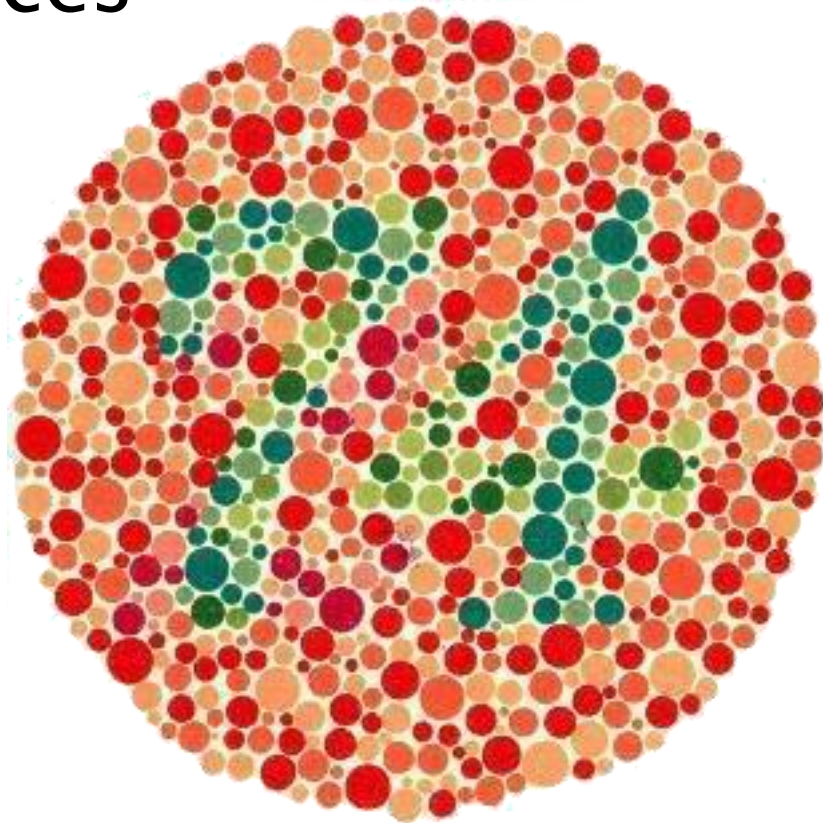
- Ordinal measure should be mapped to increasing saturation **and** intensity
 - ♦ Avoid rainbow palette
- Use sequential or diverging palette
 - ♦ E.g.



– <http://colorbrewer2.org/>

Color Blindness

- Inability to see colors or perceive color differences

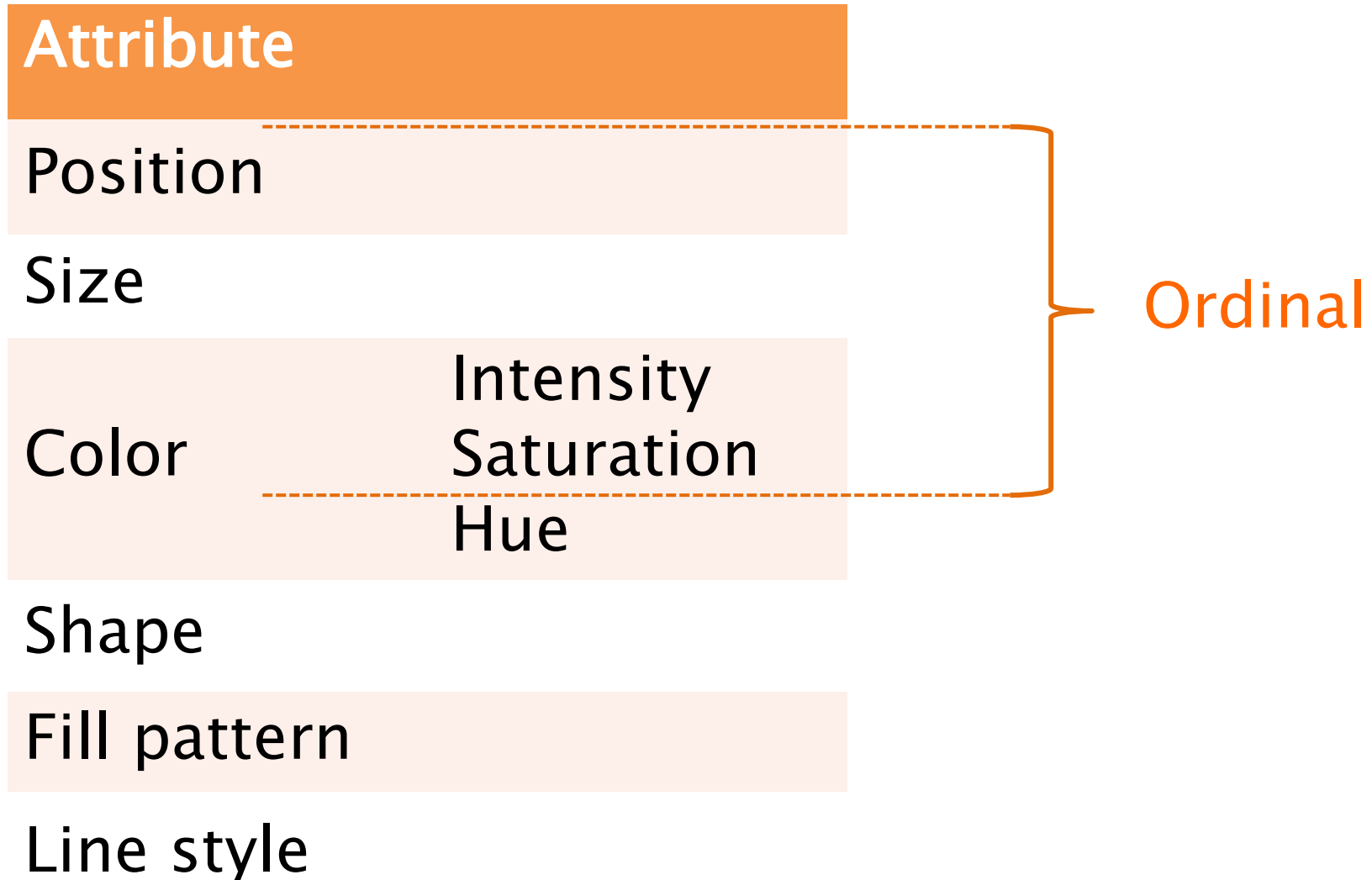


<http://www.color-blindness.com>

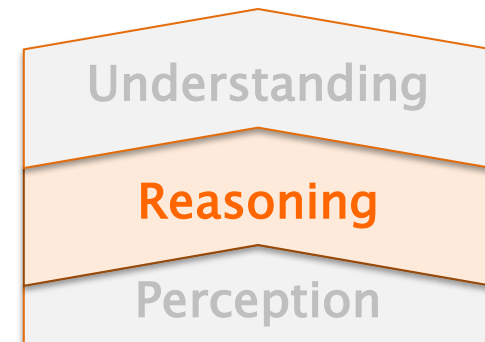
Visual Encoding: Quantitative

Object	Attribute
Point	Position (w.r.t. axis/axes)
Line	Length Position (w.r.t. axis/axes) Slope
Bar	Length
Shape	Size (area) Count

Visual Encoding: Categorical



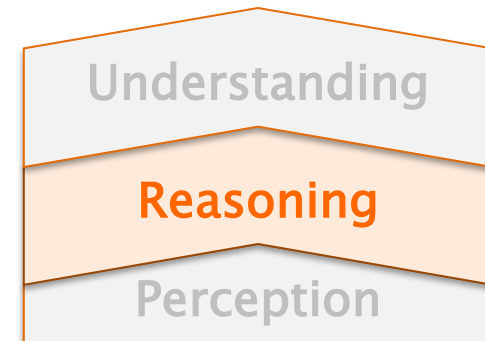
VISUAL REASONING



Graph layout

Layout and visual attributes allow:

- **Discrimination**
 - ◆ Distinguish visual objects or group of –
- **Comparison**
 - ◆ Place visual objects in order
- **Magnitude assessment**
 - ◆ Evaluate the (relative) magnitude of visual objects



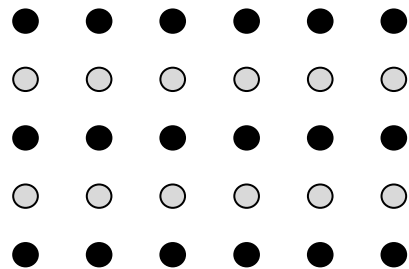
Gestalt principles

- Visual features that lead us to group visual objects together
 - ◆ Proximity
 - ◆ Similarity
 - ◆ Enclosure
 - ◆ Closure
 - ◆ Continuity
 - ◆ Connection

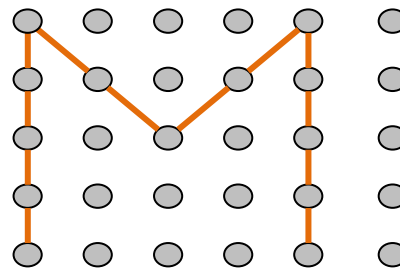
Gestalt principles

- Visual features that lead the viewer to group visual objects together

Similarity



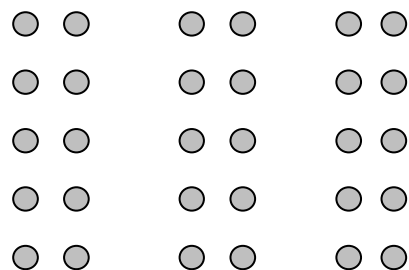
Connection



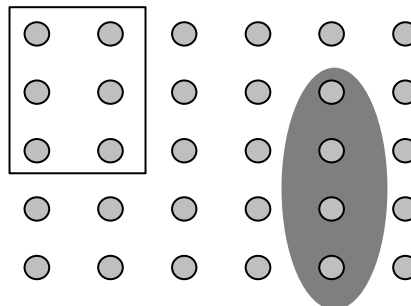
Closure



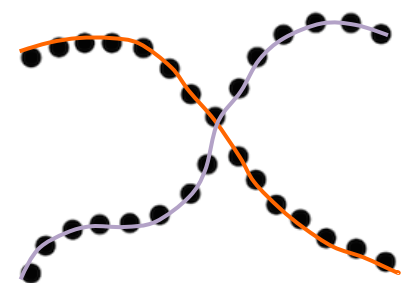
Proximity



Enclosure



Continuity



Gestalt principles

- Visual attributes/patterns that lead observer to group objects together

- ◆ Proximity

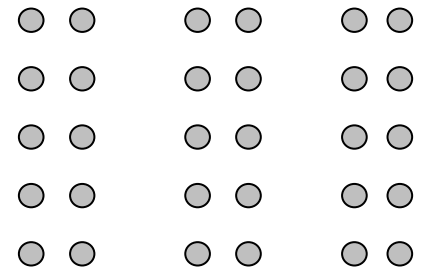
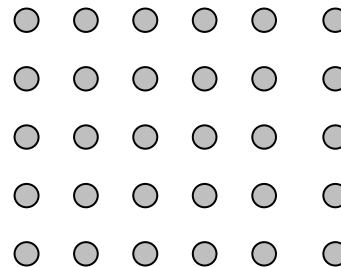
- ◆ Similarity

- ◆ Enclosure

- ◆ Closure

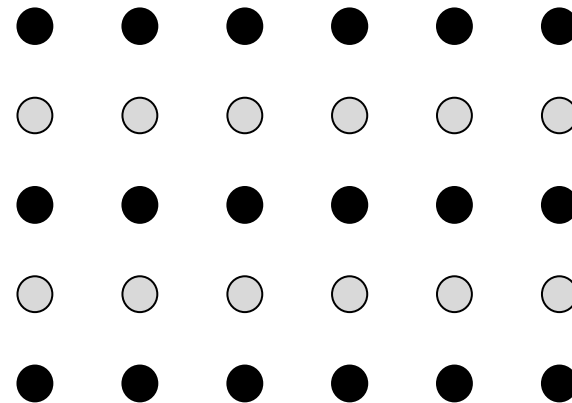
- ◆ Continuity

- ◆ Connection



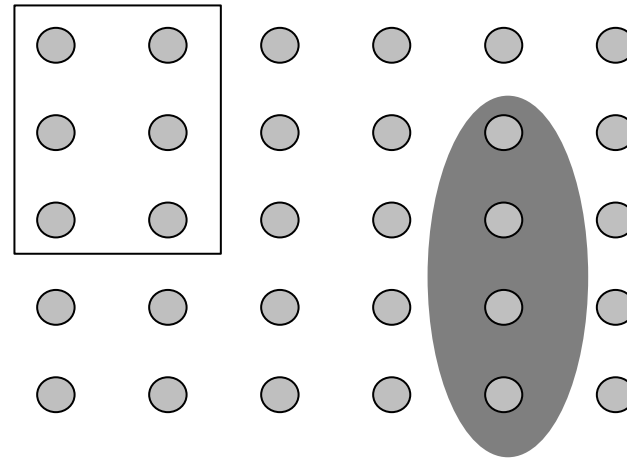
Gestalt principles

- Visual attributes/patterns that lead observer to group objects together
 - ◆ Proximity
 - ◆ **Similarity**
 - ◆ Enclosure
 - ◆ Closure
 - ◆ Continuity
 - ◆ Connection



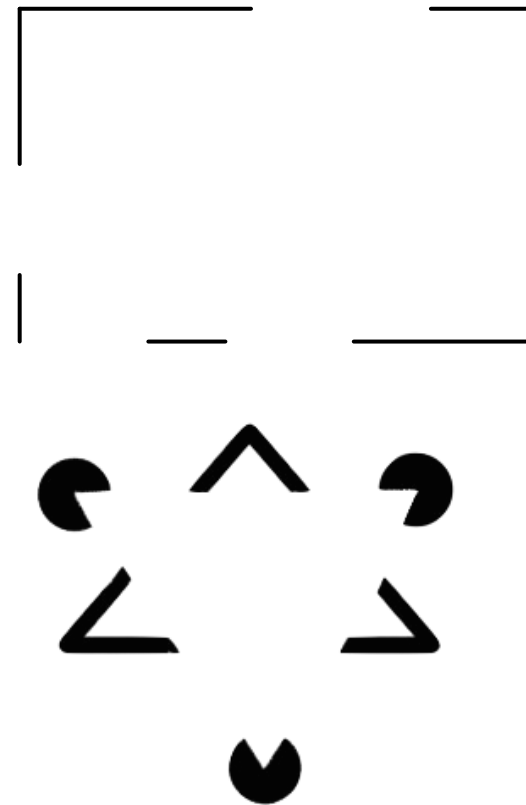
Gestalt principles

- Visual attributes/patterns that lead observer to group objects together
 - ◆ Proximity
 - ◆ Similarity
 - ◆ **Enclosure**
 - ◆ Closure
 - ◆ Continuity
 - ◆ Connection



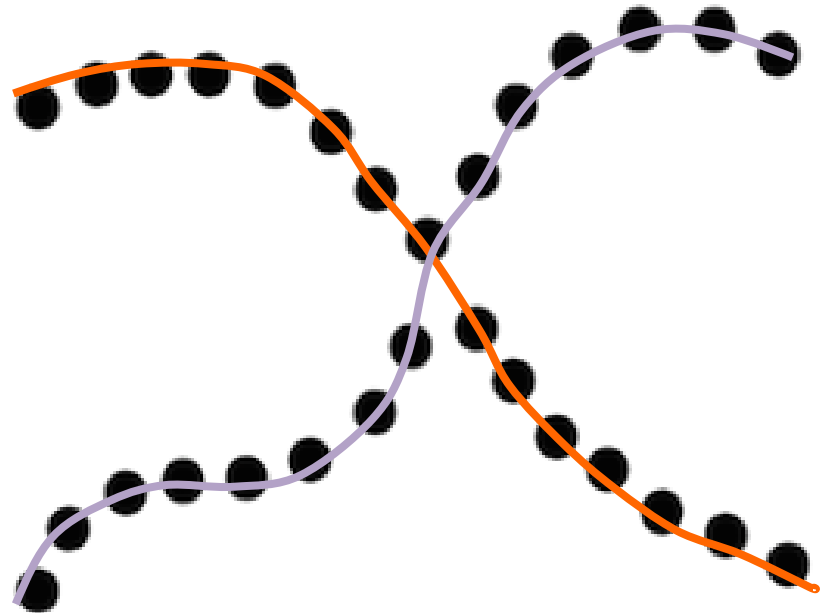
Gestalt principles

- Visual attributes/patterns that lead observer to group objects together
 - ◆ Proximity
 - ◆ Similarity
 - ◆ Enclosure
 - ◆ **Closure**
 - ◆ Continuity
 - ◆ Connection



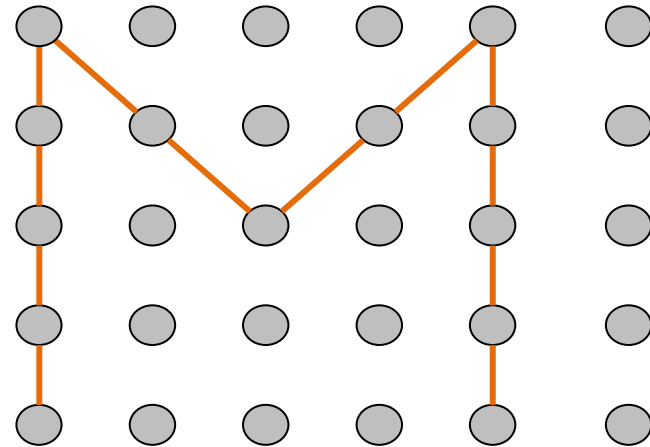
Gestalt principles

- Visual attributes/patterns that lead observer to group objects together
 - ◆ Proximity
 - ◆ Similarity
 - ◆ Enclosure
 - ◆ Closure
 - ◆ **Continuity**
 - ◆ Connection

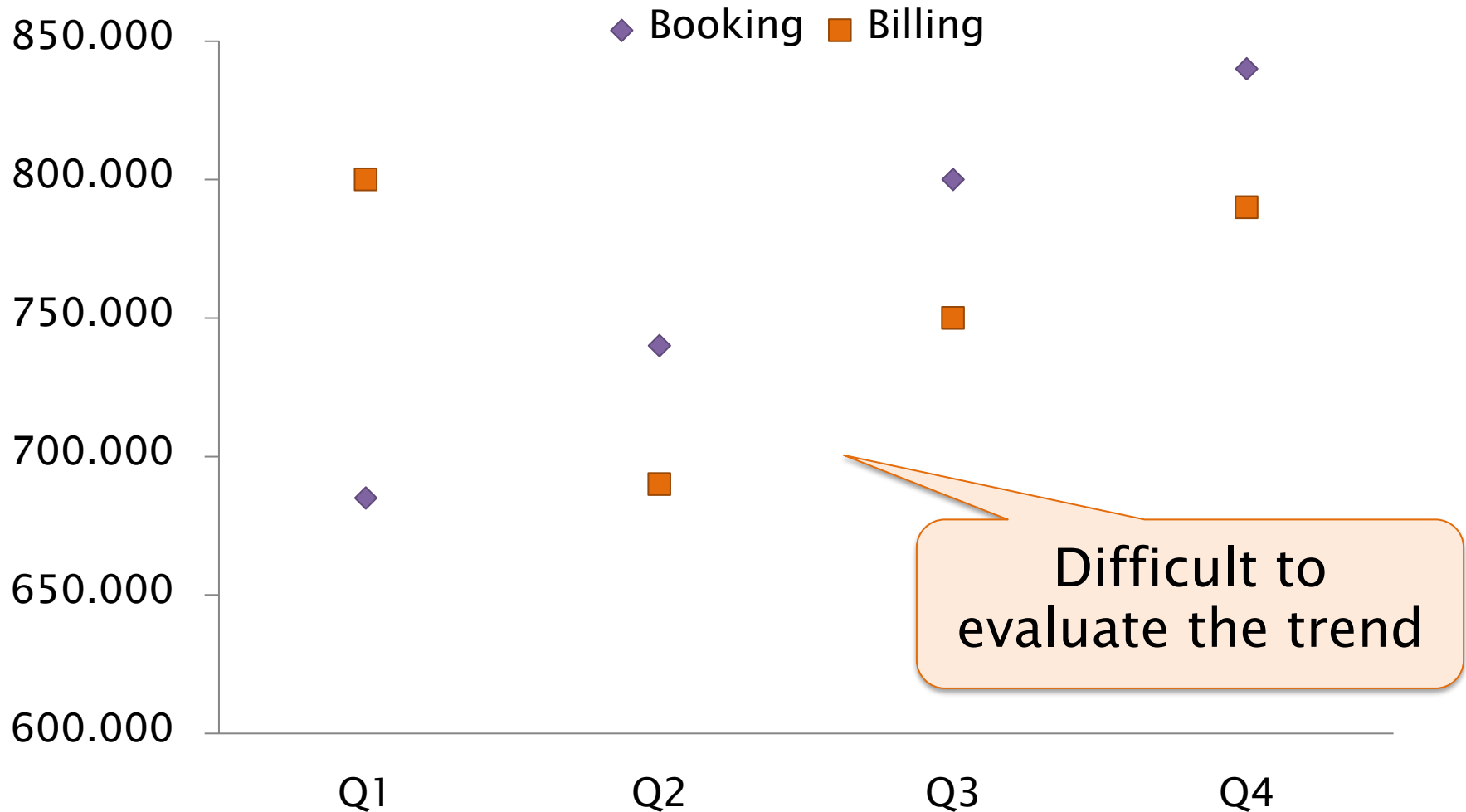


Gestalt principles

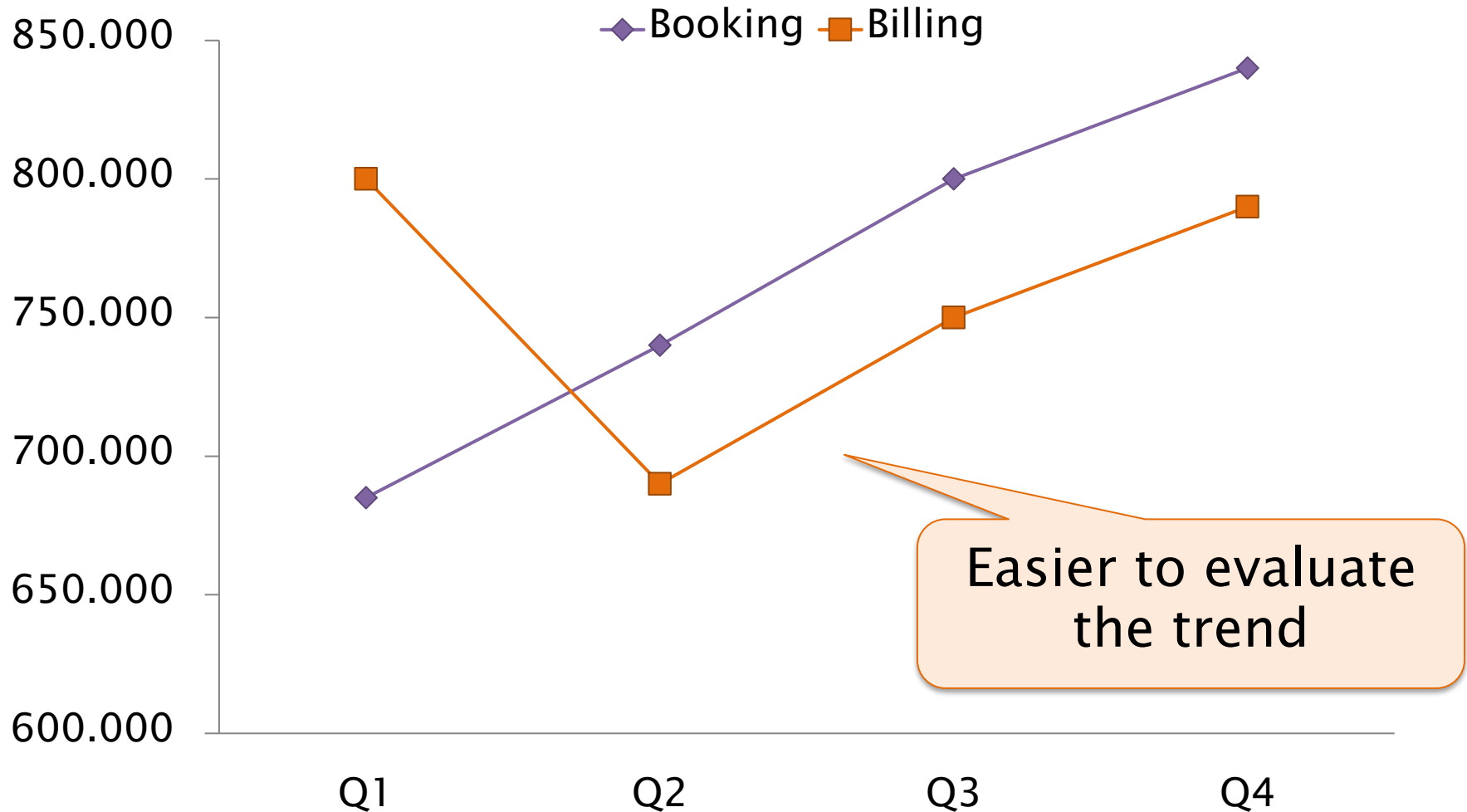
- Visual attributes/patterns that lead observer to group objects together
 - ◆ Proximity
 - ◆ Similarity
 - ◆ Enclosure
 - ◆ Closure
 - ◆ Continuity
 - ◆ **Connection**



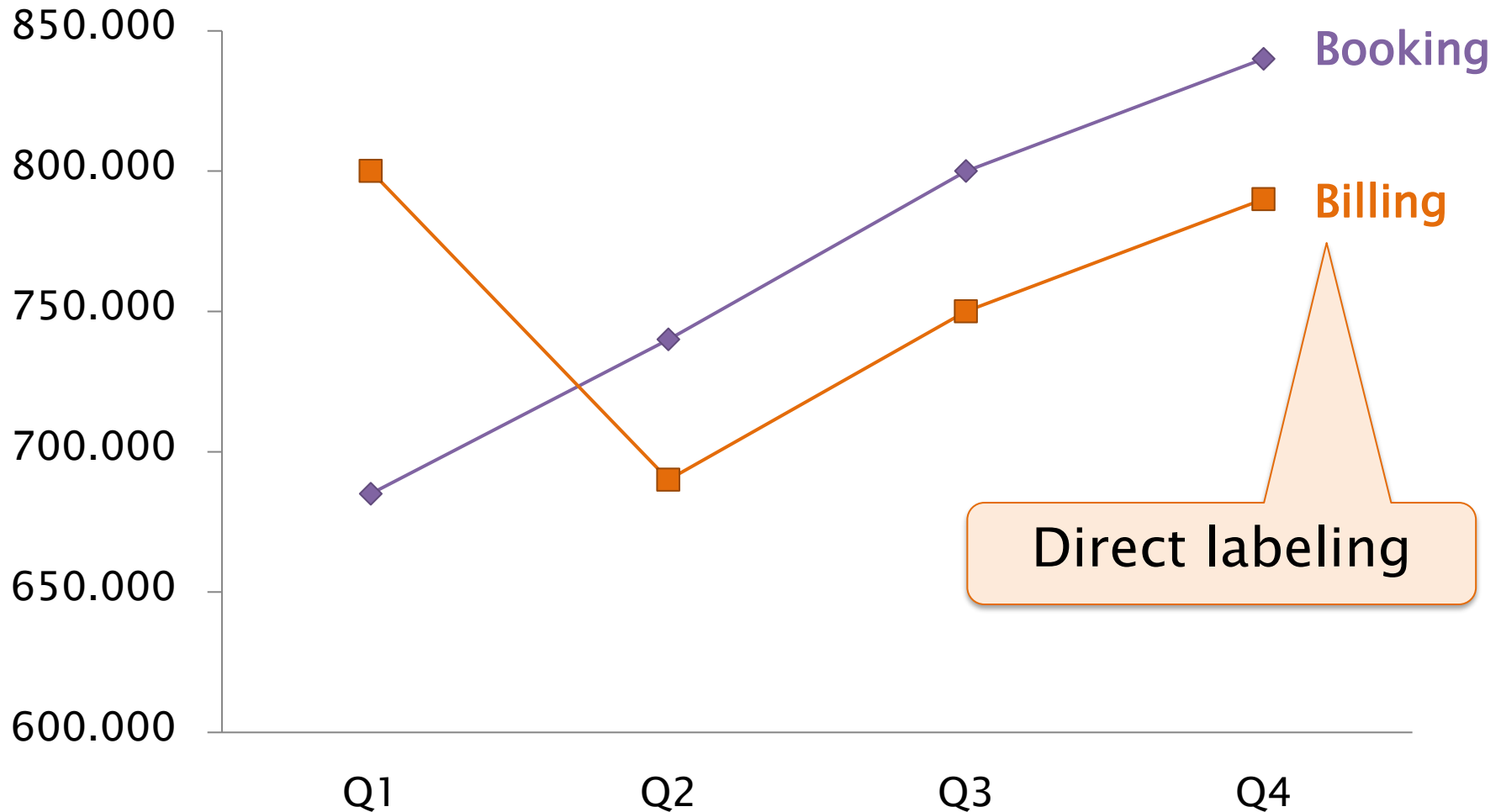
Similarity in Shape & Color



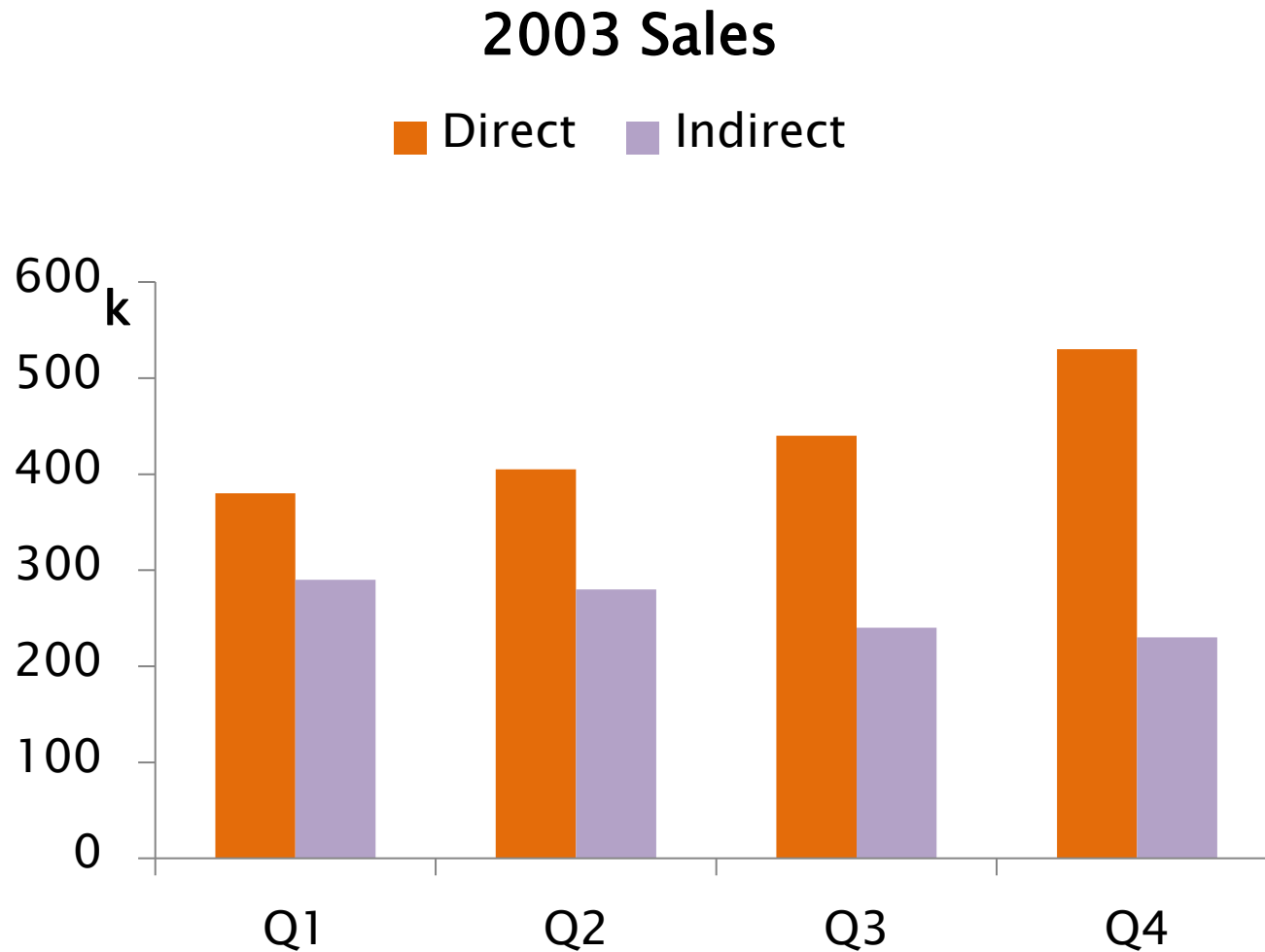
Similarity+Connection



Similarity+Connection+Proximity



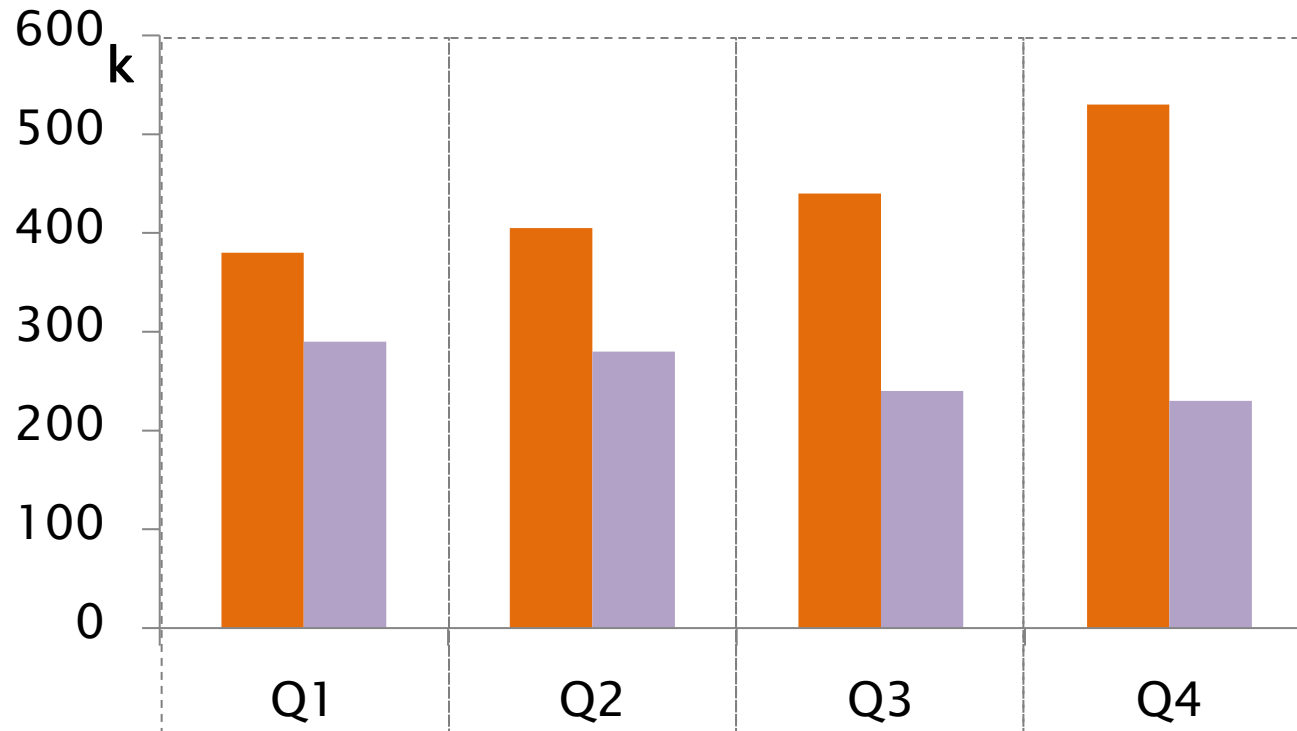
Similarity \times Proximity



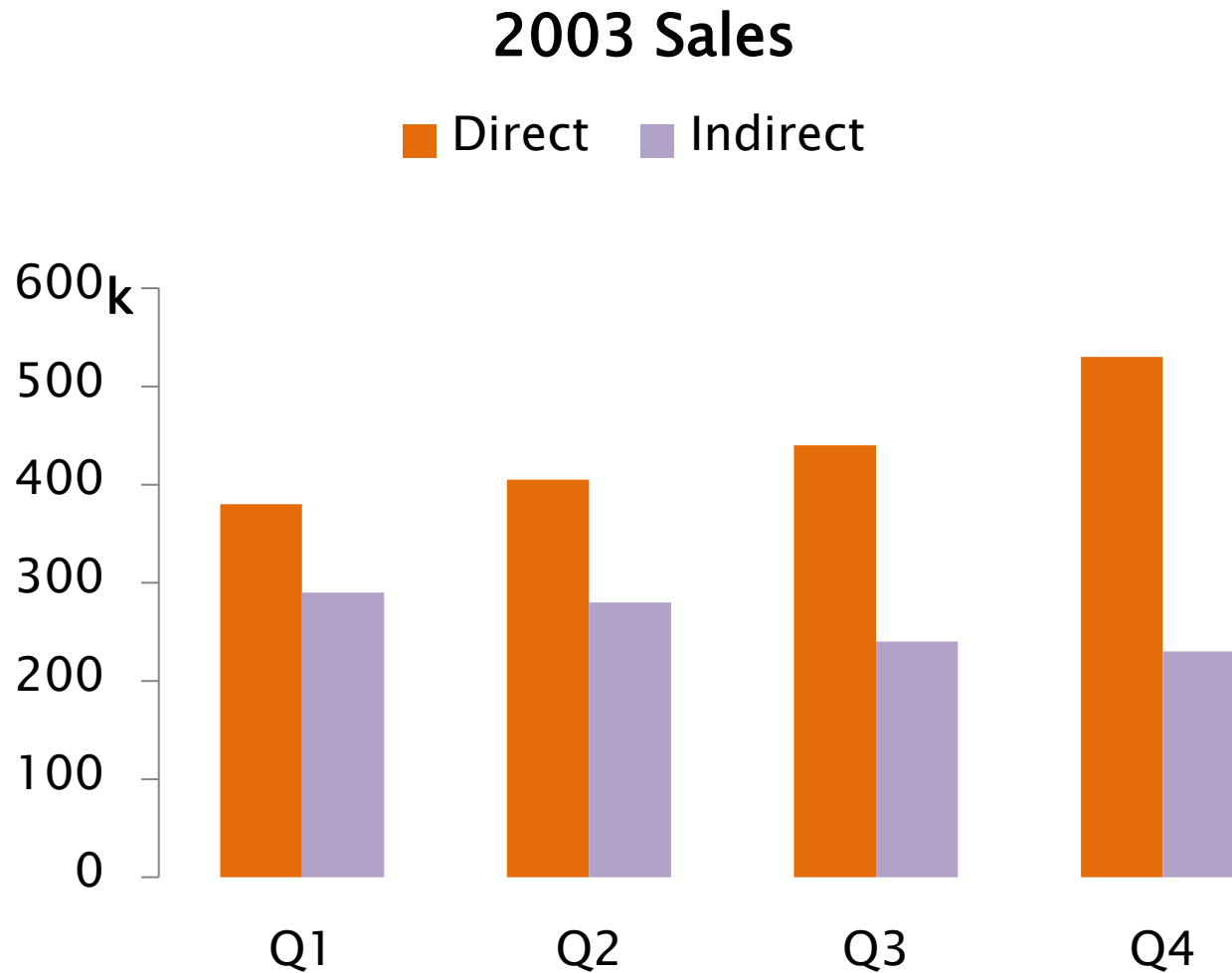
Similarity \times Proximity & Enclosure

2003 Sales

Direct Indirect



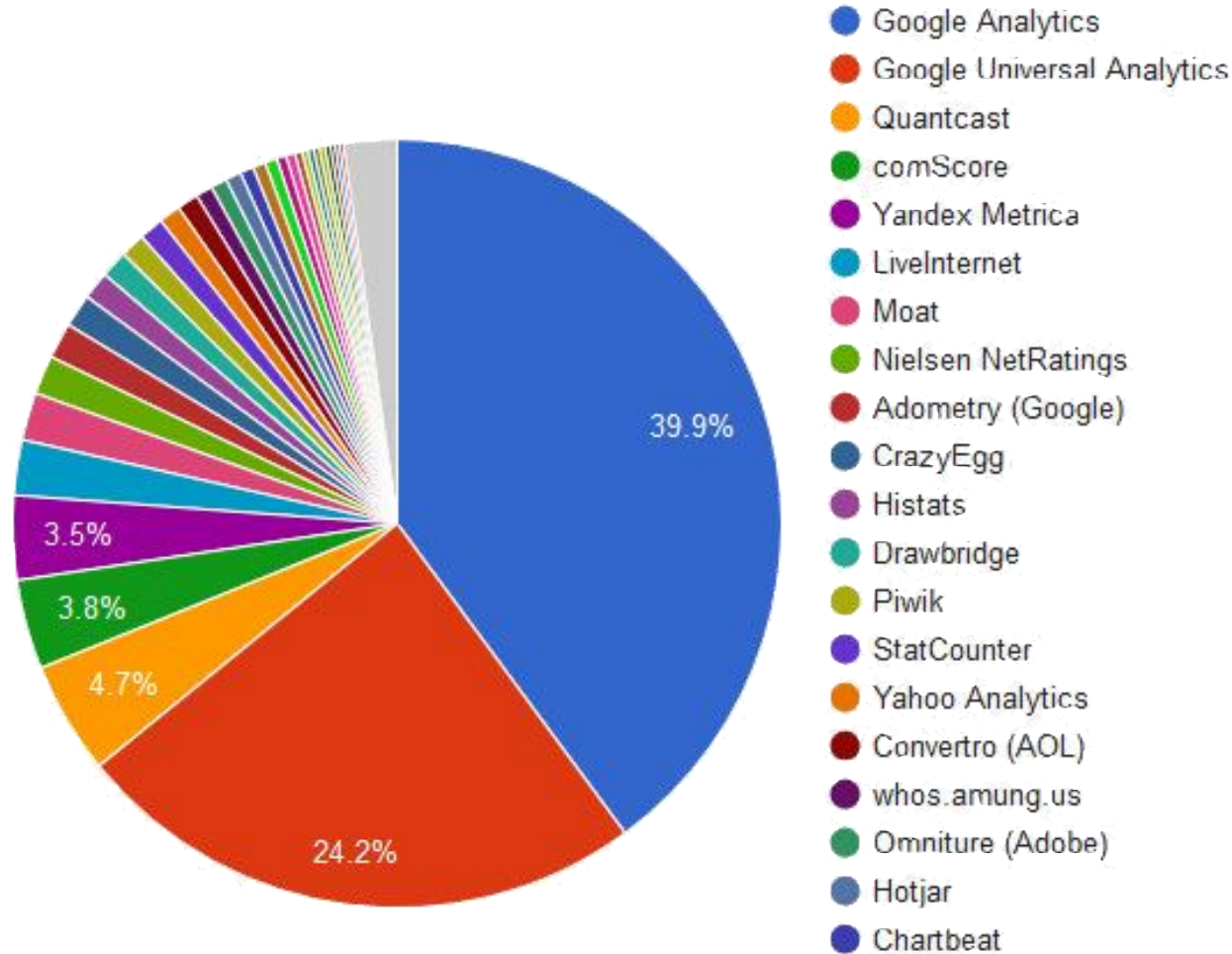
Continuity replaces axis



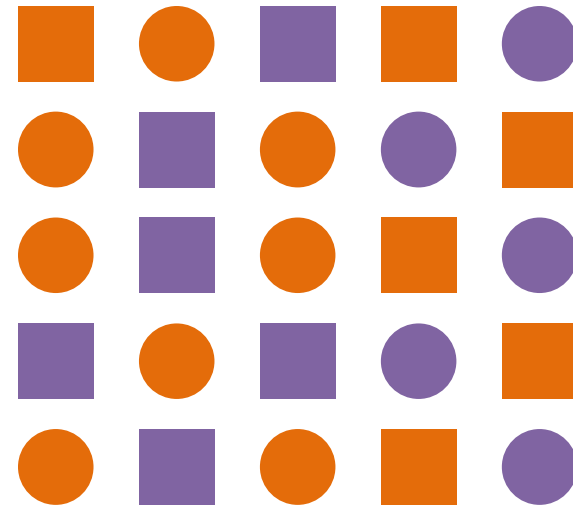
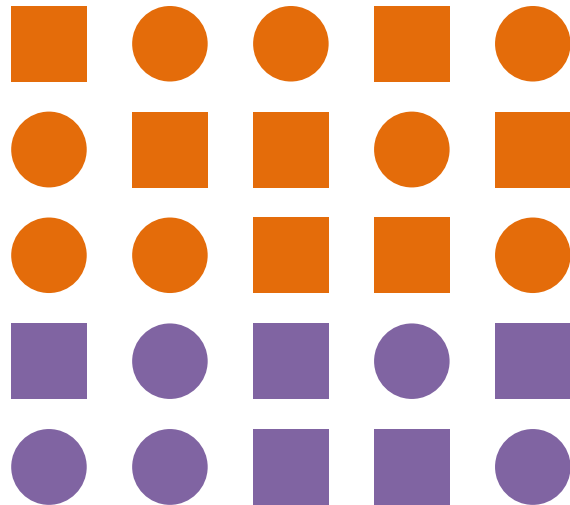
Distinct perceptions

- The immediacy of any pre-attentive cue declines as the variety of alternative patterns increases
 - ◆ Even if all the distracting patterns are individually distinct from the target
 - ◆ For each single attribute no more than **four** distinct levels are discernible

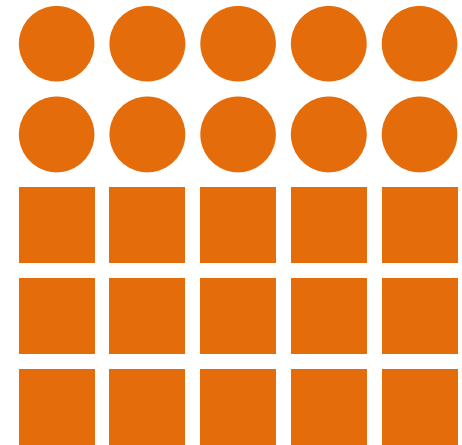
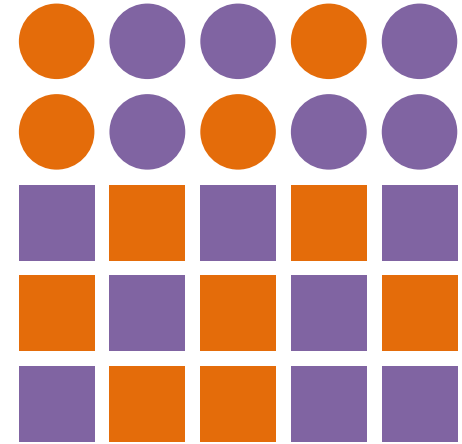
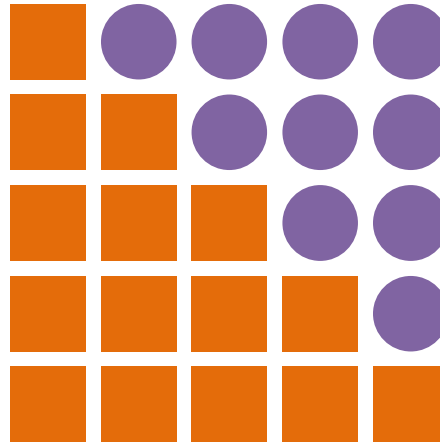
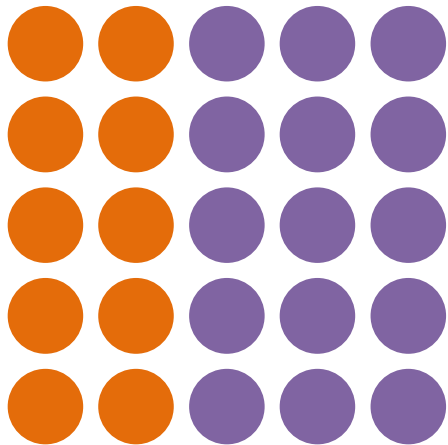
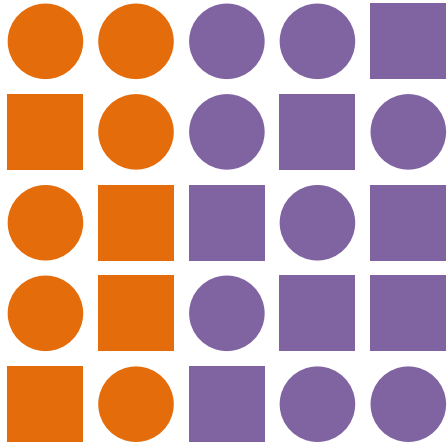
Rainbow Pies



Attribute Interference



Attribute Interference



Cultural conventions

- Reading proceed from left to right and from top to bottom
 - ◆ At least in western culture
- What is at the top (on the left) precedes what is at the bottom (on the right) in terms of
 - ◆ Importance
 - ◆ Ordering
 - ◆ Time

Emphasis

Attribute	Tables	Graphs
Line width	Boldface text	Thicker lines
Size	Bigger tables Larger fonts	Bigger graphs Wider bars Bigger symbols
Color intensity	Darker or brighter colors	
2-D position	Positioned at the top Positioned at the left Positioned in the center	

References

- C. Ware. *Information Visualization: Perception for Design*. Morgan Kaufmann Publishers, Inc., San Francisco, California, 2000
- C. Healey, and J. Enns. *Attention and Visual Memory in Visualization and Computer Graphics*. IEEE Transactions on Visualization and Computer Graphics, 18(7), 2012
- I. Inbar, N. Tractinsky and J. Meyer. Minimalism in information visualization: attitudes towards maximizing the data-ink ratio.
 - ♦ <http://portal.acm.org/citation.cfm?id=1362587>

References

- S.Few, “Practical Rules for Using Color in Charts”
 - ♦ http://www.perceptualedge.com/articles/visual_business_intelligence/rules_for_using_color.pdf
- D. Borland and R. M. Taylor II, "Rainbow Color Map (Still) Considered Harmful," in *IEEE Computer Graphics and Applications*, vol. 27, no. 2, pp. 14–17, March–April 2007.
 - ♦ http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=4118486
- <http://www.color-blindness.com>
- <http://www.csc.ncsu.edu/faculty/healey/PP/index.html>