Politecnico di Torino

1859

Data Science and Machine Learning for Engineering Applications

Numpy: Numerical Python

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- Numpy (*Numerical Python*)
  - Store and operate on **dense** data buffers
  - **Efficient** storage and operations
- Features
  - Multidimensional arrays
  - Slicing/indexing
  - Math and logic operations
- Applications
  - Computation with vectors and matrices
  - Provides fundamental Python objects for data science algorithms
    - Internally used by scikit-learn and SciPy

- **Summary**
  - Numpy and computation **efficiency**
  - Numpy **arrays**
  - **Computation** with Numpy arrays
    - Broadcasting
  - **Accessing** Numpy arrays
  - Working with arrays, other functionalities

Notebook Examples

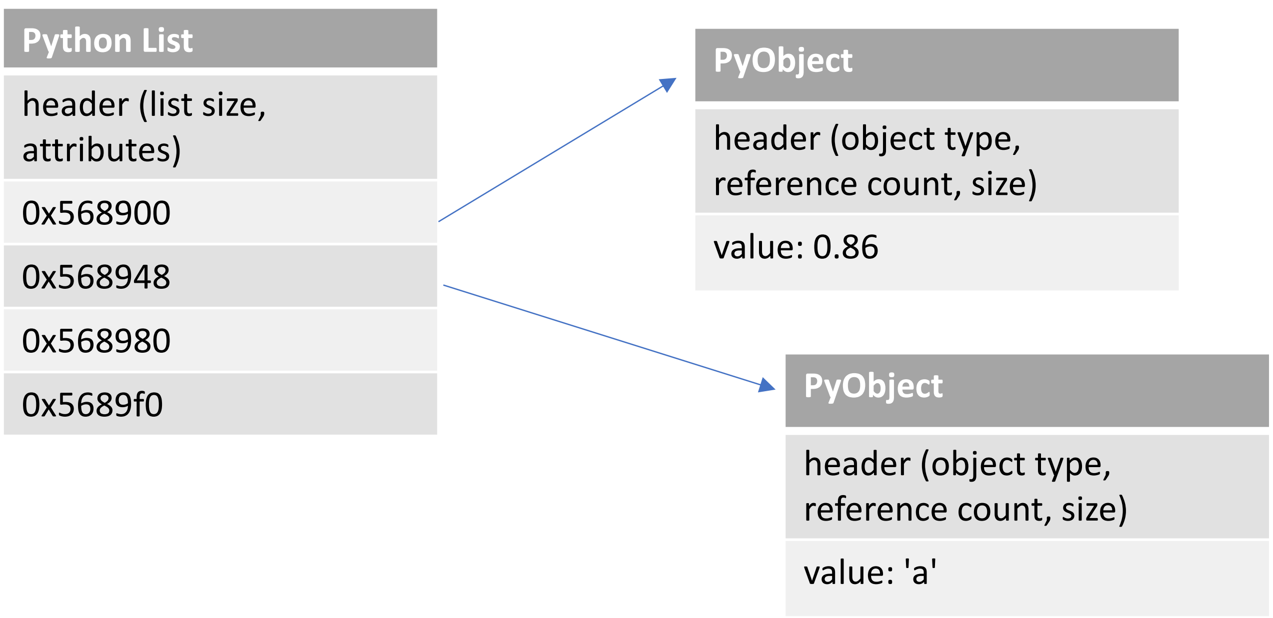
- **0-Install Numpy**

- **array** is the main object provided by Numpy
- Characteristics
  - Fixed Type
    - All its elements have the **same type**
  - Multidimensional
    - Allows representing vectors, matrices and n-dimensional arrays (i.e., a tensor)

Introduction to Numpy

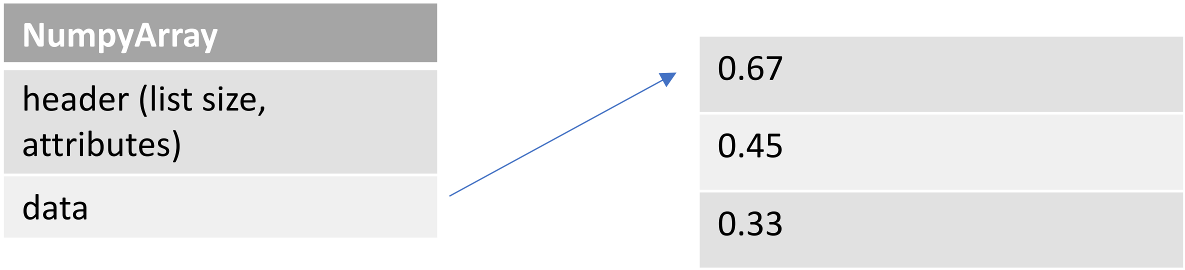
- Numpy arrays vs Python lists:
  - Also Python lists allow defining multidimensional arrays
    - E.g. my_2d_list = [[3.2, 4.0], [2.4, 6.2]]
- Numpy advantages:
  - Higher **flexibility** of indexing methods and operations
  - Higher **efficiency** of operations

- Since lists can contain heterogeneous data types, they keep **overhead** information
  - E.g. my_heterog_list = [0.86, 'a', 'b', 4]



Introduction to Numpy

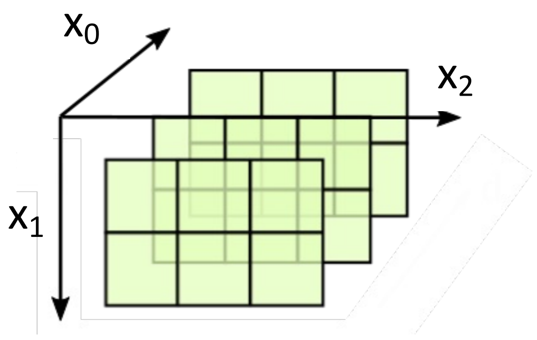
- Characteristics of numpy arrays
  - **Fixed-type** (no overhead)
  - **Contiguous** memory addresses (faster indexing)
  - E.g. my_numpy_array = np.array([0.67, 0.45, 0.33])



- Numpy data types
  - Numpy defines its own data types
  - Numerical types
    - int8, int16, int32, int64
    - uint8, … , uint64
    - float16, float32, float64
  - Boolean values
    - bool

Multidimensional arrays

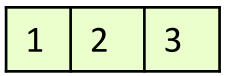
- Collections of elements organized along an arbitrary number of **dimensions**
- Multidimensional arrays can be represented with
  - Python lists
  - Numpy arrays



Multidimensional arrays

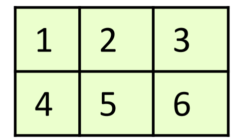
- Multidimensional arrays with **Python lists**
  - Examples:

vector



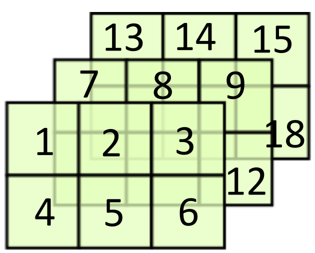

```
list1 = [1, 2, 3]
```

2D matrix




```
list2 = [[1,2,3], [4,5,6]]
```

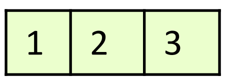
3D array

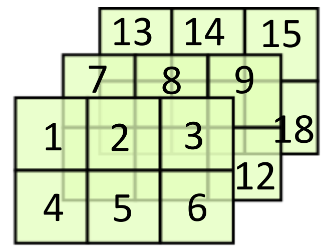



```
list3 = [[[1,2,3], [4,5,6]],
          [[7,8,9], [10,11,12]],
          [[13,14,15], [16,17,18]]]
```

Multidimensional arrays

- Multidimensional arrays with **Numpy**
  - Can be directly created from Python lists
  - Examples:

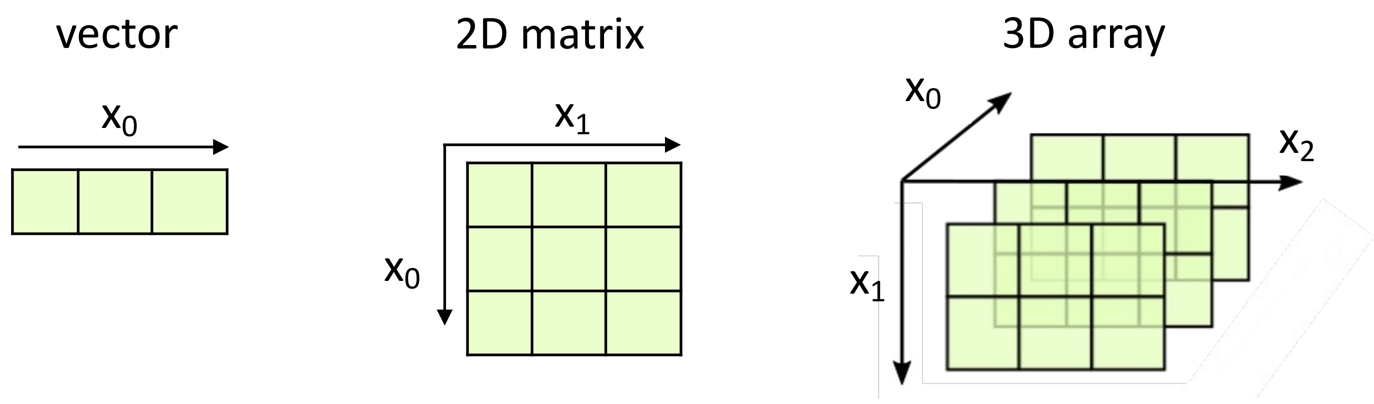




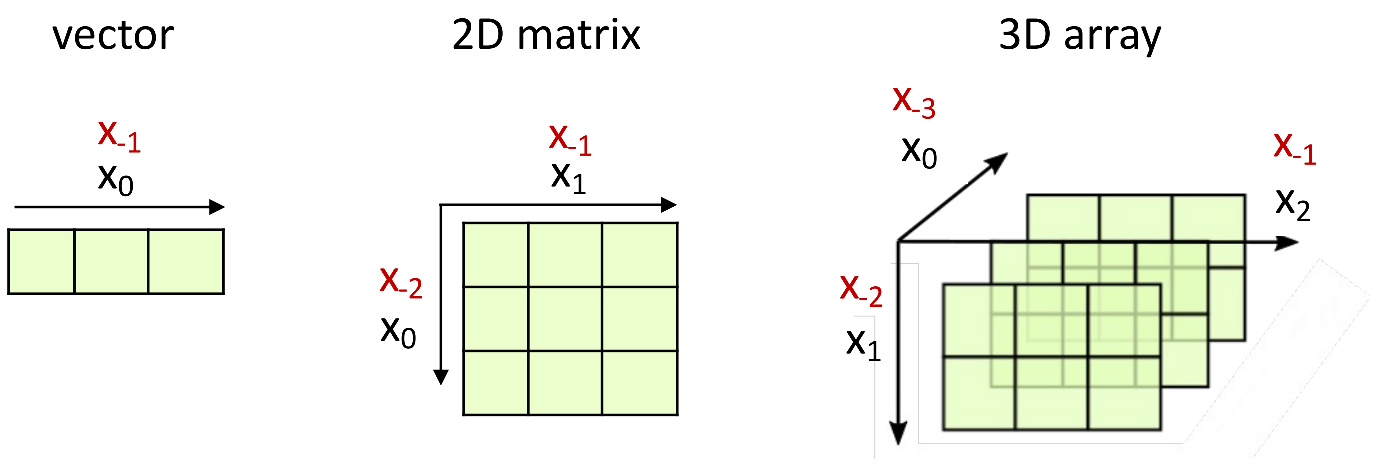

```python
import numpy as np

arr1 = np.array([1, 2, 3])
```

```python
import numpy as np

arr2 = np.array([[[1,2,3], [4,5,6]],
                 [[7,8,9], [10,11,12]],
                 [[13,14,15], [16,17,18]]])
```

- Multidimensional arrays with **Numpy**
  - Characterized by a set of **axes** and a **shape**
  - The **axes** of an array define its dimensions
    - a (row) vector has 1 axis (1 dimension)
    - a 2D matrix has 2 axes (2 dimensions)
    - a ND array has N axes

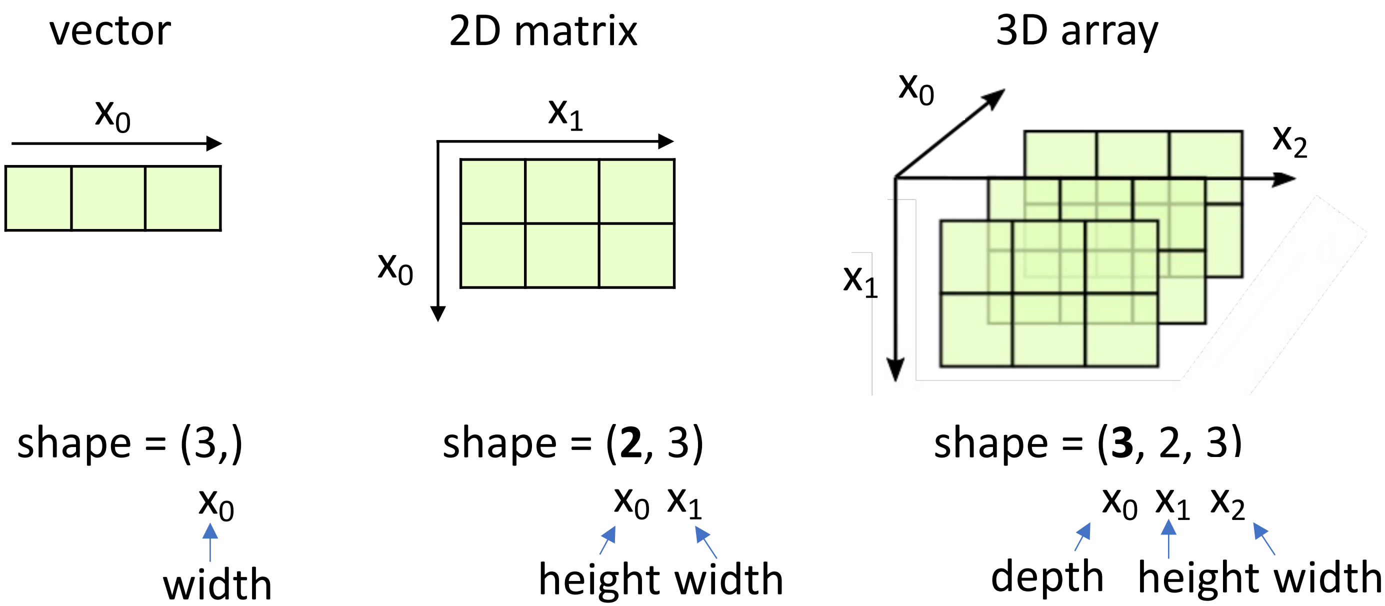


- Multidimensional arrays with **Numpy**
  - Axes can be numbered with negative values
  - Axis -1 is always along the **row**



Multidimensional arrays

- Multidimensional arrays with **Numpy**
  - The **shape** of a Numpy array is a tuple that specifies the number of elements along each axis
    - Examples:

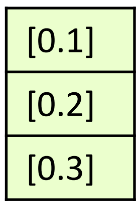


Multidimensional arrays

- Column vector vs row vector

Column Vector

e.g. np.array([[0.1], [0.2], [0.3]])

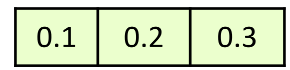


shape = (3, **1**)

Column vector is a 2D matrix!

Row Vector

e.g. np.array([0.1, 0.2, 0.3])



shape = (3,)

- Creation from list:
  - np.array(my_list, dtype=np.float16)
    - Data type inferred if not specified
- Creation from scratch:
  - np.zeros(shape)
    - Array with all 0 of the given shape
  - np.ones(shape)
    - Array with all 1 of the given shape
  - np.full(shape, value)
    - Array with all elements to the specified value, with the specified shape

Notebook Examples

- **1-Numpy Examples.ipynb**
  - **1) Create Arrays**

Numpy arrays

- Creation from scratch: examples

In [1]:
```
np.ones((2,3))
```

Out[1]:
```
[[1, 1, 1],
 [1, 1, 1]]
```

In [2]:
```
np.full((2,1)), 1.1)
```

Out[2]:
```
[[1.1],
 [1.1]]
```

Numpy arrays

- Creation from scratch:
  - np.linspace(start, stop, num)
    - Generates *num* samples from *start* to *stop* (included)
    - np.linspace(0,1,11) → [0.0, 0.1, … , 1.0]
  - np.arange(start, stop, step)
    - Generates numbers from *start* to *stop* (excluded), with step *step*
    - np.arange(1, 7, 2) → [1, 3, 5]
  - np.random.normal(mean, std, shape)
    - Generates random data with normal distribution
  - np.random.random(shape)
    - Random data uniformly distributed in [0, 1]

Numpy arrays

- Main attributes of a Numpy array
  - Consider the array
    - x = np.array([[2, 3, 4],[5,6,7]])
  - **x.ndim:** number of dimensions of the array
    - Out: 2
  - **x.shape**: tuple with the array shape
    - Out: (2,3)
  - **x.size**: array size (product of the shape values)
    - Out: 2*3=6

Summary:

- **Universal functions** (Ufuncs):
  - **Binary** operations (+,−,*,...)
  - **Unary** operations (exp(),abs(),...)
- **Aggregate** functions
- **Sorting**
- **Algebraic** operations (dot product, inner product)

- **Universal functions** (Ufuncs): element-wise operations
  - **Binary** operations with arrays of the **same shape**
    - +, -, *, /, % (modulus), // (floor division), ** (exponentiation)

Computation on Numpy

- Example:

In [1]:

```
x=np.array([[1,1],[2,2]])
y=np.array([[3, 4],[6, 5]])
x*y
```

Out[1]:

```
[[3, 4], [12, 10]]
```

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} * \begin{bmatrix} 3 & 4 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} 1*3 & 1*4 \\ 2*6 & 2*5 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 12 & 10 \end{bmatrix}$$

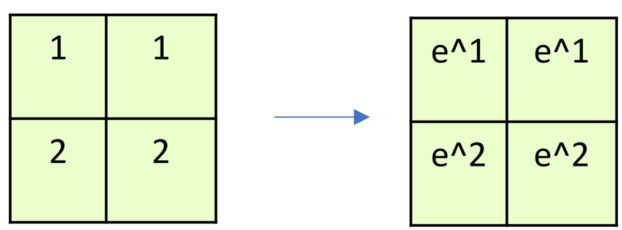
- **Universal functions** (Ufuncs):
  - **Unary** operations
    - np.abs(x)
    - np.exp(x), np.log(x), np.log2(x), np.log10(x)
    - np.sin(x), cos(x), tan(x), arctan(x), …
  - They apply the operation separately to each element of the array

Computation on Numpy

- Example:

In [1]:
```
x=np.array([[1,1],[2,2]])
np.exp(x)
```

Out[1]:
```
[[2.718, 2.718],[7.389, 7.389]]
```



- **Note: original array (x) is not modified**

- **Aggregate functions**
  - **Return** a single value from an array
    - np.min(x), np.max(x), np.mean(x), np.std(x), np.sum(x)
    - np.argmin(x), np.argmax(x)
  - Or equivalently:
    - x.min(), x.max() x.mean(), x.std(), x.sum()
    - x.argmin(), x.argmax()
  - Example

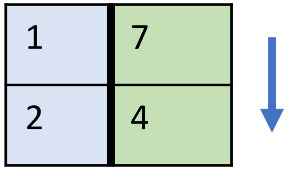
```
In [1]:  x=np.array([[1,1],[2,2]])
         x.sum()

Out[1]:  6
```

Aggregate functions along axis

- Allow specifying the **axis** along with performing the operation
- Examples

In [1]:
```
x=np.array([[1,7],[2,4]])
x.sum(axis=0)
```

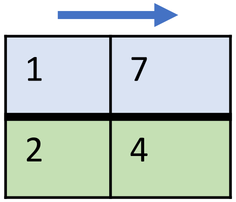
Out[1]:
```
[3, 11]
```



IMP: both aggregate functions return a row vector!

In [2]:
```
x.sum(axis=1)     # or axis=-1
```

Out[2]:
```
[8, 6]
```
→ (sum the elements of each row)



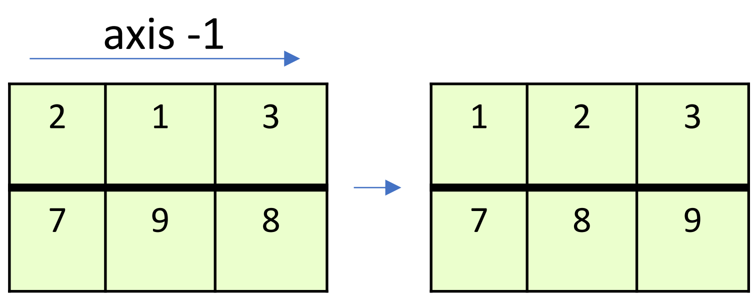
- **Sorting**
  - **np.sort(x):** creates a sorted copy of x
    - x is not modified
  - **x.sort():** sorts x inplace (x is modified)

Sorting

- Array is sorted along the last axis (-1) by default

In [1]:
```
x = np.array([[2,1,3],[7,9,8]])
np.sort(x)          # Sort along rows (axis -1)
```

Out[1]:
```
[[1,2,3],[7,8,9]]
```

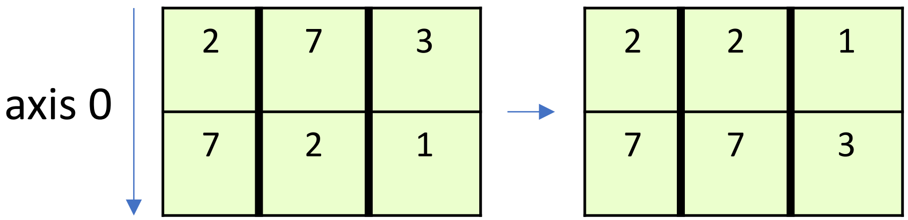


Sorting

- Allows specifying the axis being sorted

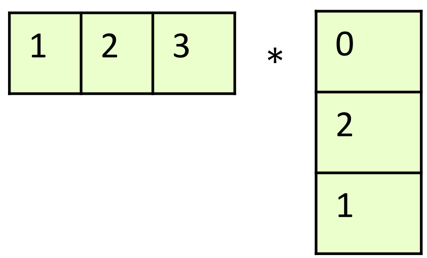
In [1]:
```
x = np.array([[2,7,3],[7,2,1]])
np.sort(x, axis=0)    # Sort along columns
```

Out[1]:
```
[[2,2,1],
[7,7,3]]
```



Algebraic operations

- np.dot(x, y)
  - inner product if x and y are two 1-D arrays



= 1*0 + 2*2 + 3*1 = 7

In [1]:
```
x=np.array([1, 2, 3])
y=np.array([0, 2, 1])  # works even if y is a row vector
np.dot(x, y)
```

Out[1]:
```
7
```

Algebraic operations

- np.dot(x, y)
  - matrix multiplied by vector

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} * \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1*2 + 1*3 & 2*2 + 2*3 \end{bmatrix} = \begin{bmatrix} 5 & 10 \end{bmatrix}$$

In [1]:

```
x=np.array([[1,1],[2,2]])
y=np.array([2, 3]) # works even if y is a row vector
np.dot(x, y)
```

Out[1]:

```
[5, 10]    # result is a row vector
```

- **Algebraic operations**
  - np.dot(x, y)
    - matrix multiplied by matrix

1	1
2	2

*

2	2
1	1

=

3	3
6	6

In [1]:

```
x=np.array([[1,1],[2,2]])
y=np.array([[2,2],[1,1]])
np.dot(x, y)
```

Out[1]:

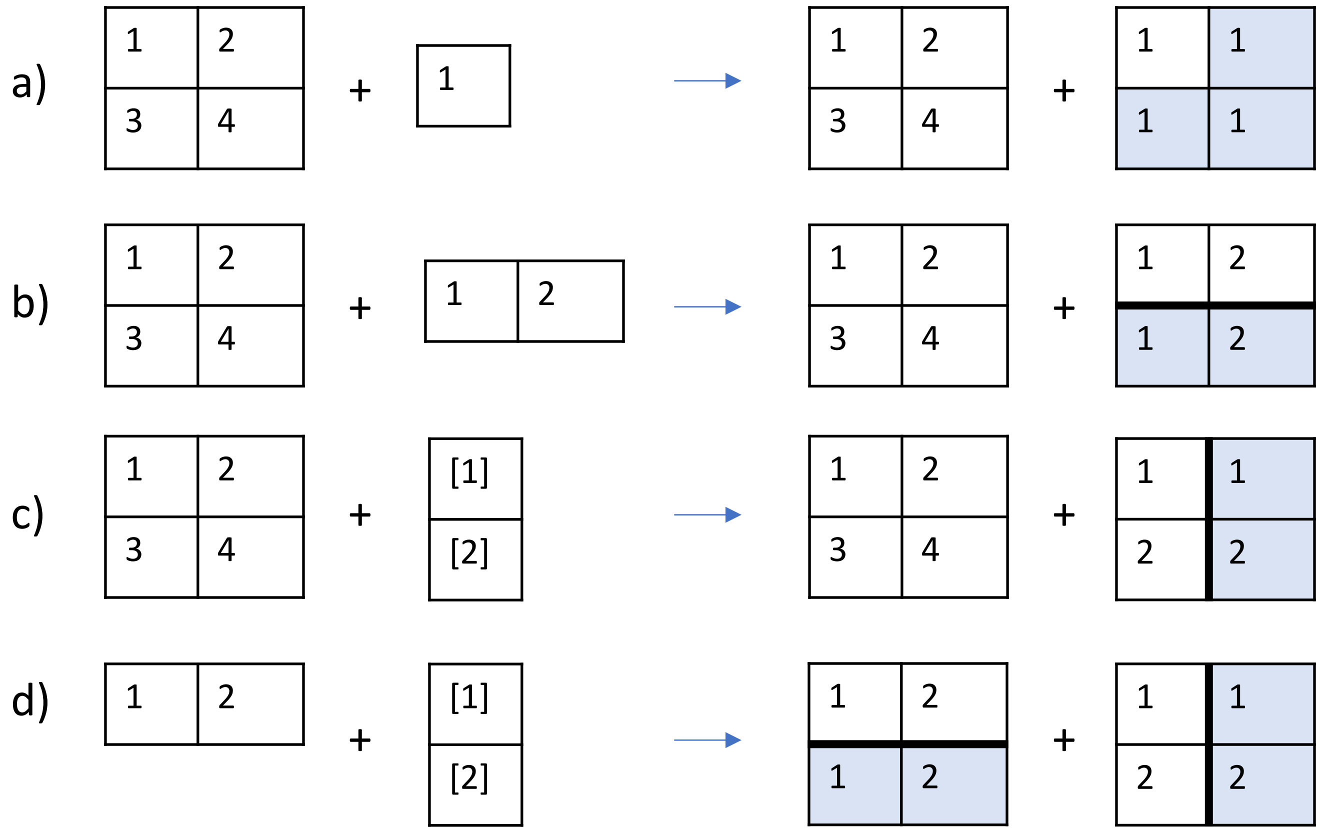
```
[[3,3],[6,6]]
```

Notebook Examples

- **1-Numpy Examples.ipynb**
  - **2) Computation with arrays**

Broadcasting

- Pattern designed to perform operations between arrays with **different shape**

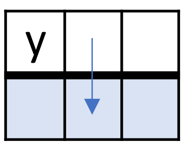


Broadcasting

- Rules of broadcasting
  1. The shape of the array with **fewer dimensions** is **padded** with <u>leading</u> ones

     x.shape = (2, 3), y.shape = (3) → y.shape = (**1**, 3)

  2. If the shape along a dimension is 1 for one of the arrays and >1 for the other, the array with shape = 1 in that dimension is **stretched to match the other array**

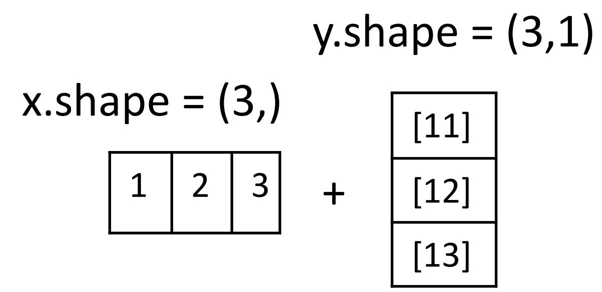


     x.shape = (2, 3), y.shape = (**1**, 3) → stretch: y.shape = (**2**, 3)

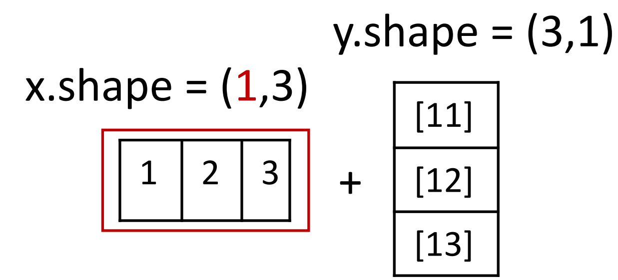
  3. If there is a dimension where both arrays have shape >1 and those shapes differ, then broadcasting **cannot be performed**

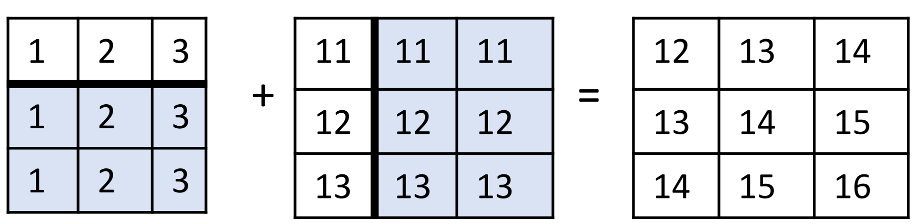
Broadcasting

- Example: compute x + y
  - x = np.array([1, 2, 3])
  - y = np.array([[11], [12], [13]])
  - z = x + y



- Apply Rule 1
  - x.shape becomes (**1**, 3): x=[[1,2,3]]
- Apply Rule 2:
  - extend x on the vertical axis, y on the horizontal one





Broadcasting

- Example: compute x + y
  - x = np.array([[1, 2],[3,4],[5,6]])   x.shape = (3, 2)
  - y = np.array([11, 12, 13])   y.shape = (3,)
  - z = x + y

  - Apply Rule 1
    - y.shape becomes (**1**, 3): y=[[11,12,13]]
  - Apply Rule 3
    - shapes (3, **2**) and (1, **3**) are incompatibles
    - Numpy will raise an **exception**

11	12	13

1	2
3	4
5	6

Notebook Examples

- **1-Numpy Examples.ipynb**
  - **3) Broadcasting examples**

Accessing Numpy Arrays

- Numpy arrays can be accessed in many ways
  - Simple indexing
  - Slicing
  - Masking
  - Fancy indexing (not covered)
  - Combined indexing (not covered)
- Slicing provides **views** on the considered array
  - Views allow **reading** and **writing** data on the **original** array
- Masking and fancy indexing provide **copies** of the array

Accessing Numpy Arrays

- **Simple indexing**: read/write access to element
  - x[i, j, k, …]

In [1]:
```
x = np.array([[2, 3, 4],[5,6,7]])
el = x[1, 2]          # read value (indexing)
print("el =", el)
```

Out[1]:
```
el = 7
```

In [2]:
```
x[1, 2] = 1           # assign value
print(x)
```

Out[2]:
```
[[2, 3, 4], [5, 6, 1]]
```

Accessing Numpy Arrays

- **Slicing**: access contiguous elements
  - x[start:stop:step, ...]
    - Creates a *view* of the elements from *start* (included) to *stop* (excluded), taken with fixed step
    - **Updates on the view yield updates on the original array**
    - Useful shortcuts**:**
      - **omit start** if you want to start from the beginning of the array
      - **omit stop** if you want to slice until the end
      - **omit step** if you don't want to skip elements

- **Slicing**: access contiguous elements
  - Select **all rows** and the **last 2 columns**:

In [1]:
```
x = np.array([[1,2,3],[4,5,6],[7,8,9]])
x[:, 1:]          # or x[0:3, 1:3]
```

Out[1]:
```
[[2,3], [5,6], [8,9]]
```

1	2	3
4	5	6
7	8	9

  - Select the **first two rows** and the **first** and **third columns**

In [2]:
```
x[:2, ::2]      # or x[0:2, 0:3:2]
```

Out[2]:
```
[[1, 3], [4, 6]]
```

1	2	3
4	5	6
7	8	9

Accessing Numpy Arrays

- **Update a sliced array**

In [1]:
```
x = np.array([[1,2,3],[4,5,6],[7,8,9]])
x[:, 1:] = 0
print(x)
```

Out[1]:
```
[[1,0,0], [4,0,0], [7,0,0]]
```

Accessing Numpy Arrays

- **Update a view**

In [1]:
```
x = np.array([[1,2,3],[4,5,6],[7,8,9]])
view = x[:,1:]
view[:,:] = 0
print(x)
```

Out[1]:
```
[[1,0,0], [4,0,0], [7,0,0]]
```

- To avoid updating the original array use .**copy**()
  - x1=x[:,1:].copy()

Accessing Numpy Arrays

- **Masking**: use boolean masks to select elements
  - x[mask]
    - mask
      - **boolean** numpy array that specifies which elements should be selected (select if True)
      - **same shape** as the original array
  - The result is a **one-dimensional vector** that is a **copy** of the original array elements selected by the mask

Accessing Numpy Arrays

- **Mask creation**
  - x *op* value (e.g *x==4*)
  - where *op* can be >, >=, <, <=, ==, !=
- Examples

In [1]:
```
x = np.array([1.2, 4.1, 1.5, 4.5])
x > 4
```

Out[1]:
```
[False, True, False, True]
```

In [2]:
```
x2 = np.array([[1.2, 4.1], [1.5, 4.5]])
x2 >= 4
```

Out[2]:
```
[[False, True], [False, True]]
```

- **Operations with masks (boolean arrays)**
  - Numpy allows boolean operations between masks with the same shape (bitwise operators)
    - & (and), | (or), ^ (xor), ~ (negation)
  - Example
    - mask = ~((x < 1) | (x > 5)) ⇔ ((x >= 1) & (x <= 5))
    - elements that are between 1 and 5 (included)

Accessing Numpy Arrays

- **Masking** examples

In [1]:
```
x = np.array([1.2, 4.1, 1.5, 4.5])
x[x > 4]
```

Out[1]:
```
[4.1, 4.5]
```

In [2]:
```
x2 = np.array([[1.2, 4.1], [1.5, 4.5]])
x2[x2 >= 4]
```

Out[2]:
```
[4.1, 4.5]
```

- Even if the shape of x2 is (2, 2), the result is **a one-dimensional** array containing the elements that satisfy the condition

- **Update a masked array**

In [1]:
```
x = np.array([1.2, 4.1, 1.5, 4.5])
x[x > 4] = 0        # Assignment is allowed
x
```

Out[1]:
```
[1.2, 0, 1.5, 0]
```

Accessing Numpy Arrays

- **Masking does not create views, but copies**

In [2]:
```
x = np.array([1.2, 4.1, 1.5, 4.5])
masked = x[x > 4]  # Masked is a copy of x
masked[:] = 0      # Assignment does not affect x
x
```

Out[2]:
```
[1.2, 4.1, 1.5, 4.5]
```

Notebook Examples

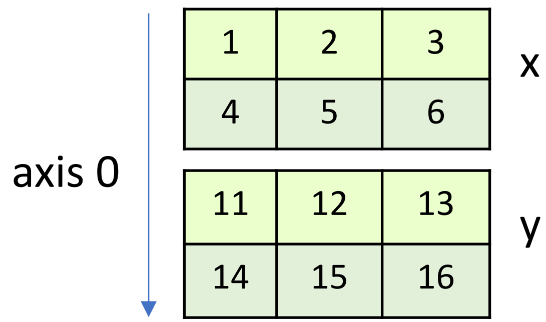
- **1-Numpy Examples.ipynb**
  - **4) Accessing Numpy Arrays**

Summary:

- Array concatenation
- Array splitting
- Array reshaping
- Adding new dimensions

Working with arrays

- Array concatenation along **existing axis**
  - The result has the **same number of dimensions** of the input arrays



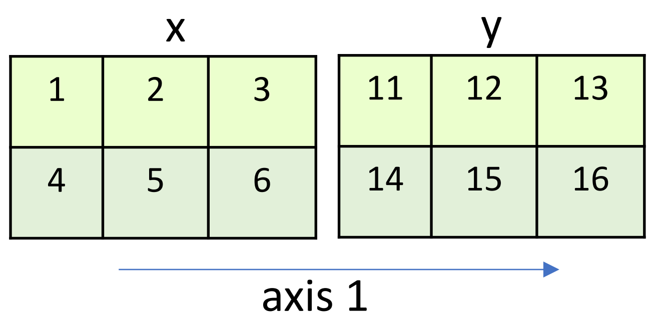
In [1]:

```python
x = np.array([[1,2,3],[4,5,6]])
y = np.array([[11,12,13],[14,15,16]])
np.concatenate((x, y))      # Default axis: 0
```

Out[1]:

```
[[1,2,3],[4,5,6],[11,12,13],[14,15,16]]
```

- **Array concatenation along existing axis**
  - Concatenation along **rows (axis=1)**



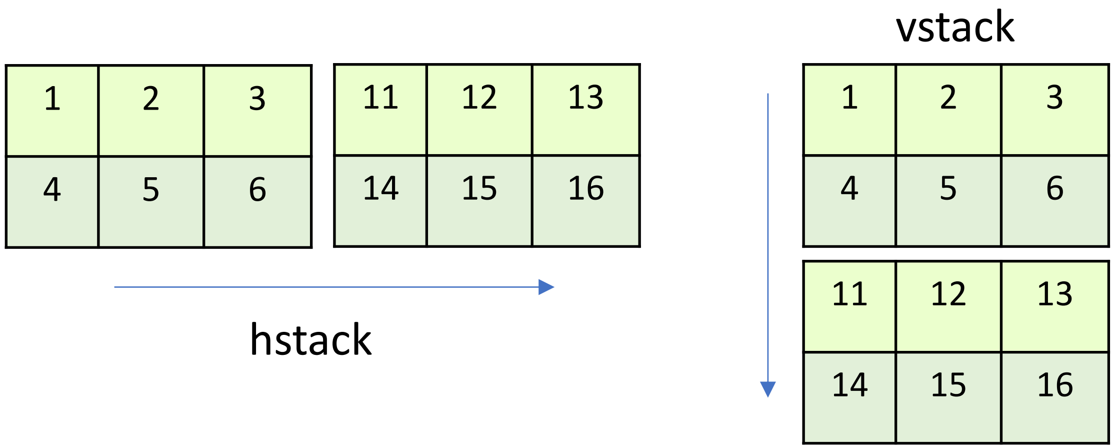
In [1]:

```
x = np.array([[1,2,3],[4,5,6]])
y = np.array([[11,12,13],[14,15,16]])
np.concatenate((x, y), axis=1)
```

Out[1]:

```
[[1,2,3,11,12,13],[4,5,6,14,15,16]]
```

- **Array concatenation: hstack, vstack**
  - Similar to np.concatenate()

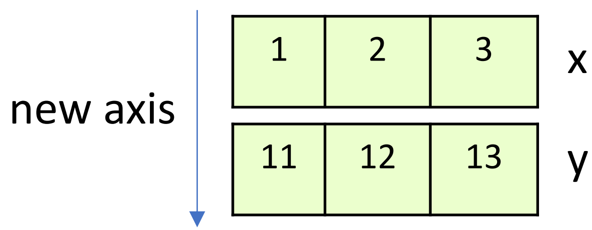


In [1]:

```
x = np.array([[1,2,3],[4,5,6]])
y = np.array([[11,12,13],[14,15,16]])
h = np.hstack((x, y))      # along rows (horizontal)
v = np.vstack((x, y))      # along columns (vertical)
```

Array concatenation: hstack, vstack

- **vstack** allows concatenating 1-D vectors along **new axis** (not possible with np.concatenate)



In [1]:

```
x = np.array([1,2,3])
y = np.array([11,12,13])
v = np.vstack((x, y))        # vertically
```

- **Splitting arrays (split, hsplit, vsplit)**
  - **np.split(arr, N, axis=0)**
    - outputs a **list** of Numpy arrays
    - If N is integer: divide *arr* into *N* equal arrays (along axis), if possible!
    - if N is a 1d array: specify the entries where the array is split (along *axis*)

x

index	0	1	**2**	3	**4**	5
values	7	7	9	9	8	8

In [1]:

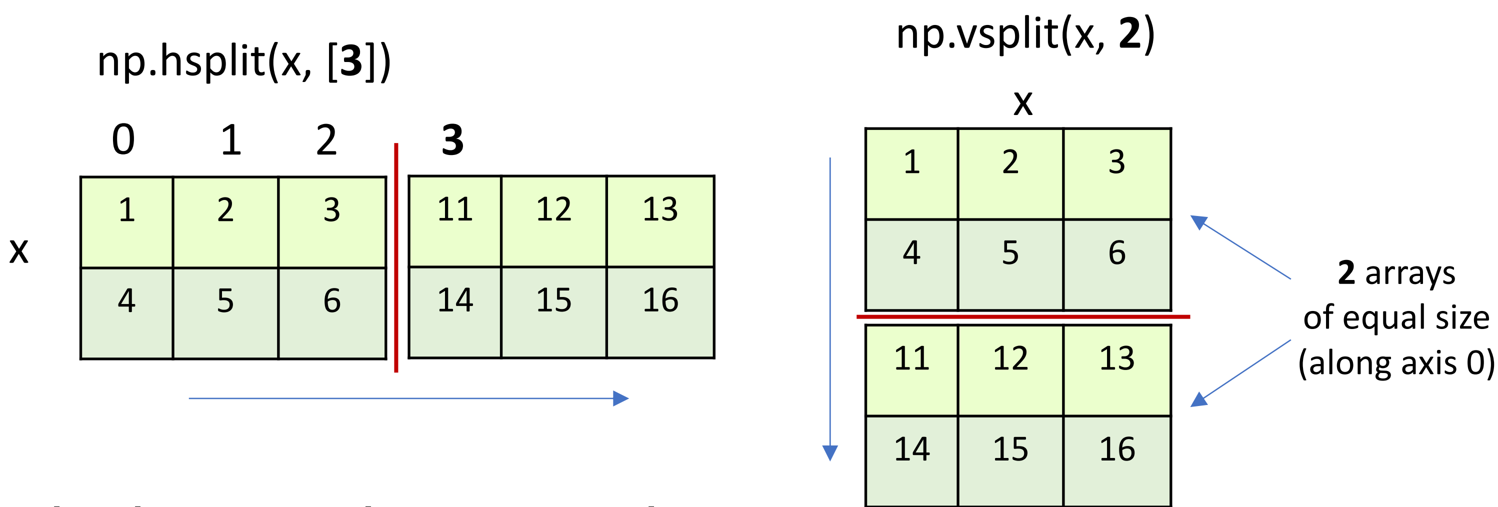
```
x = np.array([7, 7, 9, 9, 8, 8])
np.split(x,[2,4])          # split before element 2 and 4
                           # same as passing N = 3
```

Out[1]:

```
[array([7, 7]), array([9, 9]), array([8, 8])]
```

Working with arrays

- **Splitting arrays (split, hsplit, vsplit)**
  - **hsplit, vsplit** with 2D arrays
    - return a **list** with the arrays after the split

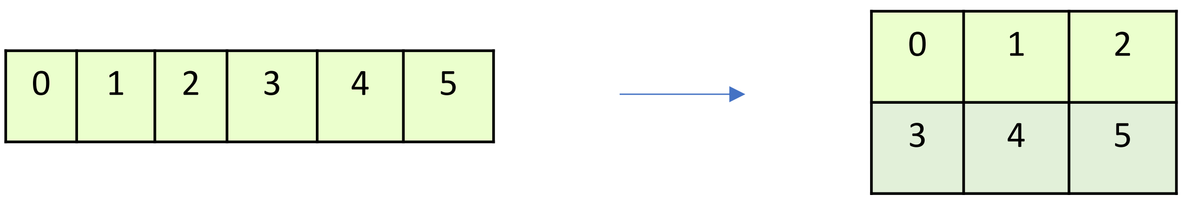


  - In both examples output is:

Out: [array([[1,2,3],[4,5,6]]), array([[11,12,13],[14,15,16]])]

Reshaping arrays

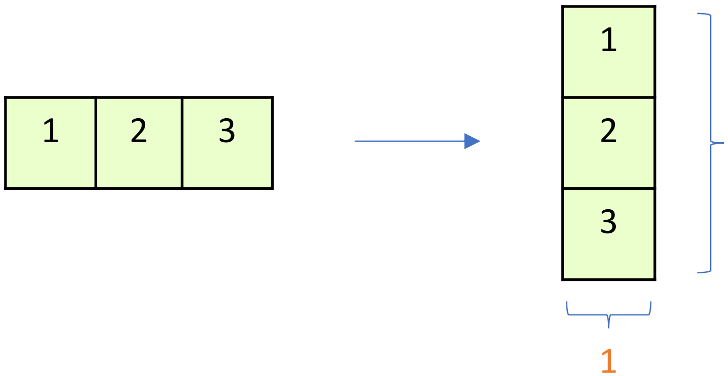
In [1]:
```
x = np.arange(6)
y = x.reshape((2,3))
```



- y is filled following the index order:
  - y[0,0] = x[0], y[0,1] = x[1], y[0,2] = x[2]
  - y[1,0] = x[3], y[1,1] = x[4], y[1,2] = x[5]

- **Reshaping arrays**
  - At most one dimension can be −1 ("unknown")
  - If present, the size is inferred from
    - The source array
    - The other dimensions

In [1]:
```
x = np.array([1,2,3])
y = x.reshape(-1,1)
```



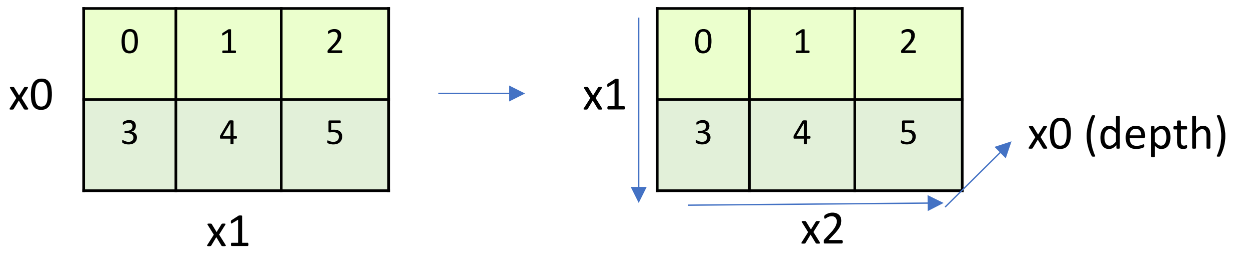
The first dimension (rows) is inferred to be 3, considering that the second dimension (columns) is 1 and x.size = 3

Adding new dimensions

- **np.newaxis** adds a new dimension with **shape=1** at the specified position

In [1]:
```
arr = np.array([[1,2,3],[4,5,6]])
res = arr[np.newaxis, :, :] # output shape = (1,2,3)
print(res)
```

Out[1]:
```
[[[1,2,3],[4,5,6]]]
```



Adding new dimensions

- **Application:** row vector to column vector
  - Alternative approach to .reshape(−1,1)

In [1]:
```
arr = np.array([1,2,3])
res = arr[:, np.newaxis]    # output shape = (3,1)
print(res)
```

Out[1]:
```
[[1],[2],[3]]
```

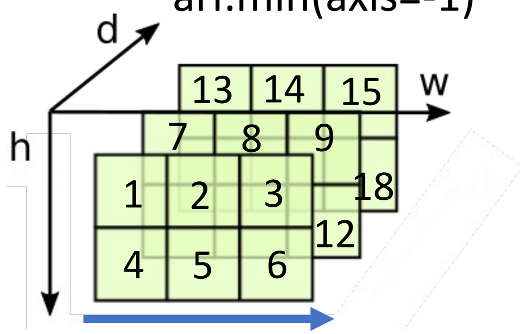
Uncovered Topics

Computation on Numpy

Aggregate functions along axis

- The aggregation dimension is **removed** from the output

arr.min(axis=-1)



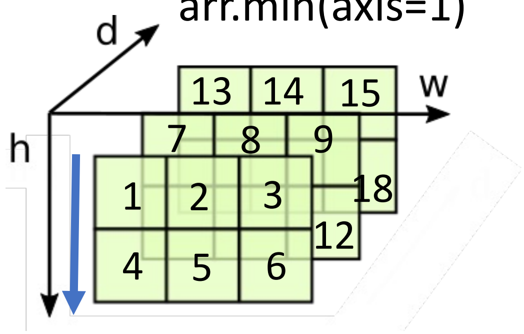
shape = (3, 2, 1)
[[[1], [4]],
[[7], [10]],
[[13], [16]]]

→

Final output

shape = (3, 2)
[[1, 4],
 [7, 10],
 [13, 16]]

arr.min(axis=1)



shape = (3, 1, 3)
[[[1,2,3]],
 [[7,8,9]],
 [[13,14,15]]]

→

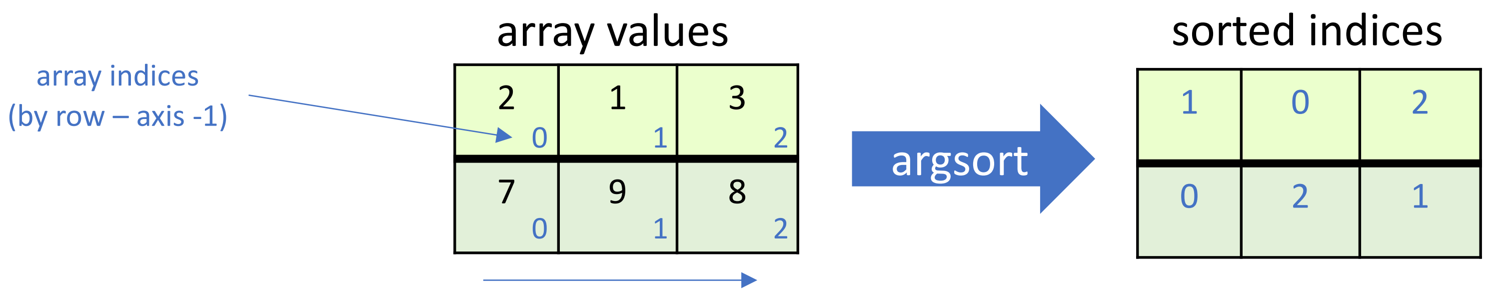
shape = (3, 3)
[[1, 2, 3],
 [7, 8, 9],
 [13, 14, 15]]

Sorting

- **np.argsort(x):** return the position of the indices of the sorted array (sorts by default on axis -1)

In [1]:
```
x = np.array([[2,1,3],[7,9,8]])
np.argsort(x)      # Sort along rows (axis -1)
```

Out[1]:
```
[[1,0,2],[0,2,1]]
```



Accessing Numpy Arrays

- **Simple indexing:** returning elements **from the end**
- Consider the array
  - x = np.array([[2, 3, 4],[5,6,7]])
- x[0, **-1**]
  - Get last element of the first row: 4
- x[0, **-2**]
  - Get second element from the end of the first row: 3

- **Fancy indexing**: specify the **index** of elements to be selected
  - Example: select elements from 1-dimensional array

```
                   x[1]      x[3]
In [1]:  x = np.array([7.0, 9.0, 6.0, 5.0])
         x[[1, 3]]

Out[1]:  [9.0, 5.0]
```

- **Fancy indexing:** selection of **rows** from a 2-dimensional array

	0.0	1.0	2.0
x[1,:]	3.0	4.0	5.0
x[2,:]	6.0	7.0	8.0

In [1]:

```
x = np.array([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0],
              [6.0, 7.0, 8.0]])
x[[1, 2]]
```

Out[1]:

```
[[3.0, 4.0, 5.0], [6.0, 7.0, 8.0]]
```

- **Fancy indexing**: selection of elements with coordinates
  - Result contains a 1-dimensional array with selected elements

0,0	0,1	0,2
1,0	1,1	1,2
2,0	2,1	2,2

In [1]:

```
x = np.array([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0],
              [6.0, 7.0, 8.0]])
x[[1, 2], [0, 2]]
```

→ | 1, 0 | 2, 2 | (indices being selected)

Out[1]:

```
[3.0, 8.0]
```

Accessing Numpy Arrays

- Similarly to masking, fancy indexing provides **copies** (not views) of the original array

In [1]:

```
x = np.array([1.2, 4.1, 1.5, 4.5])
x[[1, 3]] = 0      # Assignment is allowed
x
```

Out[1]:

```
[1.2, 0, 1.5, 0]
```

In [2]:

```
x = np.array([1.2, 4.1, 1.5, 4.5])
sel = x[[1, 3]]    # sel is a copy of x
sel[:] = 0         # Assignment does not affect x
x
```

Out[2]:

```
[1.2, 4.1, 1.5, 4.5]
```

- **Combined indexing:**
  - Allows mixing the indexing types described so far
  - Important rule:
    - The number of dimensions of selected data is:
      - **The same as the input** if you mix:
        - masking+slicing, fancy+slicing
      - **Reduced by one** for each axis where simple indexing is used
        - Because simple indexing takes only 1 **single** element from an axis

Accessing Numpy Arrays

- **Combined indexing:** masking+slicing, fancy+slicing
  - Output has the same numer of dimensions as input

```
x = np.array([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]])
```

```
x[[True,False,True], 1:]
# Masking + Slicing: [[1.0,2.0],[7.0,8.0]]
```

0.0	1.0	2.0
3.0	4.0	5.0
6.0	7.0	8.0

```
x[[0,2], :2]
# Fancy + Slicing: [[0.0,1.0],[6.0,7.0]]
```

0.0	1.0	2.0
3.0	4.0	5.0
6.0	7.0	8.0

- **Combined indexing:** simple+slicing, simple+masking
  - Simple indexing **reduces** the number of dimensions

```
x = np.array([[0.0, 1.0, 2.0], [3.0, 4.0, 5.0], [6.0, 7.0, 8.0]])
```

```
x[0, 1:]
# Simple + Slicing: [1.0, 2.0]
```

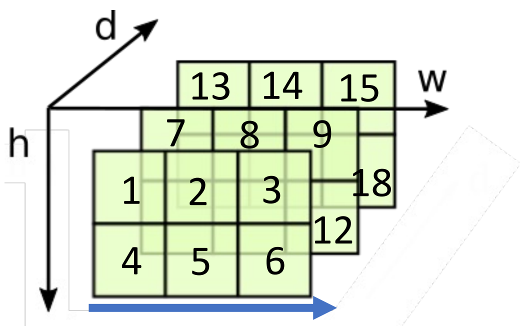
0.0	1.0	2.0
3.0	4.0	5.0
6.0	7.0	8.0

```
x[[True, False, True], 0]
# Simple + Masking: [0.0, 6.0]
```

0.0	1.0	2.0
3.0	4.0	5.0
6.0	7.0	8.0

- **Simple indexing + slicing**
  - The dimension selected with simple indexing is **removed** from the output

arr[:,:,0]



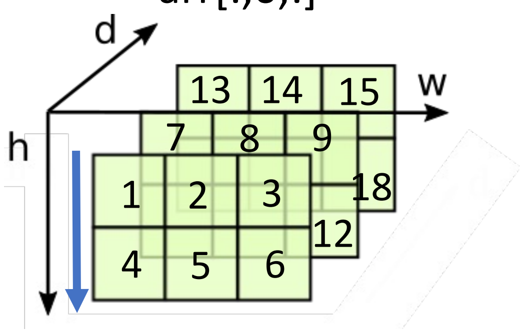
shape = (3, 2, 1)
[[[1], [4]],
[[7], [10]],
[[13], [16]]]

→

Final output

shape = (3, 2)
[[1, 4],
[7, 10],
[13, 16]]

arr[:,0,:]



shape = (3, 1, 3)
[[[1,2,3]],
[[7,8,9]],
[[13,14,15]]]

→

shape = (3, 3)
[[1, 2, 3],
[7, 8, 9],
[13, 14, 15]]