Data preprocessing

Data Base and Data Mining Group of Politecnico di Torino

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Politecnico di Torino

Outline

- Data types and properties
- Data preparation
- Data preparation for document data
- Similarity and dissimilarity
- Correlation

Data types and properties

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What is Data?

- Collection of ***data objects*** and their ***attributes***
- An ***attribute*** is a property or characteristic of an object
  - Examples: eye color of a person, temperature, etc.
  - Attribute is also known as variable, field, characteristic, dimension, or feature
- A collection of attributes describes an ***object***
  - Object is also known as record, point, case, sample, entity, or instance

Attributes

Objects

Tid	**Refund**	**Marital Status**	**Taxable Income**	**Cheat**
1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	95K	**Yes**
6	No	Married	60K	**No**
7	Yes	Divorced	220K	**No**
8	No	Single	85K	**Yes**
9	No	Married	75K	**No**
10	No	Single	90K	**Yes**

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Attribute Values

- ***Attribute values*** are numbers or symbols assigned to an attribute for a particular object
- Distinction between attributes and attribute values
  - Same attribute can be mapped to different attribute values
    - Example: height can be measured in feet or meters
  - Different attributes can be mapped to the same set of values
    - Example: Attribute values for ID and age are integers
    - But properties of attribute values can be different

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Attribute types

- There are different types of attributes
  - Nominal
    - Examples: ID numbers, eye color, zip codes
  - Ordinal
    - Examples: rankings (e.g., taste of potato chips on a scale from 1-10), grades, height in {tall, medium, short}
  - Interval
    - Examples: calendar dates
  - Ratio
    - Examples: temperature in Kelvin, length, time, counts

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Properties of Attribute Values

- The type of an attribute depends on which of the following properties it possesses:
  - Distinctness: = ≠
  - Order: < >
  - Addition: + -
  - Multiplication: * /

  - Nominal attribute: distinctness
  - Ordinal attribute: distinctness & order
  - Interval attribute: distinctness, order & addition
  - Ratio attribute: all 4 properties

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Discrete and Continuous Attributes

- Discrete Attribute
  - Has only a finite or countably infinite set of values
  - Examples: zip codes, counts, or the set of words in a collection of documents
  - Often represented as integer variables.
  - Note: binary attributes are a special case of discrete attributes
- Continuous Attribute
  - Has real numbers as attribute values
  - Examples: temperature, height, or weight.
  - Practically, real values can only be measured and represented using a finite number of digits.
  - Continuous attributes are typically represented as floating-point variables.

More Complicated Examples

- ID numbers
  - Nominal, ordinal, or interval?

- Number of cylinders in an automobile engine
  - Nominal, ordinal, or ratio?

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Key Messages for Attribute Types

- The types of operations you choose should be "meaningful" for the type of data you have
  - Distinctness, order, meaningful intervals, and meaningful ratios are only four properties of data
  - The data type you see – often numbers or strings – may not capture all the properties or may suggest properties that are not there
  - Analysis may depend on these other properties of the data
    - Many statistical analyses depend only on the distribution
  - Many times what is meaningful is measured by statistical significance
  - But in the end, what is meaningful is measured by the domain

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Data set types

- Record
  - Tables
  - Document Data
  - Transaction Data
- Graph
  - World Wide Web
  - Molecular Structures
- Ordered
  - Spatial Data
  - Temporal Data
  - Sequential Data
  - Genetic Sequence Data

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Tabular Data

- A collection of records
  - Each record is characterized by a fixed set of attributes

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From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Document data

- It includes textual data that can be **semi-structured** or **unstructured**
  - Plain text can be organized in sentences, paragraphs, sections, documents
- Text acquired in different contexts may have a structure and/or a semantics
  - **Web pages** are enriched with tags
  - **Documents in digital libraries** are enriched with metadata
  - **E-learning documents** can be annotated or partly highglihted

Document Data

- Each document becomes a `term' vector
  - each term is a component (attribute) of the vector,
  - the value of each component is the number of times the corresponding term occurs in the document.

	team	coach	pla y	ball	score	game	wi n	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Transaction Data

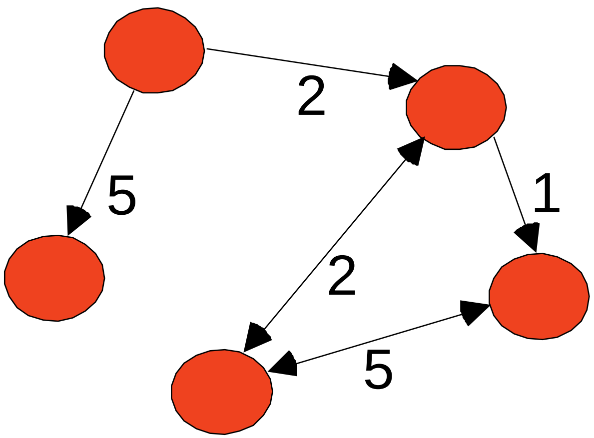
- A special type of record data, where
  - each record (transaction) involves a set of items.
  - For example, consider a grocery store. The set of products purchased by a customer during one shopping trip constitute a transaction, while the individual products that were purchased are the items.

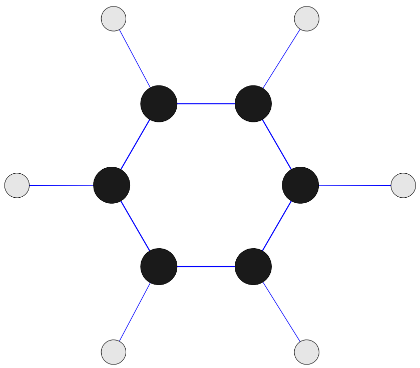
TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

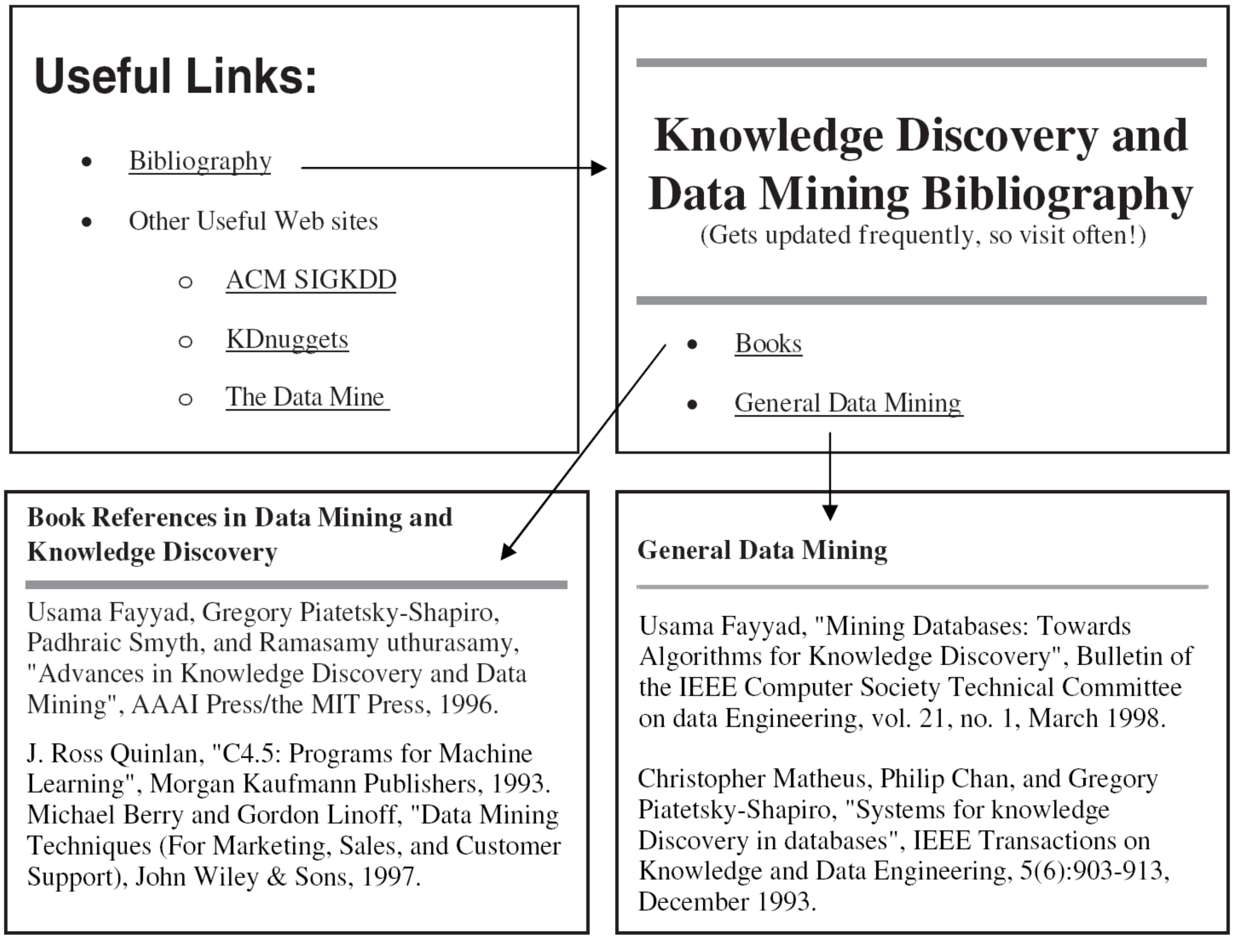
Graph Data

- Examples: Generic graph, a molecule, and webpages





Benzene Molecule: $C6H6$



From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Ordered Data

- Sequences of transactions

Items/Events

(A B) (D) (C E)
(B D) (C) (E)
(C D) (B) (A E)

An element of
the sequence

Ordered Data

- Genomic sequence data

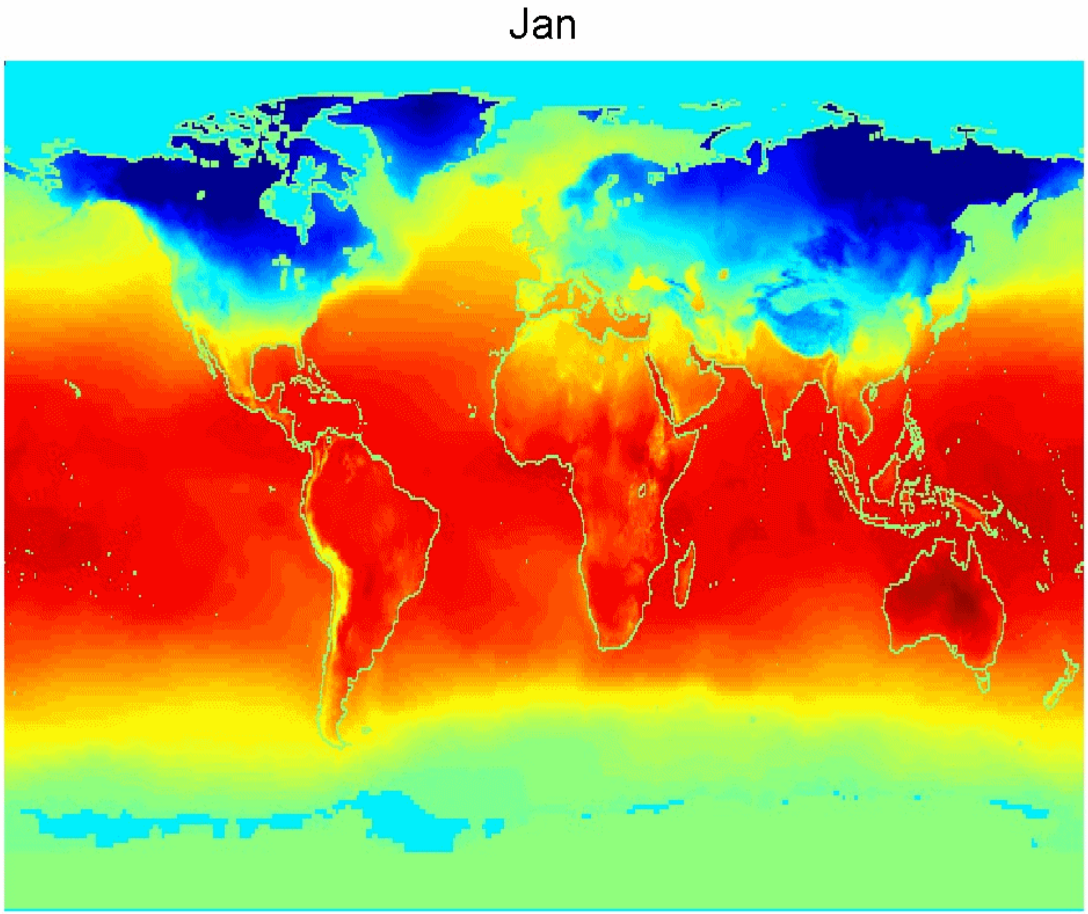
```
GGTTCCGCCTTCAGCCCCGCGCC
CGCAGGGCCCGCCCCGCGCCGTC
GAGAAGGGCCCGCCTGGCGGGCG
GGGGGAGGCGGGGCCGCCCGAGC
CCAACCGAGTCCGACCAGGTGCC
CCCTCTGCTCGGCCTAGACCTGA
GCTCATTAGGCGGCAGCGGACAG
GCCAAGTAGAACACGCGAAGCGC
TGGGCTGCCTGCTGCGACCAGGG
```


From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Ordered Data

- Spatio-Temporal Data

Average Monthly Temperature of land and ocean



From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Data Quality

- Poor data quality negatively affects many data processing efforts

  "The most important point is that poor data quality is an unfolding disaster. Poor data quality costs the typical company at least ten percent (10%) of revenue; twenty percent (20%) is probably a better estimate."

  Thomas C. Redman, DM Review, August 2004

- Data mining example: a classification model for detecting people who are loan risks is built using poor data
  - Some credit-worthy candidates are denied loans
  - More loans are given to individuals that default

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Data Quality

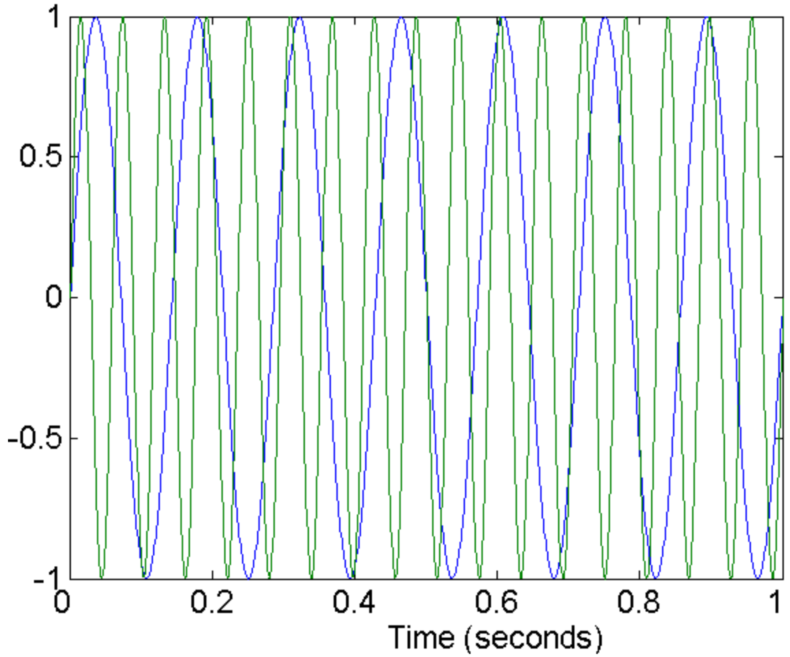
- What kinds of data quality problems?
- How can we detect problems with the data?
- What can we do about these problems?

- Examples of data quality problems
  - Noise and outliers
  - Missing values
  - Duplicate data
  - Wrong data

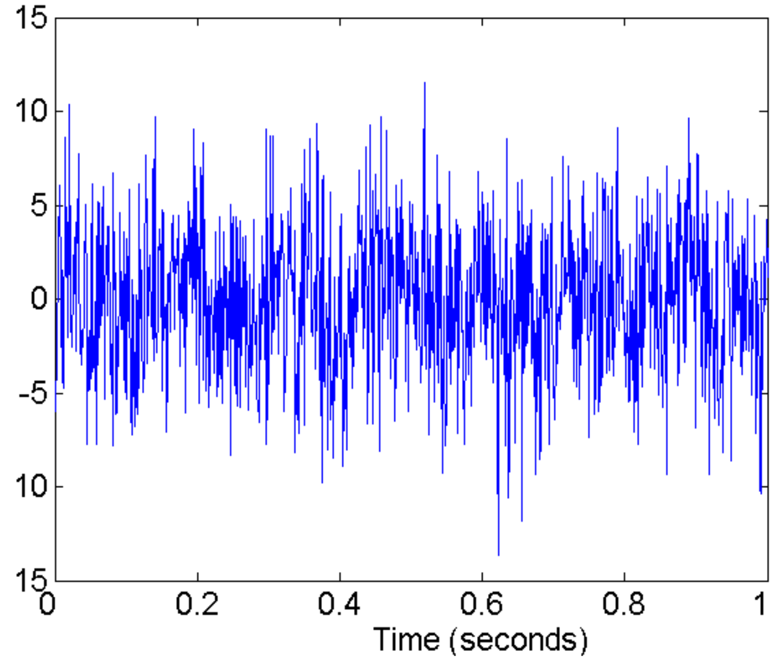
From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Noise

- Noise refers to modification of original values
  - Examples: distortion of a person's voice when talking on a poor phone and "snow" on television screen



Two Sine Waves

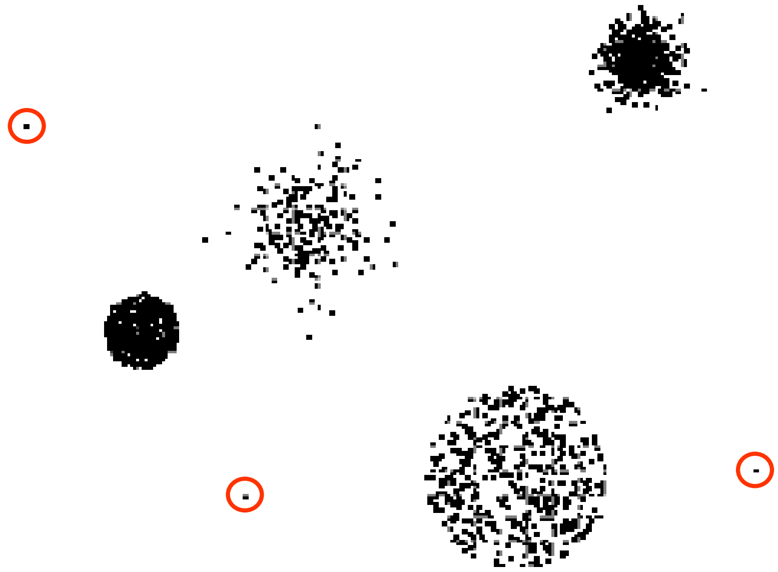


Two Sine Waves + Noise

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Outliers

- ***Outliers*** are data objects with characteristics that are considerably different than most of the other data objects in the data set
  - **Case 1:** Outliers are noise that interferes with data analysis
  - **Case 2:** Outliers are the goal of our analysis
    - Credit card fraud
    - Intrusion detection



From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Missing Values

- Reasons for missing values
  - Information is not collected
    (e.g., people decline to give their age and weight)
  - Attributes may not be applicable to all cases
    (e.g., annual income is not applicable to children)
- Handling missing values
  - Eliminate data objects or variables
  - Estimate missing values
    - Example: time series of temperature
  - Ignore the missing value during analysis

Duplicate Data

- Data set may include data objects that are duplicates, or almost duplicates of one another
  - Major issue when merging data from heterogeneous sources
- Examples
  - Different words/abbreviations for the same concept (e.g., Street, St.)
- Data cleaning
  - Process of dealing with duplicate data issues
- When should duplicate data not be removed?

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Data preparation

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Data Preprocessing

- Aggregation
- Sampling
- Dimensionality Reduction
- Feature subset selection
- Feature creation
- Discretization and Binarization
- Attribute Transformation

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Aggregation

- Combining two or more attributes (or objects) into a single attribute (or object)
- Purpose
  - Data reduction
    - Reduce the number of attributes or objects
  - Change of scale
    - Cities aggregated into regions, states, countries, etc
  - More "stable" data
    - Aggregated data tends to have less variability

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

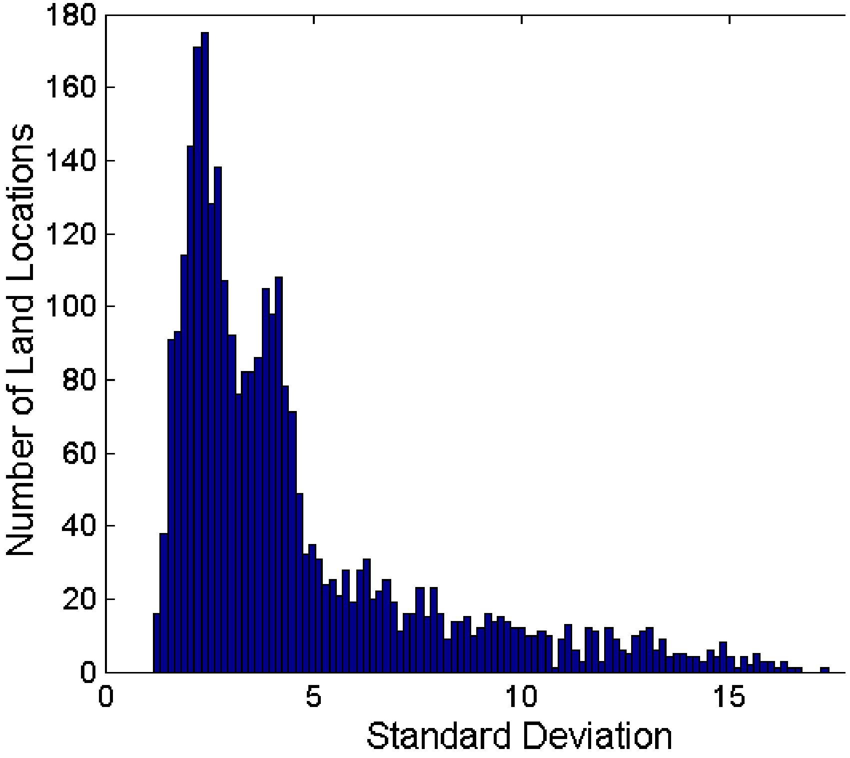
Example: Precipitation in Australia

- The next slide shows precipitation in Australia from the period 1982 to 1993
  - A histogram for the standard deviation of average monthly precipitation for 3,030 0.5° by 0.5° grid cells in Australia
  - A histogram for the standard deviation of the average yearly precipitation for the same locations.
- The average yearly precipitation has less variability than the average monthly precipitation.
- All precipitation measurements (and their standard deviations) are in centimeters.

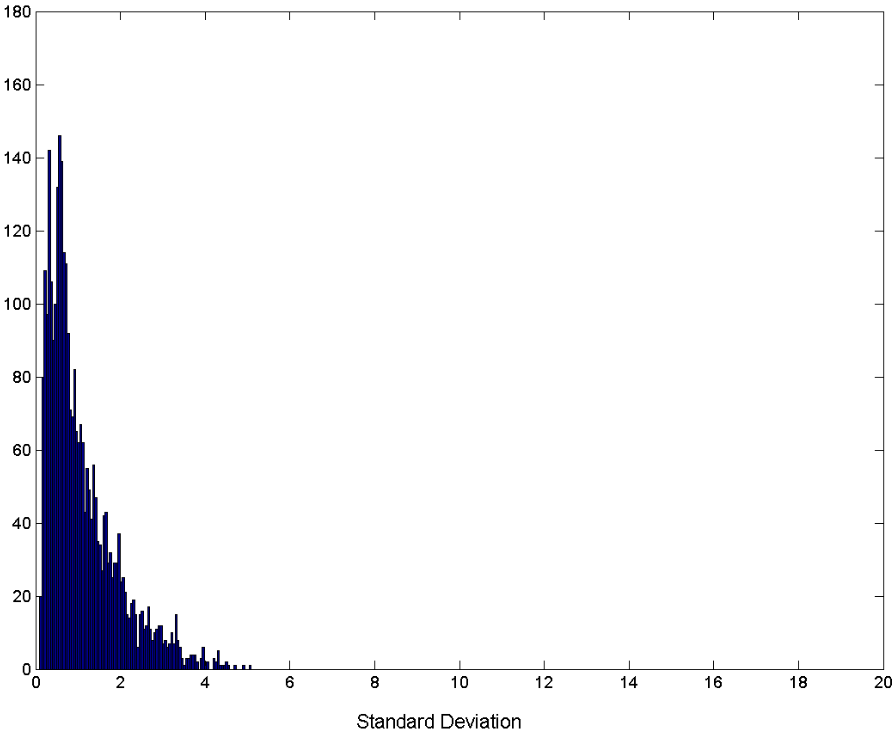
From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Aggregation

Variation of Precipitation in Australia



Standard Deviation of Average Monthly Precipitation



Standard Deviation of Average Yearly Precipitation

Data reduction

- Generates a reduced representation of the dataset
- This representation is smaller in volume, but it can provide similar analytical results
  - sampling
    - reduces the cardinality of the set
  - feature selection
    - reduces the number of attributes
  - discretization
    - reduces the cardinality of the attribute domain

Sampling

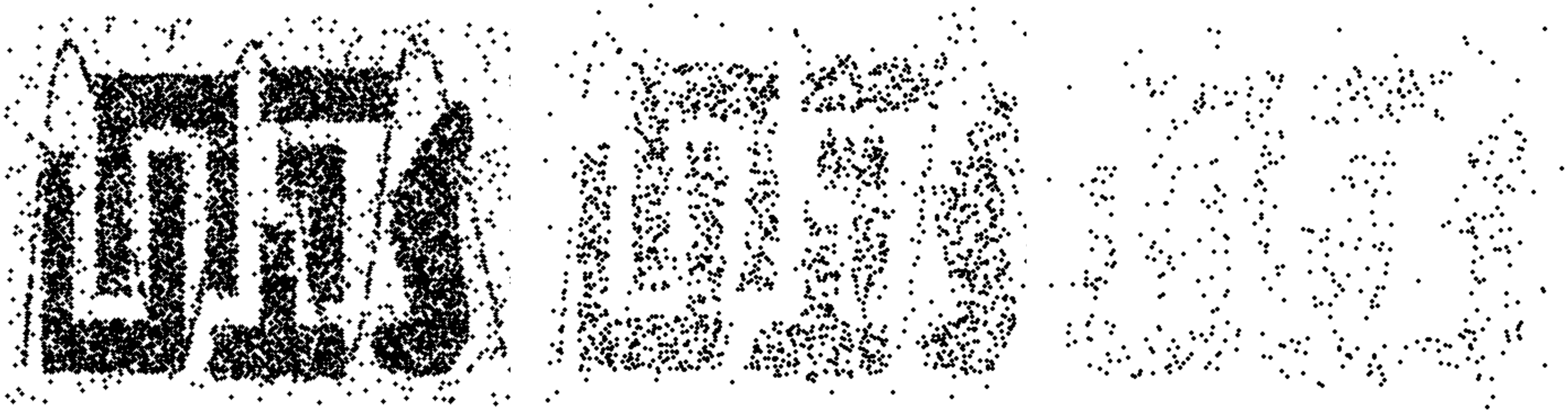
- Sampling is the main technique employed for data selection.
  - It is often used for both the preliminary investigation of the data and the final data analysis.
- Statisticians sample because obtaining the entire set of data of interest is too expensive or time consuming.
- Sampling is used in data mining because processing the entire set of data of interest is too expensive or time consuming.

Sampling …

- The key principle for effective sampling is the following:
  - using a sample will work almost as well as using the entire data set, if the sample is representative
  - A sample is representative if it has approximately the same property (of interest) as the original set of data

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Sample Size: examples



8000 points

2000 Points

500 Points

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

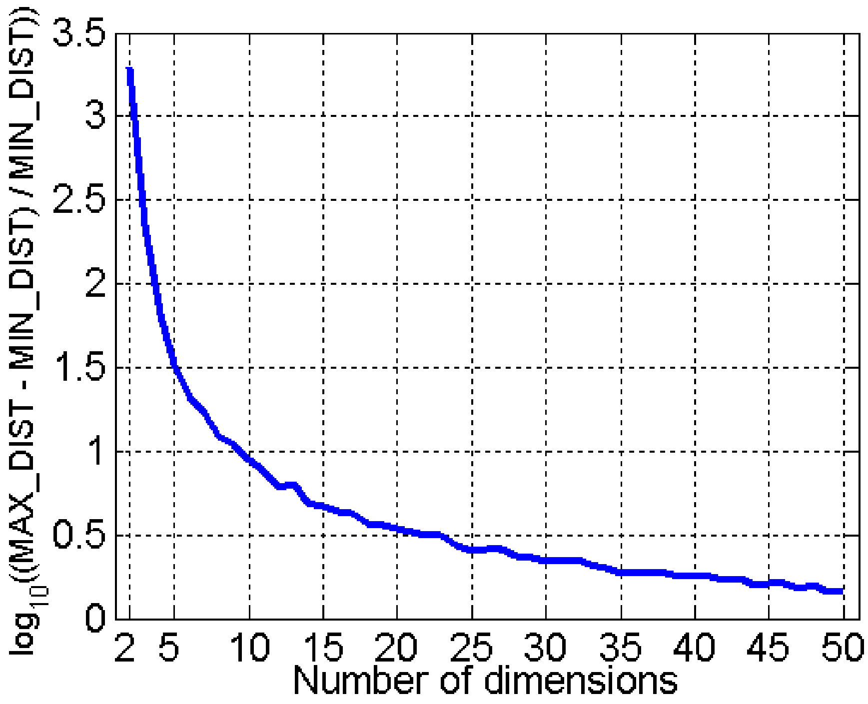
Types of Sampling

- Simple Random Sampling
  - There is an equal probability of selecting any particular item
  - Sampling without replacement
    - As each item is selected, it is removed from the population
  - Sampling with replacement
    - Objects are not removed from the population as they are selected for the sample.
    - In sampling with replacement, the same object can be picked up more than once
- Stratified sampling
  - Split the data into several partitions; then draw random samples from each partition

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Curse of Dimensionality

- When dimensionality increases, data becomes increasingly sparse in the space that it occupies
- Definitions of density and distance between points, which is critical for clustering and outlier detection, become less meaningful



- Randomly generate 500 points
- Compute difference between max and min distance between any pair of points

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Dimensionality Reduction

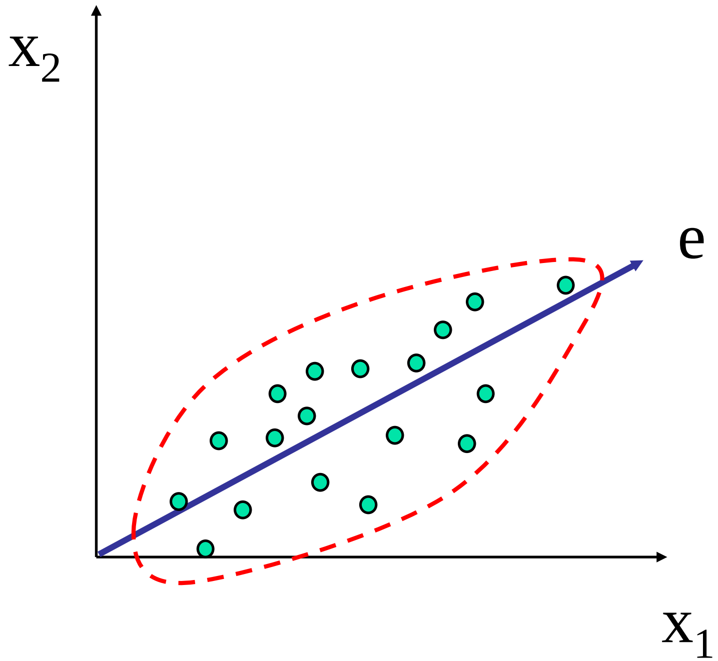
- Purpose
  - Avoid curse of dimensionality
  - Reduce amount of time and memory required by data mining algorithms
  - Allow data to be more easily visualized
  - May help to eliminate irrelevant features or reduce noise

- Techniques
  - Principal Component Analysis (PCA)
  - Singular Value Decomposition
  - Others: supervised and non-linear techniques

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Dimensionality Reduction: PCA

- Goal is to find a projection that captures the largest amount of variation in data



Dimensionality Reduction: PCA

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Feature Subset Selection

- Another way to reduce dimensionality of data

- Redundant features
  - duplicate much or all of the information contained in one or more other attributes
  - Example: purchase price of a product and the amount of sales tax paid

- Irrelevant features
  - contain no information that is useful for the data mining task at hand
  - Example: students' ID is irrelevant to the task of predicting students' GPA

Feature Subset Selection

- Techniques
  - Brute-force approach
    - Try all possible feature subsets as input to data mining algorithm
  - Embedded approaches
    - Feature selection occurs naturally as part of the data mining algorithm
  - Filter approaches
    - Features are selected before data mining algorithm is run
  - Wrapper approaches
    - Use the data mining algorithm as a black-box to find best subset of attributes

Feature Creation

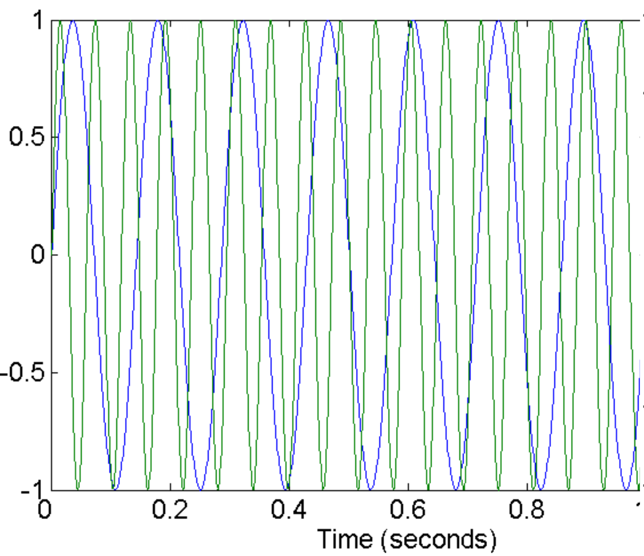
- Create new attributes that can capture the important information in a data set much more efficiently than the original attributes

- Three general methodologies
  - Feature Extraction
    - domain-specific
  - Mapping Data to New Space
  - Feature Construction
    - combining features

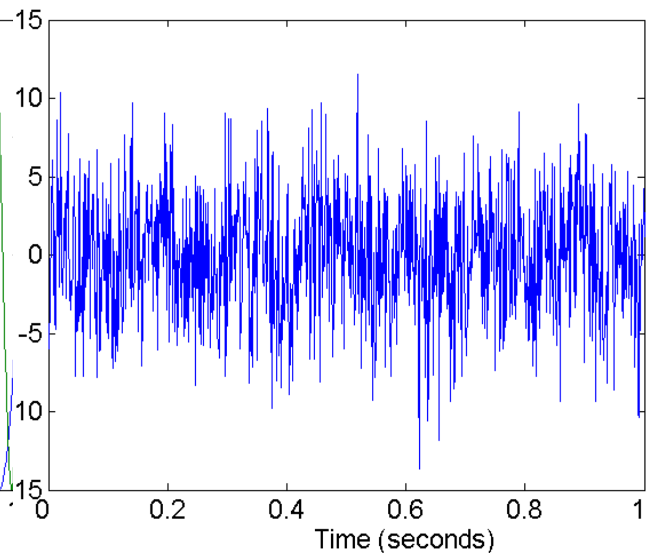
From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Mapping Data to a New Space

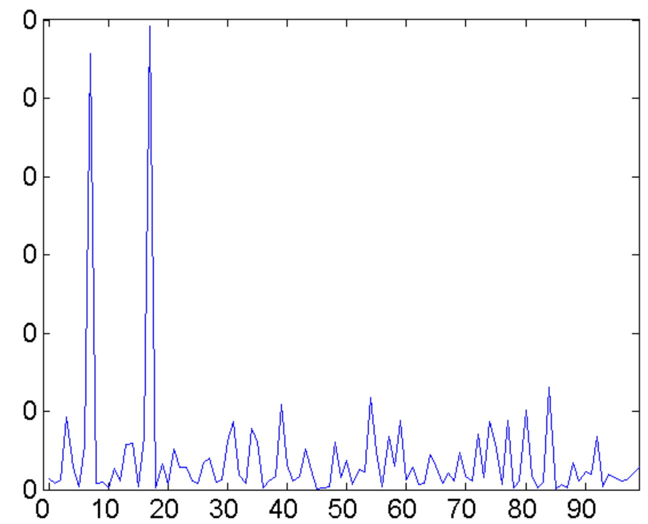
- Fourier transform
- Wavelet transform



Two Sine Waves



Two Sine Waves + Noise



Frequency

Discretization

- Discretization is the process of converting a continuous attribute into an ordinal attribute
  - A potentially infinite number of values are mapped into a small number of categories
  - Discretization is commonly used in classification
    - Many classification algorithms work best if both the independent and dependent variables have only a few values

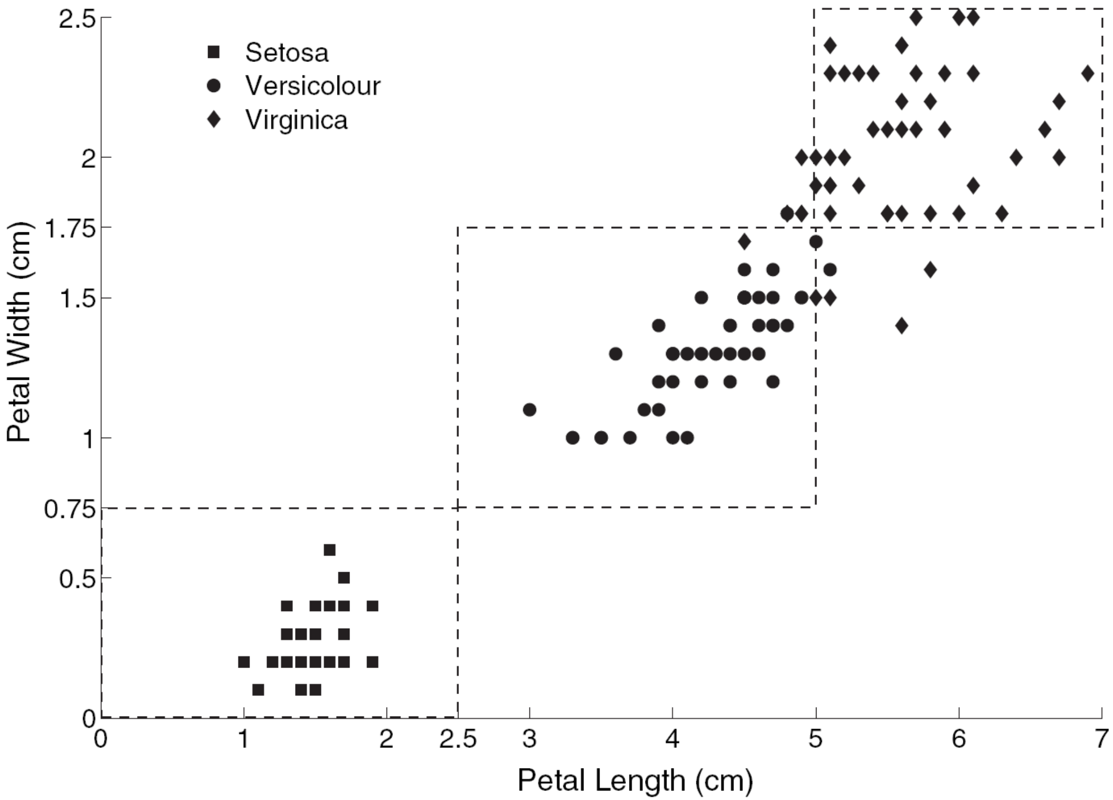
From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Iris Sample Data Set

- Iris Plant data set
  - Can be obtained from the UCI Machine Learning Repository http://www.ics.uci.edu/~mlearn/MLRepository.html
  - From the statistician Douglas Fisher
  - Three flower types (classes)
    - Setosa
    - Versicolour
    - Virginica
  - Four (non-class) attributes
    - Sepal width and length
    - Petal width and length

Virginica. Robert H. Mohlenbrock. USDA NRCS. 1995. Northeast wetland flora: Field office guide to plant species. Northeast National Technical Center, Chester, PA. Courtesy of USDA NRCS Wetland Science Institute.

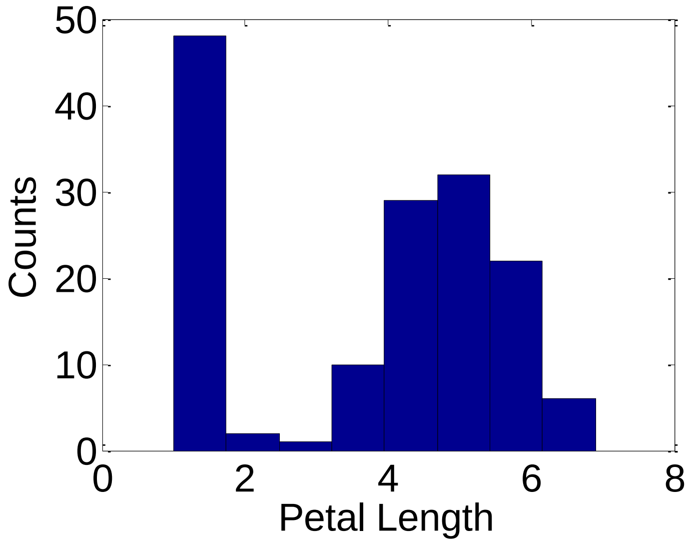
Discretization: Iris Example



Petal width low or petal length low implies Setosa.
Petal width medium or petal length medium implies Versicolour.
Petal width high or petal length high implies Virginica.

Discretization: Iris Example …

- How can we tell what the best discretization is?
  - Unsupervised discretization: find breaks in the data values
    - Example: Petal Length

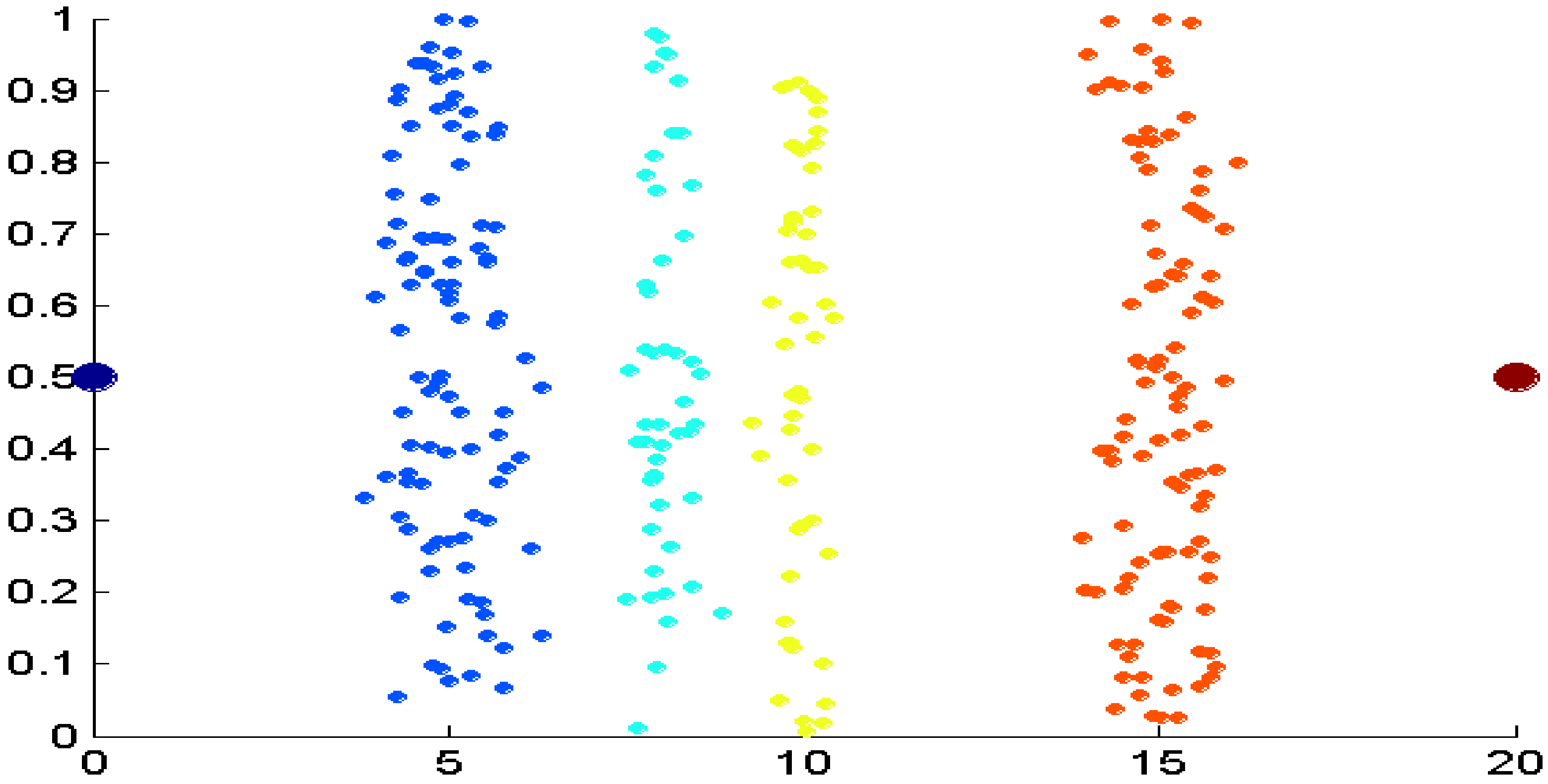


  - Supervised discretization: Use class labels to find breaks

Discretization

- Examples of unsupervised discretization techniques
  - N intervals with the same width $W=(v_{max} - v_{min})/N$
    - Easy to implement
    - It can be badly affected by outliers and sparse data
    - Incremental approach
  - N intervals with (approximately) the same cardinality
    - It better fits sparse data and outliers
    - Non incremental approach
  - clustering
    - It fits well sparse data and outliers
  - analysis of data distribution
    - e.g., 4 intervals, one for each quartile

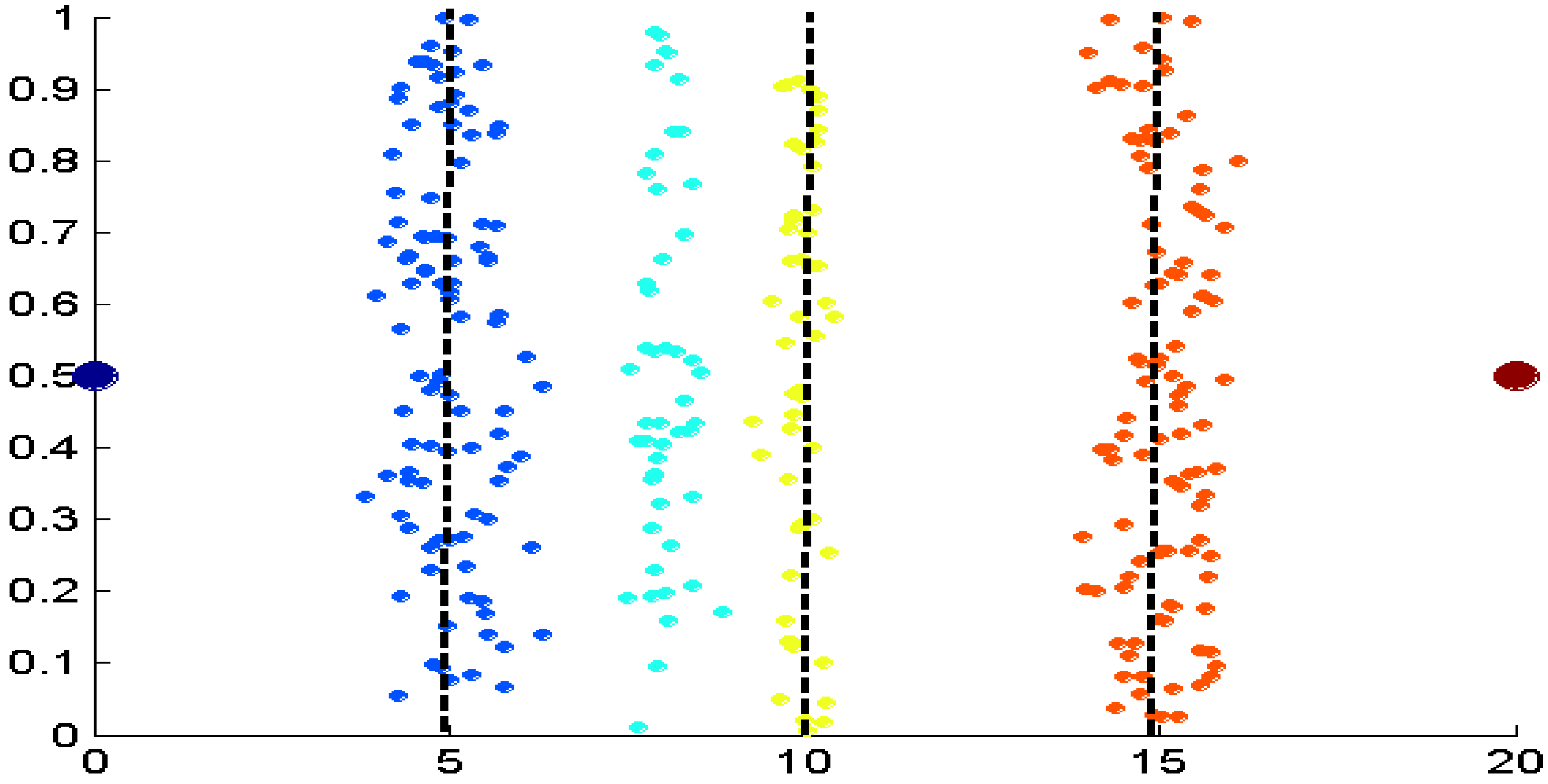
Example: unsupervised discretization technique



Data consists of four groups of points and two outliers. Data is one-dimensional, but a random y component is added to reduce overlap.

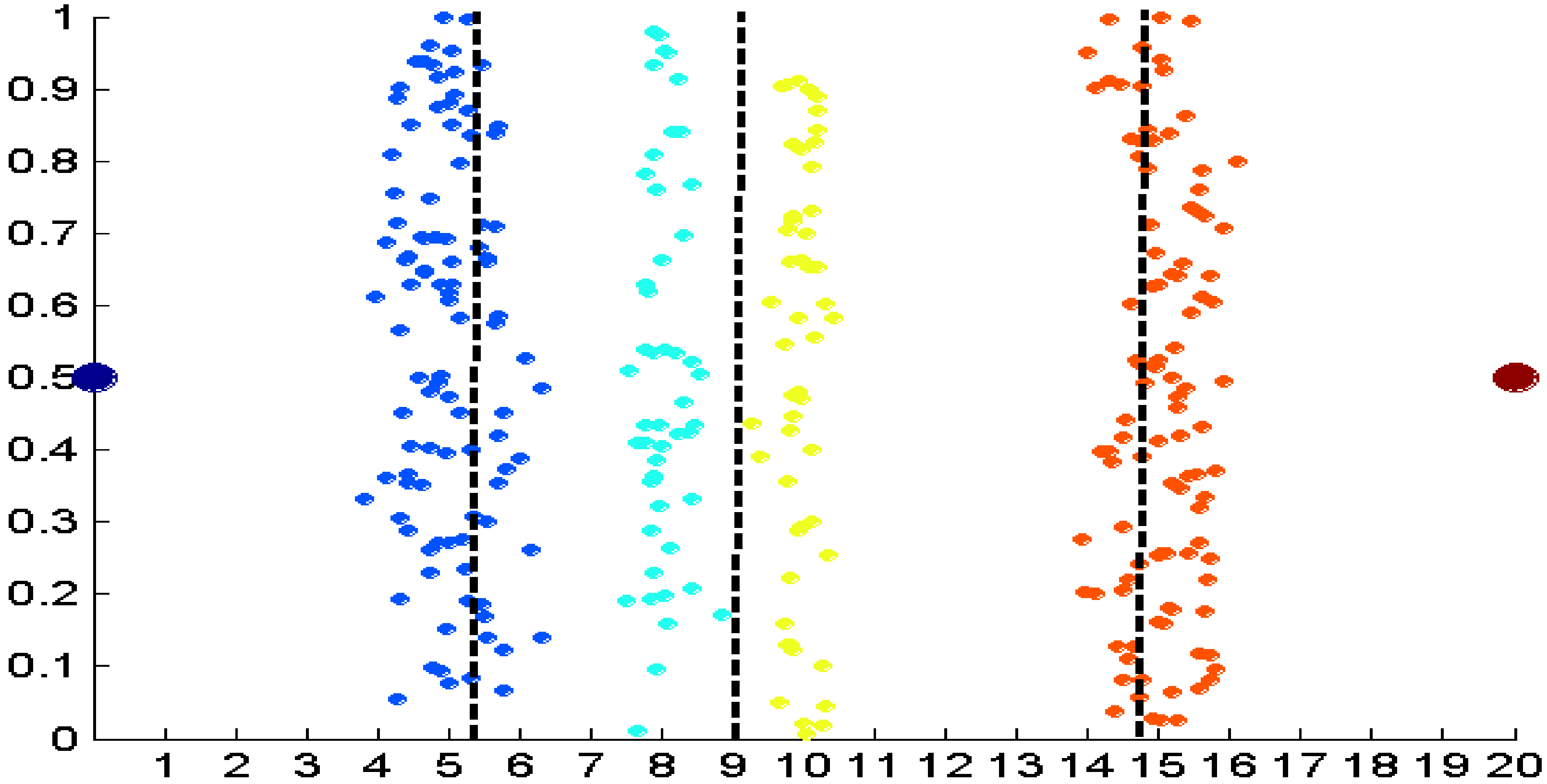
From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Example: unsupervised discretization technique



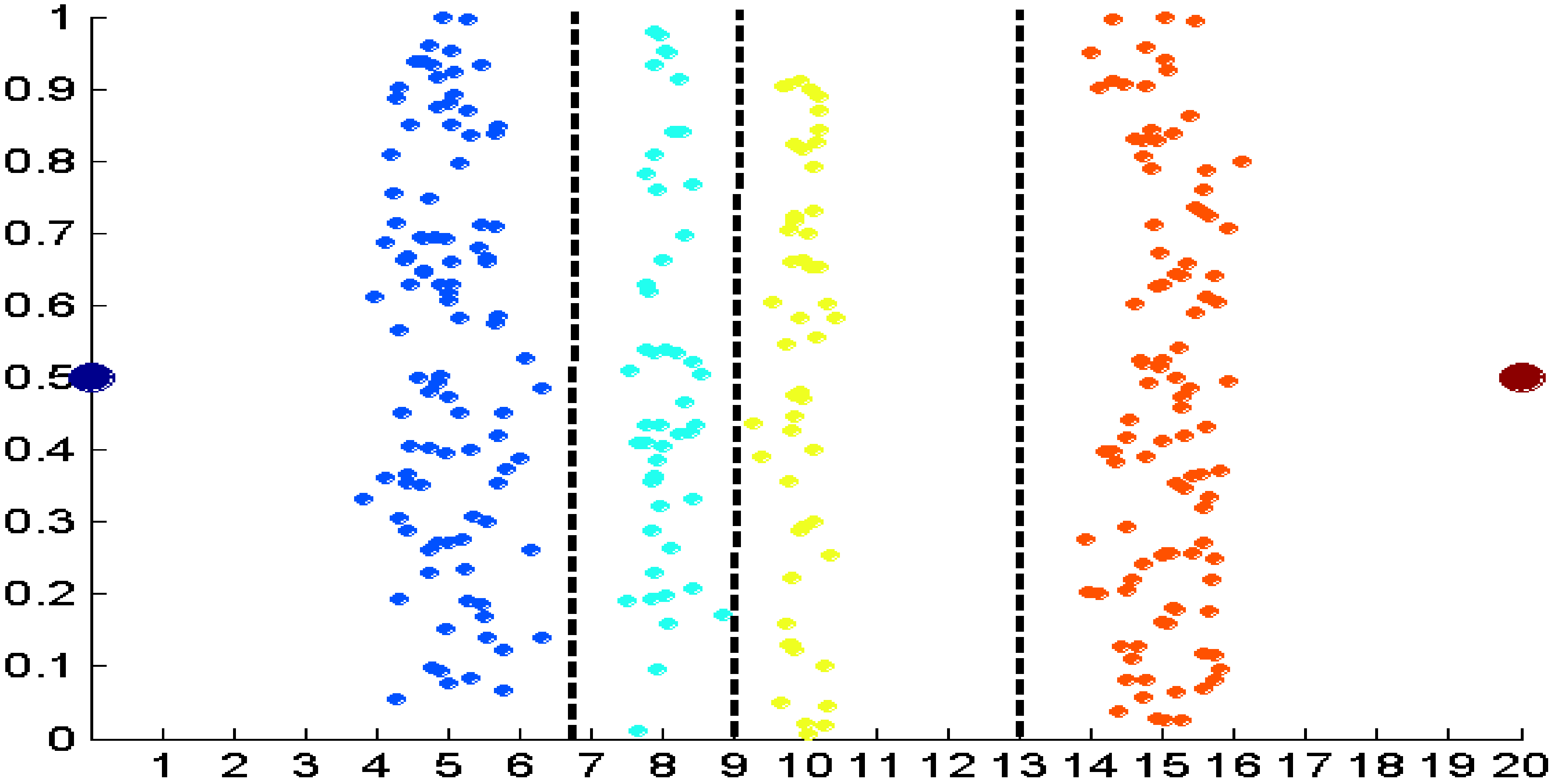
Equal interval width approach used to obtain 4 values.

Example: unsupervised discretization technique



Equal frequency approach used to obtain 4 values.

Example: unsupervised discretization technique



K-means approach to obtain 4 values.

Binarization

- Binarization maps an attribute into one or more binary variables
- Continuous attribute: first map the attribute to a categorical one
  - Example: height measured as {low, medium, high}
- Categorical attribute
  - Mapping to a set of binary attributes
  - Example: Low, medium, high as 1 0 0, 0 1 0, 0 0 1
  - One-hot encoding
    - Only 1 bit takes value 1
    - It represents the specific value taken by the attribute

Attribute Transformation

- An attribute transform is a function that maps the entire set of values of a given attribute to a new set of replacement values such that each old value can be identified with one of the new values
  - Simple functions: x^k, $\log(x)$, e^x, $|x|$
- Normalization
  - Refers to various techniques to adjust to differences among attributes in terms of frequency of occurrence, mean, variance, range
  - Take out unwanted, common signal, e.g., seasonality
- In statistics, standardization refers to subtracting off the means and dividing by the standard deviation

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Normalization

- It is a type of data transformation
  - The values of an attribute are scaled so as to fall within a small specified range, typically [-1,+1] or [0,+1]
- Techniques
  - min-max normalization

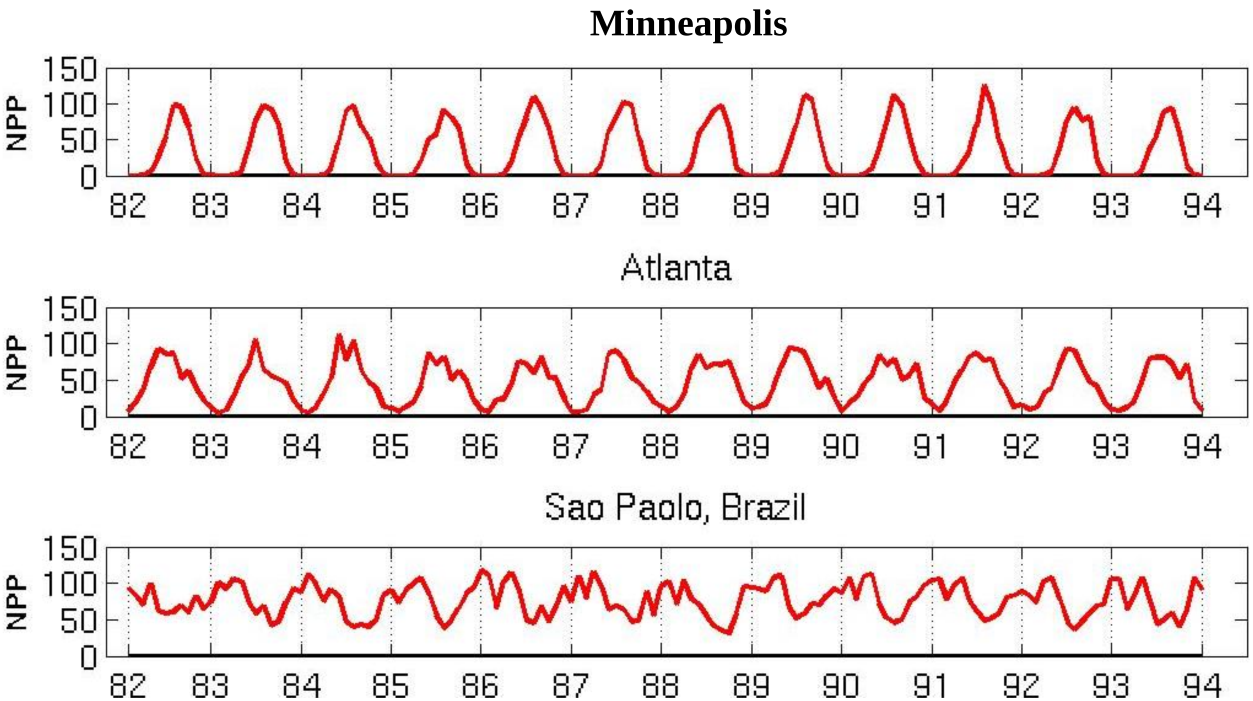
$$v' = \frac{v - min_A}{max_A - min_A}(new\_max_A - new\_min_A) + new\_min_A$$

  - z-score normalization $v' = \frac{v - mean_A}{stand\_dev_A}$
  - decimal scaling

$$v' = \frac{v}{10^j}$$

j is the smallest integer such that $\max(|v'|) < 1$

Example: Sample Time Series of Plant Growth



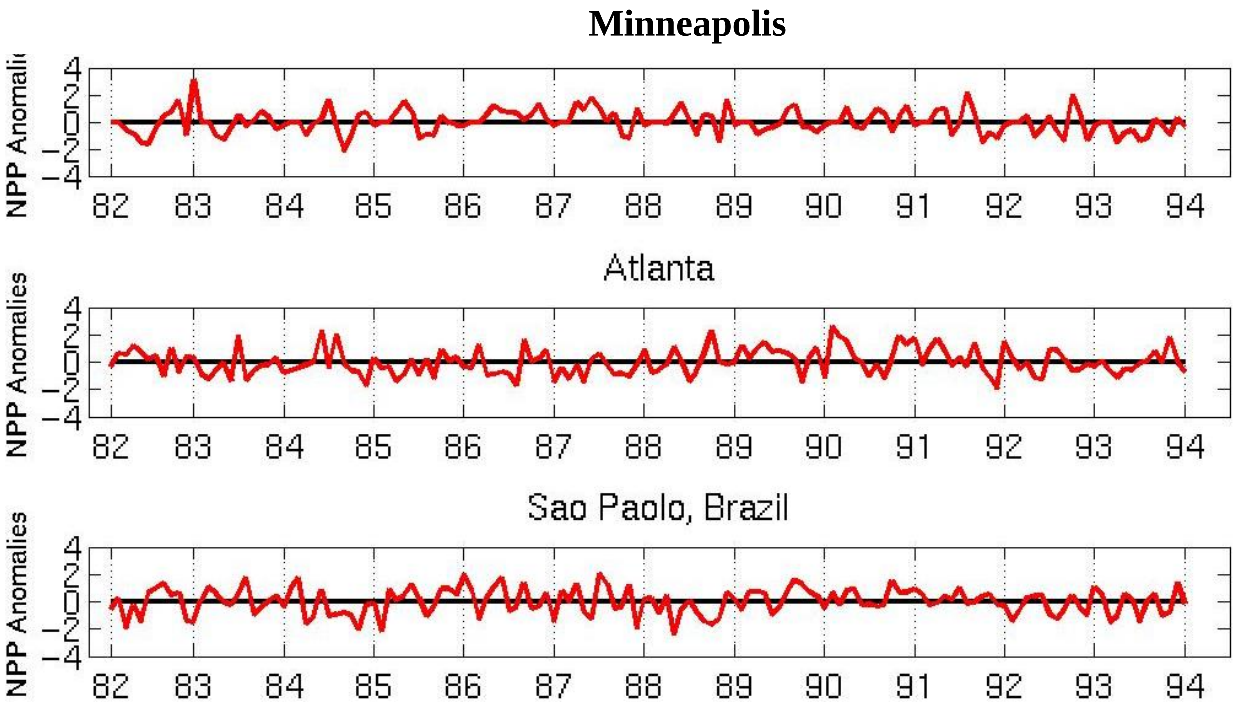
Net Primary Production (NPP) is a measure of plant growth used by ecosystem scientists.

Correlations between time series

	Minneapolis	Atlanta	Sao Paolo
Minneapolis	1.0000	0.7591	-0.7581
Atlanta	0.7591	1.0000	-0.5739
Sao Paolo	-0.7581	-0.5739	1.0000

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Example: Sample Time Series of Plant Growth



Normalized using monthly Z Score:

Subtract off monthly mean and divide by monthly standard deviation

Correlations between time series

	Minneapolis	Atlanta	Sao Paolo
Minneapolis	1.0000	0.0492	0.0906
Atlanta	0.0492	1.0000	-0.0154
Sao Paolo	0.0906	-0.0154	1.0000

Similarity and dissimilarity

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Similarity and Dissimilarity

- Similarity
  - Numerical measure of how alike two data objects are
  - Is higher when objects are more alike
  - Often falls in the range [0,1]
- Dissimilarity
  - Numerical measure of how different are two data objects
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- Proximity refers to a similarity or dissimilarity

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Similarity/Dissimilarity for Simple Attributes

The following table shows the similarity and dissimilarity between two objects, x and y, with respect to a single, simple attribute.

Attribute Type	**Dissimilarity**	**Similarity**
Nominal	$d = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$	$s = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{if } x \neq y \end{cases}$
Ordinal	$d = \lvert x - y \rvert / (n - 1)$ (values mapped to integers 0 to $n-1$, where n is the number of values)	$s = 1 - d$
Interval or Ratio	$d = \lvert x - y \rvert$	$s = -d$, $s = \frac{1}{1+d}$, $s = e^{-d}$, $s = 1 - \frac{d - min\_d}{max\_d - min\_d}$

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2[nd] Ed., McGraw Hill 2018

Euclidean Distance

- Euclidean Distance

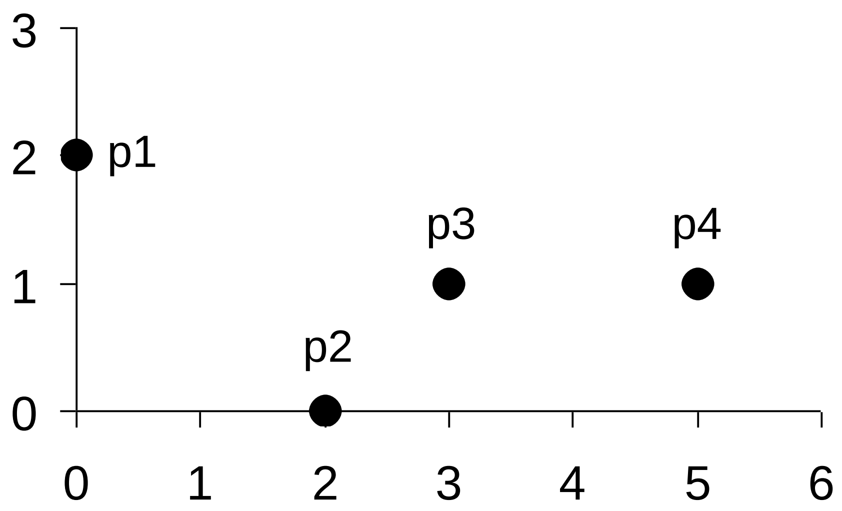
$$d(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{k=1}^{n} (x_k - y_k)^2}$$

where *n* is the number of dimensions (attributes) and x_k and y_k are, respectively, the k^{th} attributes (components) of data objects $\mathbf{x}$ and $\mathbf{y}$.

- Standardization is necessary, if scales differ.

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Euclidean Distance



point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

Distance Matrix

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Minkowski Distance

- Minkowski Distance is a generalization of Euclidean Distance

$$d(\mathbf{x},\mathbf{y}) = \left(\sum_{k=1}^{n} |x_k - y_k|^r \right)^{1/r}$$

Where r is a parameter, n is the number of dimensions (attributes) and x_k and y_k are, respectively, the k^{th} attributes (components) or data objects $\boldsymbol{x}$ and $\boldsymbol{y}$.

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Minkowski Distance: Examples

- $r = 1$. City block (Manhattan, taxicab, L_1 norm) distance
  - A common example of this is the Hamming distance, which is just the number of bits that are different between two binary vectors
- $r = 2$. Euclidean distance
- $r \rightarrow \infty$. "supremum" (L_{max} norm, L_∞ norm) distance
  - This is the maximum difference between any component of the vectors
- Do not confuse *r* with *n*, i.e., all these distances are defined for all numbers of dimensions.

Minkowski Distance

point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

L1	p1	p2	p3	p4
p1	0	4	4	6
p2	4	0	2	4
p3	4	2	0	2
p4	6	4	2	0

L2	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

L_∞	p1	p2	p3	p4
p1	0	2	3	5
p2	2	0	1	3
p3	3	1	0	2
p4	5	3	2	0

Distance Matrix

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Common Properties of a Distance

- Distances, such as the Euclidean distance, have some well-known properties.

1. $d(\mathbf{x}, \mathbf{y}) \geq 0$ for all *x* and *y* and $d(\mathbf{x}, \mathbf{y}) = 0$ only if $\mathbf{x} = \mathbf{y}$. (Positive definiteness)
2. $d(\mathbf{x}, \mathbf{y}) = d(\mathbf{y}, \mathbf{x})$ for all $\mathbf{x}$ and $\mathbf{y}$. (Symmetry)
3. $d(\mathbf{x}, \mathbf{z}) \leq d(\mathbf{x}, \mathbf{y}) + d(\mathbf{y}, \mathbf{z})$ for all points $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$. (Triangle Inequality)

where $d(\mathbf{x}, \mathbf{y})$ is the distance (dissimilarity) between points (data objects) $\mathbf{x}$ and $\mathbf{y}$.

- A distance that satisfies these properties is a metric

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Common Properties of a Similarity

- Similarities also have some well known properties

1. $s(\mathbf{x}, \mathbf{y}) = 1$ (or maximum similarity) only if $\mathbf{x} = \mathbf{y}$

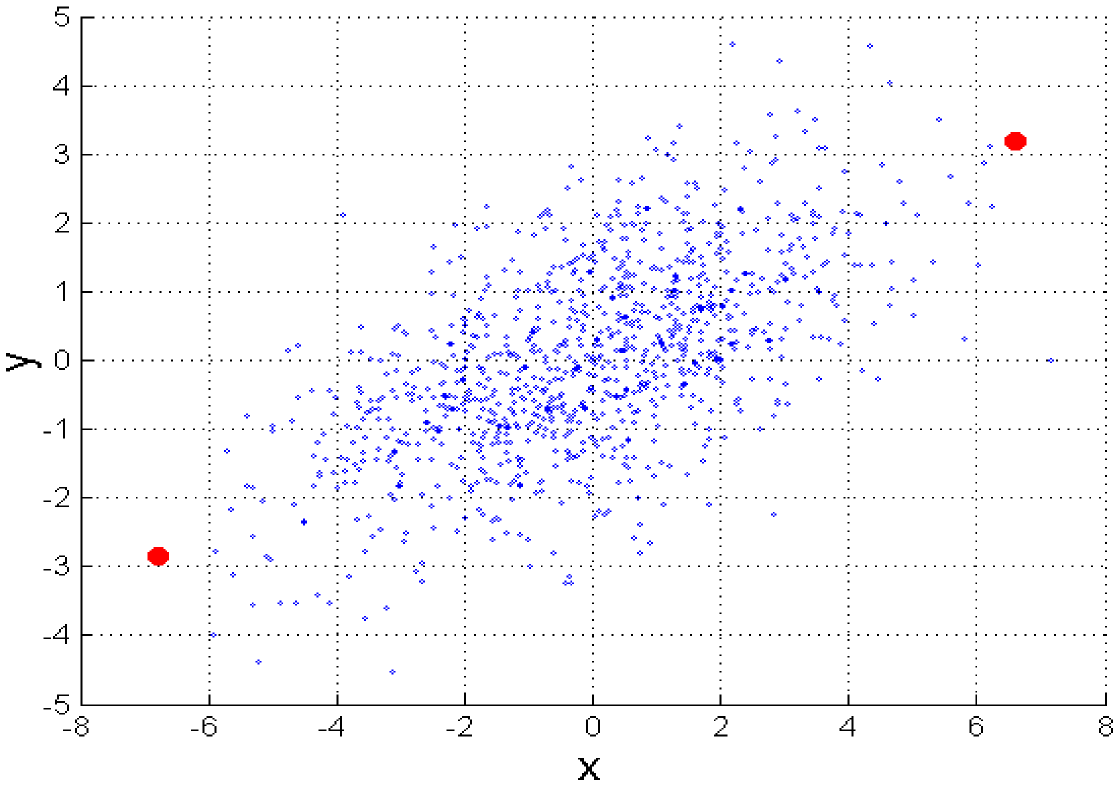
2. $s(\mathbf{x}, \mathbf{y}) = s(\mathbf{y}, \mathbf{x})$ for all $\mathbf{x}$ and $\mathbf{y}$ (symmetry)

where $s(\mathbf{x}, \mathbf{y})$ is the similarity between points (data objects) $\mathbf{x}$ and $\mathbf{y}$

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Mahalanobis Distance

$$\mathbf{mahalanobis}(\mathbf{x}, \mathbf{y}) = (\mathbf{x} - \mathbf{y})^T \Sigma^{-1} (\mathbf{x} - \mathbf{y})$$

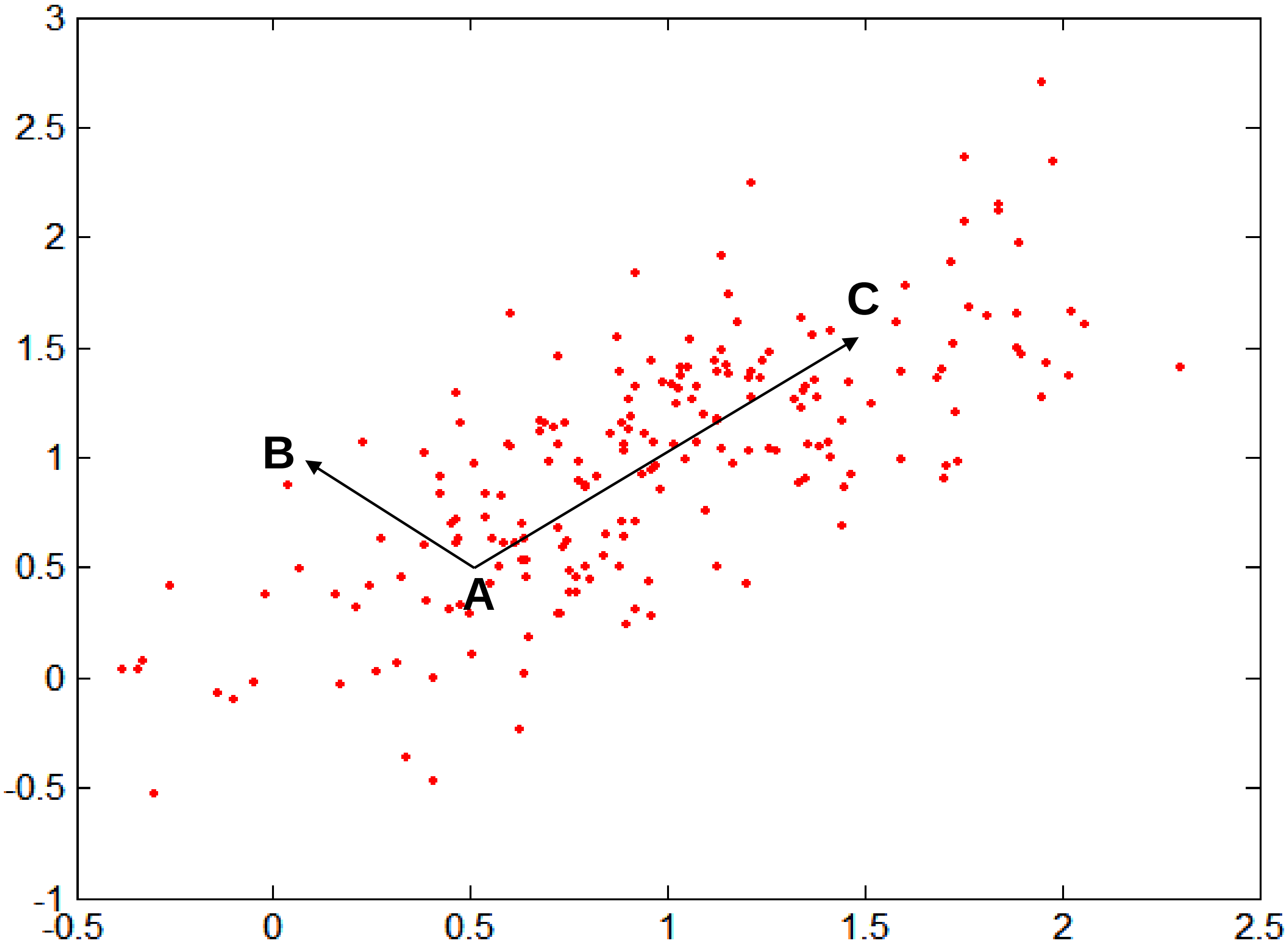


Σ is the covariance matrix

For red points, the Euclidean distance is 14.7, Mahalanobis distance is 6.

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Mahalanobis Distance



Covariance Matrix:

$$\Sigma = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$$

A: (0.5, 0.5)

B: (0, 1)

C: (1.5, 1.5)

Mahal(A,B) = 5

Mahal(A,C) = 4

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Similarity Between Binary Vectors

- Common situation is that objects *p* and *q* have only binary attributes
- Compute similarities using the following quantities
  M_{01} = the number of attributes where p was 0 and q was 1
  M_{10} = the number of attributes where p was 1 and q was 0
  M_{00} = the number of attributes where p was 0 and q was 0
  M_{11} = the number of attributes where p was 1 and q was 1
- Simple Matching and Jaccard Coefficients
  SMC = number of matches / number of attributes
  $= (M_{11} + M_{00}) / (M_{01} + M_{10} + M_{11} + M_{00})$

  J = number of 11 matches / number of not-both-zero attributes values
  $= (M_{11}) / (M_{01} + M_{10} + M_{11})$

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

SMC versus Jaccard: Example

$p =$ 1 0 0 0 0 0 0 0 0 0
$q =$ 0 0 0 0 0 0 1 0 0 1

$M_{01} = 2$ (the number of attributes where p was 0 and q was 1)
$M_{10} = 1$ (the number of attributes where p was 1 and q was 0)
$M_{00} = 7$ (the number of attributes where p was 0 and q was 0)
$M_{11} = 0$ (the number of attributes where p was 1 and q was 1)

$SMC = (M_{11} + M_{00})/(M_{01} + M_{10} + M_{11} + M_{00}) = (0+7) / (2+1+0+7) = 0.7$

$J = (M_{11}) / (M_{01} + M_{10} + M_{11}) = 0 / (2 + 1 + 0) = 0$

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Cosine Similarity

- If d_1 and d_2 are two document vectors, then

  $$\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1|| \, ||d_2|| ,$$

  where $\bullet$ indicates vector dot product and $|| d ||$ is the norm of vector d

- Example:

  d_1 = **3 2 0 5 0 0 0 2 0 0**
  d_2 = **1 0 0 0 0 0 0 1 0 2**

$d_1 \bullet d_2$= 3*1 + 2*0 + 0*0 + 5*0 + 0*0 + 0*0 + 0*0 + 2*1 + 0*0 + 0*2 = 5

$||d_1||$ = (3*3+2*2+0*0+5*5+0*0+0*0+0*0+2*2+0*0+0*0)$^{0.5}$ = (42) $^{0.5}$ = 6.481

$||d_2||$ = (1*1+0*0+0*0+0*0+0*0+0*0+0*0+1*1+0*0+2*2) $^{0.5}$ = (6) $^{0.5}$ = 2.245

$\cos(d_1, d_2)$ = .3150

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

General Approach for Combining Similarities

- Sometimes attributes are of many different types, but an overall similarity is needed.

1: For the k^{th} attribute, compute a similarity, $s_k(\mathbf{x}, \mathbf{y})$, in the range [0, 1].

2: Define an indicator variable, δ_k, for the k^{th} attribute as follows:

δ_k = 0 if the k^{th} attribute is an asymmetric attribute and both objects have a value of 0, or if one of the objects has a missing value for the kth attribute

δ_k = 1 otherwise

3. Compute $$\text{similarity}(\mathbf{x}, \mathbf{y}) = \frac{\sum_{k=1}^{n} \delta_k s_k(\mathbf{x}, \mathbf{y})}{\sum_{k=1}^{n} \delta_k}$$

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Using Weights to Combine Similarities

- May not want to treat all attributes the same.
  - Use non-negative weights ω_k
  - $similarity(\mathbf{x}, \mathbf{y}) = \frac{\sum_{k=1}^{n} \omega_k \delta_k s_k(\mathbf{x},\mathbf{y})}{\sum_{k=1}^{n} \omega_k \delta_k}$
- Can also define a weighted form of distance

$$d(\mathbf{x}, \mathbf{y}) = \left(\sum_{k=1}^{n} w_k |x_k - y_k|^r \right)^{1/r}$$

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Correlation

Data Base and Data Mining Group of Politecnico di Torino

Data correlation

- Measure of the linear relationship between two data objects
  - having binary or continuous variables
- Useful during the data exploration phase
  - To be better aware of data properties
- Analysis of feature correlation
  - Correlated features should be removed
    - simplifying the next analytics steps
    - improving the performance of the data-driven algorithms

Pearson's correlation

$$\text{corr}(\mathbf{x}, \mathbf{y}) = \frac{\text{covariance}(\mathbf{x}, \mathbf{y})}{\text{standard\_deviation}(\mathbf{x}) * \text{standard\_deviation}(\mathbf{y})} = \frac{s_{xy}}{s_x \, s_y},$$

where we are using the following standard statistical notation and definitions

$$\text{covariance}(\mathbf{x}, \mathbf{y}) = s_{xy} = \frac{1}{n-1} \sum_{k=1}^{n} (x_k - \overline{x})(y_k - \overline{y})$$

$$\text{standard\_deviation}(\mathbf{x}) = s_x = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} (x_k - \overline{x})^2}$$

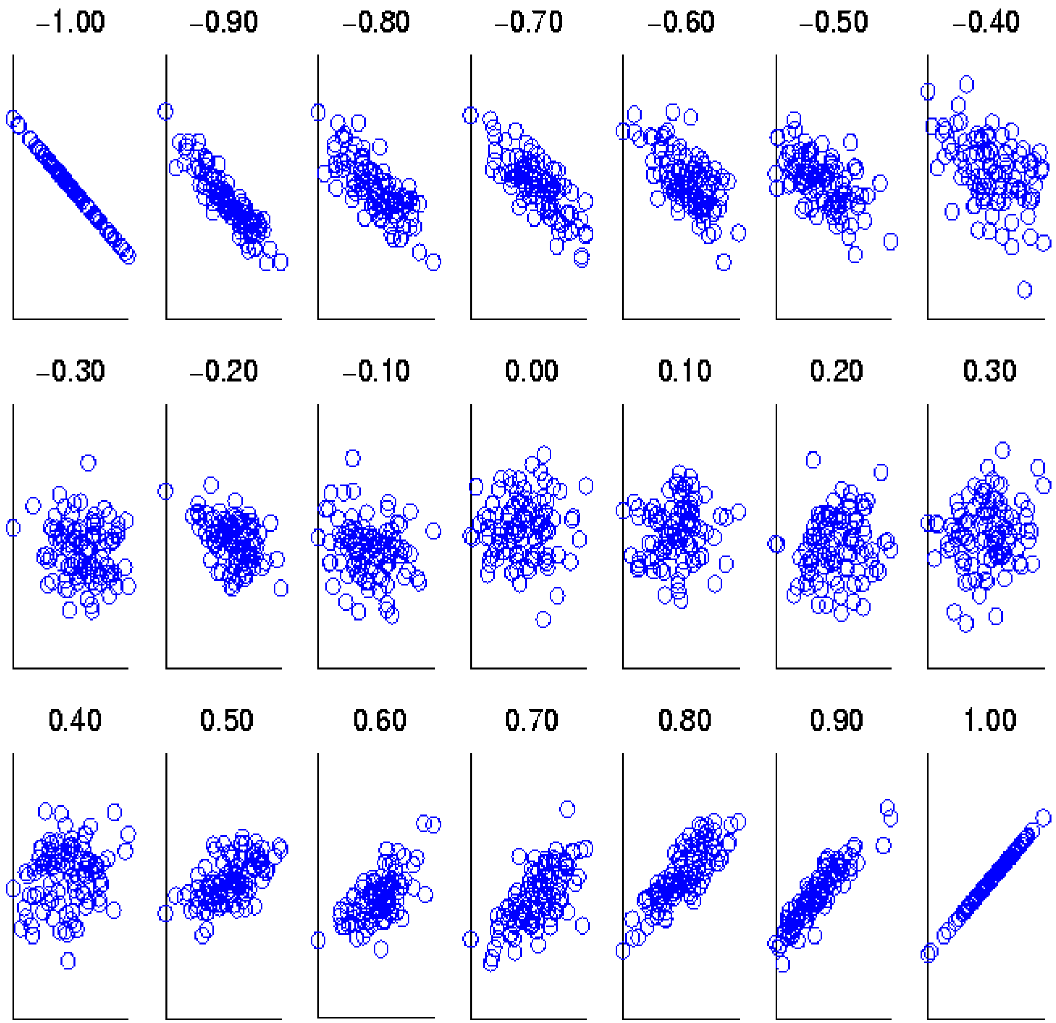
$$\text{standard\_deviation}(\mathbf{y}) = s_y = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} (y_k - \overline{y})^2}$$

$$\overline{x} = \frac{1}{n} \sum_{k=1}^{n} x_k \text{ is the mean of } \mathbf{x}$$

$$\overline{y} = \frac{1}{n} \sum_{k=1}^{n} y_k \text{ is the mean of } \mathbf{y}$$

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Visually Evaluating Correlation



Scatter plots showing the similarity from –1 to 1.

Perfect linear correlation when value is 1 or -1

From: Tan, Steinbach, Karpatne, Kumar, Introduction to Data Mining 2nd Ed., McGraw Hill 2018

Drawback of Correlation

- $\mathbf{x}$ = (-3, -2, -1, 0, 1, 2, 3)
- $\mathbf{y}$ = (9, 4, 1, 0, 1, 4, 9)

$y_i = x_i^2$

- mean($\mathbf{x}$) = 0, mean($\mathbf{y}$) = 4
- std($\mathbf{x}$) = 2.16, std($\mathbf{y}$) = 3.74

corr = (-3)(5)+(-2)(0)+(-1)(-3)+(0)(-4)+(1)(-3)+(2)(0)+3(5) / (6 * 2.16 * 3.74)

= 0