Association Rules Fundamentals

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Association rules

- Objective
  - extraction of frequent correlations or pattern from a transactional database

Tickets at a supermarket counter

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk
…	…

- Association rule

  diapers $\Rightarrow$ beer

  - 2% of transactions contains both items
  - 30% of transactions containing diapers also contains beer

Association rule mining

- A collection of transactions is given
  - a transaction is a set of items
  - items in a transaction are *not ordered*
- Association rule

$$A, B \Rightarrow C$$

  - A, B = items in the rule body
  - C = item in the rule head
- The $\Rightarrow$ means co-occurrence
  - *not* causality
- Example
  - coke, diapers $\Rightarrow$ milk

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk
…	…

Transactional formats

- Association rule extraction is an *exploratory technique* that can be applied to any data type
- A transaction can be any set of items
  - Market basket data
  - Textual data
  - Structured data
  - …

Transactional formats

- Textual data
  - A document is a transaction
  - Words in a document are items in the transaction

- Data example
  - Doc1: algorithm analysis customer data mining relationship
  - Doc2: customer data management relationship
  - Doc3: analysis customer data mining relationship social
- Rule example

customer, relationship $\Rightarrow$ data, mining

Transactional formats

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	**No**
2	No	Married	100K	**No**
3	No	Single	70K	**No**
4	Yes	Married	120K	**No**
5	No	Divorced	95K	**Yes**
6	No	Married	60K	**No**
7	Yes	Divorced	220K	**No**
8	No	Single	85K	**Yes**
9	No	Married	75K	**No**
10	No	Single	90K	**Yes**

- Structured data
  - A table row is a transaction
  - Pairs (attribute, value) are items in the transaction
- Data example

Refund	Marital Status	Taxable Income	Cheat
No	Married	< 80K	**No**

  - Transaction

    Refund=no, MaritalStatus=married, TaxableIncome<80K, Cheat=No
- Rule example

  Refund=No, MaritalStatus=Married $\Rightarrow$ Cheat = No

Example from: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Definitions

- *Itemset* is a set including one or more items
  - Example: {Beer, Diapers}
- *k-itemset* is an itemset that contains k items
- *Support count* (#) is the frequency of occurrence of an itemset
  - Example: #{Beer,Diapers} = 2
- *Support* is the fraction of transactions that contain an itemset
  - Example: sup({Beer, Diapers}) = 2/5
- *Frequent itemset* is an itemset whose support is greater than or equal to a *minsup* threshold

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk

Rule quality metrics

- Given the association rule

$$A \Rightarrow B$$

  - A, B are itemsets
- *Support* is the fraction of transactions containing both A and B

$$\frac{\#\{A,B\}}{|T|}$$

  - |T| is the cardinality of the transactional database
  - a priori probability of itemset AB
  - rule frequency in the database
- *Confidence* is the frequency of B in transactions containing A

$$\frac{sup(A,B)}{sup(A)}$$

  - conditional probability of finding B having found A
  - "strength" of the "$\Rightarrow$"

Rule quality metrics: example

- From itemset {Milk, Diapers} the following rules may be derived

- Rule: Milk $\Rightarrow$ Diapers
  - support
    sup=#{Milk,Diapers}/#trans. =3/5=60%
  - confidence
    conf=#{Milk,Diapers}/#{Milk}=3/4=75%
- Rule: Diapers $\Rightarrow$ Milk
  - same support
    s=60%
  - confidence
    conf=#{Milk,Diapers}/#{Diapers}=3/3 =100%

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk

Association rule extraction

- Given a set of transactions T, association rule mining is the extraction of the rules satisfying the constraints
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold
- The result is
  - complete (*all* rules satisfying both constraints)
  - correct (*only* the rules satisfying both constraints)
- May add other more complex constraints

Association rule extraction

- Brute-force approach
  - enumerate all possible permutations (i.e., association rules)
  - compute support and confidence for each rule
  - prune the rules that do not satisfy the *minsup* and *minconf* constraints
- Computationally *unfeasible*
- Given an itemset, the extraction process may be split
  - first generate frequent itemsets
  - next generate rules from each frequent itemset
- Example
  - Itemset

    {Milk, Diapers} sup=60%
  - Rules

    Milk $\Rightarrow$ Diapers (conf=75%)

    Diapers $\Rightarrow$ Milk (conf=100%)

Association rule extraction

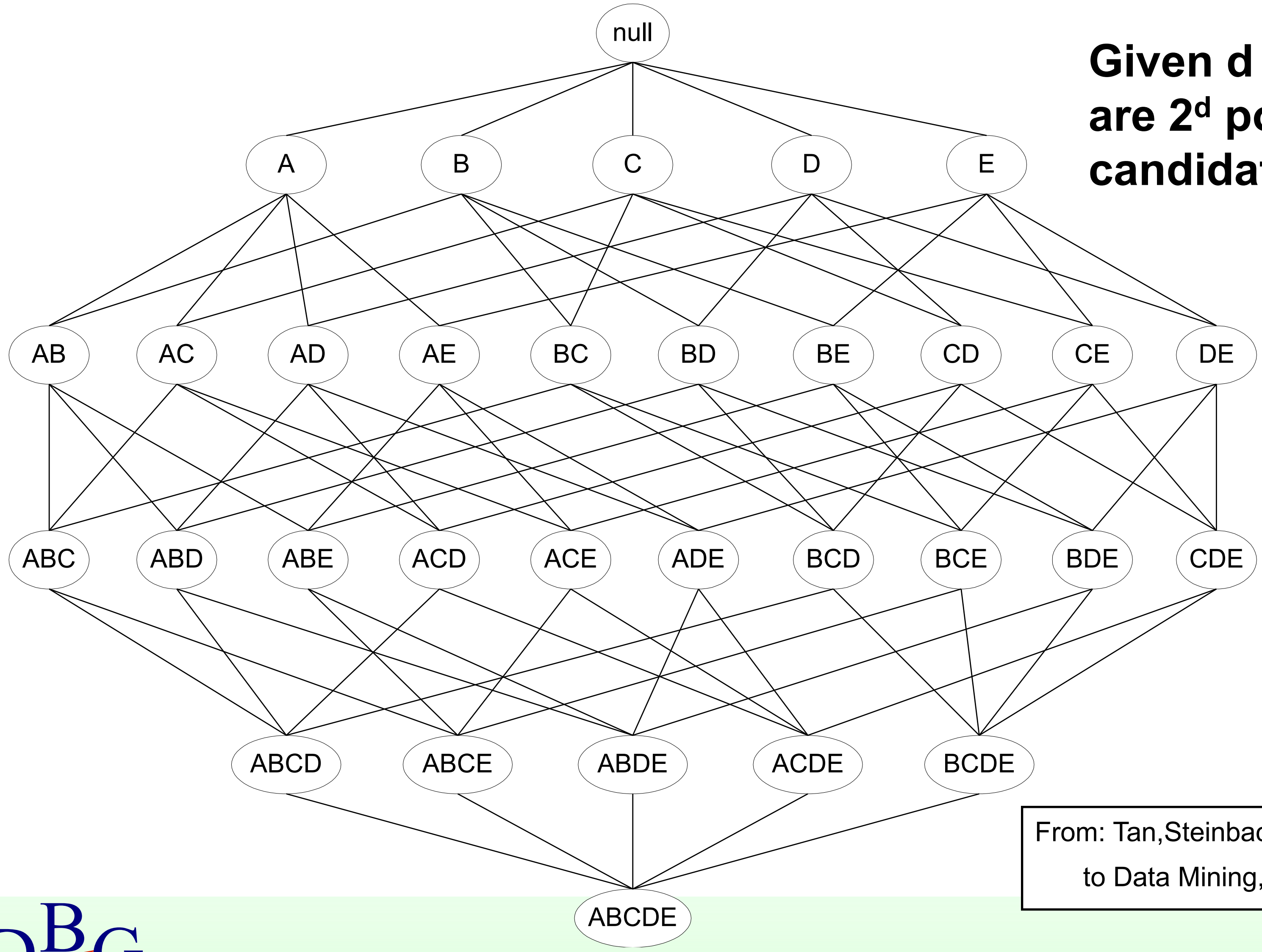
(1) Extraction of frequent itemsets

- many different techniques
  - level-wise approaches (Apriori, ...)
  - approaches without candidate generation (FP-growth, ...)
  - other approaches
- most computationally expensive step
  - limit extraction time by means of support threshold

(2) Extraction of association rules

- generation of all possible binary partitioning of each frequent itemset
  - possibly enforcing a confidence threshold

Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Frequent Itemset Generation

- Brute-force approach
  - each itemset in the lattice is a *candidate* frequent itemset
  - scan the database to count the support of each candidate
    - match each transaction against every candidate
  - Complexity ~ $O(|T|\ 2^d\ w)$
    - $|T|$ is number of transactions
    - d is number of items
    - w is transaction length

Improving Efficiency

- Reduce the number of candidates
  - Prune the search space
    - complete set of candidates is 2^d
- Reduce the number of transactions
  - Prune transactions as the size of itemsets increases
    - reduce $|T|$
- Reduce the number of comparisons
  - Equal to $|T|\ 2^d$
  - Use efficient data structures to store the candidates or transactions

The Apriori Principle

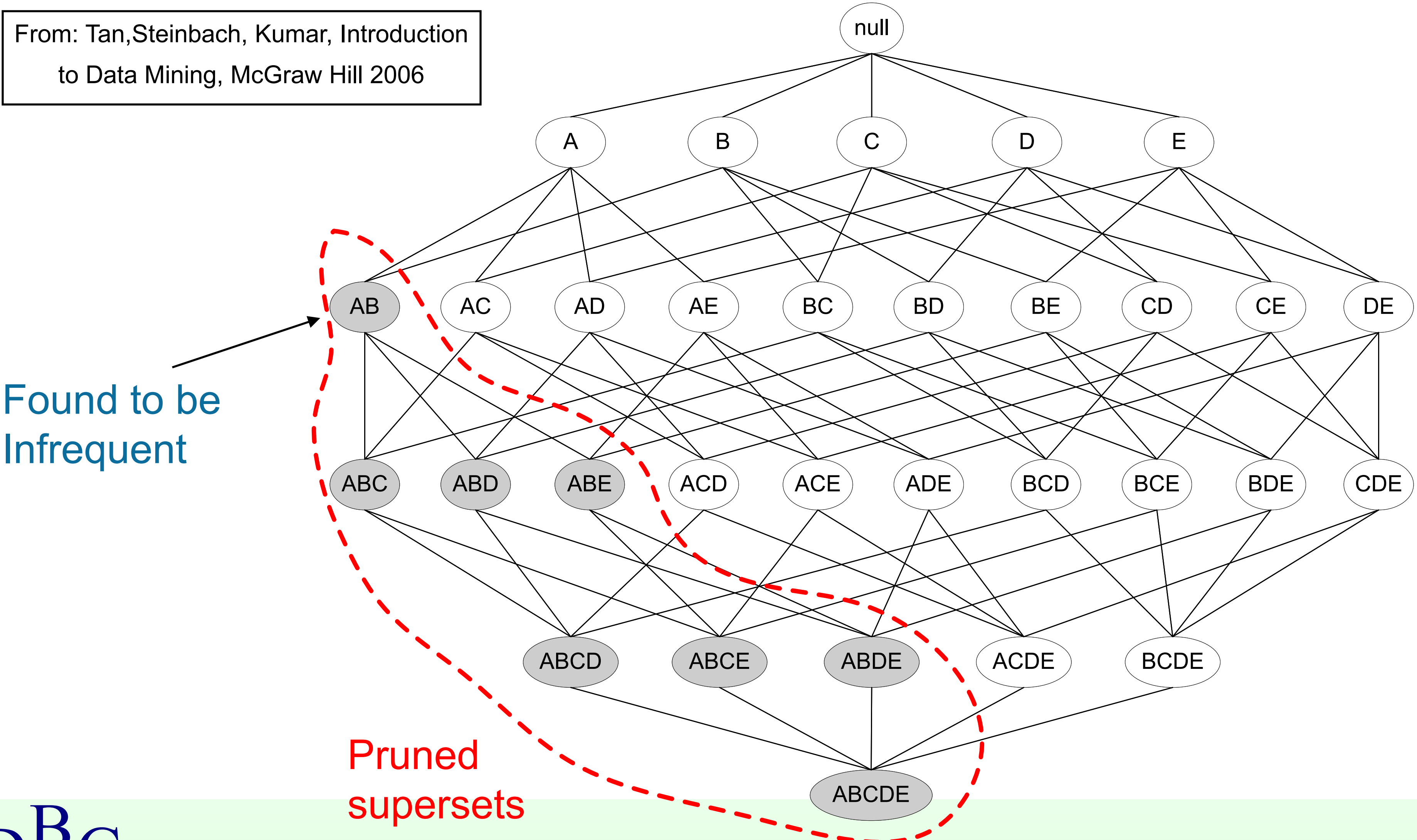
"If an itemset is frequent, then all of its subsets must also be frequent"

- The support of an itemset can never exceed the support of any of its subsets
- It holds due to the antimonotone property of the support measure
  - Given two arbitrary itemsets A and B

    if $A \subseteq B$ then $sup(A) \geq sup(B)$
- It reduces the number of candidates

The Apriori Principle

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006



Apriori Algorithm [Agr94]

- Level-based approach
  - at each iteration extracts itemsets of a given length k
- Two main steps for each level
  - (1) Candidate generation
    - Join Step
      - generate candidates of length k+1 by joining frequent itemsets of length k
    - Prune Step
      - apply Apriori principle: prune length k+1 candidate itemsets that contain at least one k-itemset that is not frequent
  - (2) Frequent itemset generation
    - scan DB to count support for k+1 candidates
    - prune candidates below minsup

Apriori Algorithm [Agr94]

- Pseudo-code

C_k: Candidate itemset of size k
L_k : frequent itemset of size k

L_1 = {frequent items};
for (k = 1; L_k !=$\emptyset$; k++) **do**
begin
C_{k+1} = candidates generated from L_k;
for each transaction t in database do
increment the count of all candidates in C_{k+1} that are contained in t
L_{k+1} = candidates in C_{k+1} satisfying *minsup*
end
return $\cup_k L_k$;

Generating Candidates

- Sort L_k candidates in lexicographical order
- For each candidate of length k
  - Self-join with each candidate sharing same L_{k-1} prefix
  - Prune candidates by applying Apriori principle
- Example: given L_3={*abc, abd, acd, ace, bcd*}
  - Self-join
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Prune by applying Apriori principle
    - *acde* is removed because *ade, cde* are not in L_3
    - C_4={*abcd*}

Apriori Algorithm: Example

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup>1

Generate candidate 1-itemsets

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

1st DB scan

C_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

minsup>1

Prune infrequent candidates in C_1

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup>1

1st DB scan

C_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

$L_1 \equiv C_1$

- All itemsets in set C_1 are frequent according to minsup>1

Generate candidates from L_1

L_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

C_2

itemsets
{A,B}
{A,C}
{A,D}
{A,E}
{B,C}
{B,D}
{B,E}
{C,D}
{C,E}
{D,E}

Count support for candidates in C_2

L_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

C_2

itemsets
{A,B}
{A,C}
{A,D}
{A,E}
{B,C}
{B,D}
{B,E}
{C,D}
{C,E}
{D,E}

2nd DB scan

C_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{B,E}	1
{C,D}	3
{C,E}	2
{D,E}	2

Prune infrequent candidates in C_2

L_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

C_2

itemsets
{A,B}
{A,C}
{A,D}
{A,E}
{B,C}
{B,D}
{B,E}
{C,D}
{C,E}
{D,E}

2nd DB scan

C_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
~~{B,E}~~	~~1~~
{C,D}	3
{C,E}	2
{D,E}	2

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

Generate candidates from L_2

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

C_3

itemsets
{A,B,C}
{A,B,D}
{A,B,E}
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

Apply Apriori principle on C_3

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

C_3

itemsets
{A,B,C}
{A,B,D}
~~{A,B,E}~~
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

- Prune {A,B,E}
  - Its subset {B,E} is infrequent ({B,E} is not in L_2)

Count support for candidates in C_3

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

C_3

itemsets
{A,B,C}
{A,B,D}
~~{A,B,E}~~
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

3rd DB scan

C_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,C,E}	1
{A,D,E}	2
{B,C,D}	2
{C,D,E}	1

Prune infrequent candidates in C_3

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

C_3

itemsets
{A,B,C}
{A,B,D}
~~{A,B,E}~~
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

3rd DB scan

C_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
~~{A,C,E}~~	~~1~~
{A,D,E}	2
{B,C,D}	2
~~{C,D,E}~~	~~1~~

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

- {A,C,E} and {C,D,E} are actually infrequent
  - They are discarded from C_3

Generate candidates from L_3

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

C_4

itemsets
{A,B,C,D}

Apply Apriori principle on C_4

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

C_4

itemsets
{A,B,C,D}

- Check if {A,C,D} and {B,C,D} belong to L_3
  - L_3 contains all 3-itemset subsets of {A,B,C,D}
  - {A,B,C,D} is potentially frequent

Count support for candidates in C_4

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

C_4

itemsets
{A,B,C,D}

4th DB scan

C_4

itemsets	sup
{A,B,C,D}	1

Prune infrequent candidates in C_4

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

C_4

itemsets
{A,B,C,D}

4th DB scan

C_4

itemsets	sup
~~{A,B,C,D}~~	~~1~~

$L_4 = \emptyset$

- {A,B,C,D} is actually infrequent
  - {A,B,C,D} is discarded from C_4

Final set of frequent itemsets

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup>1

L_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

Counting Support of Candidates

- Scan transaction database to count support of each itemset
  - total number of candidates may be large
  - one transaction may contain many candidates
- Approach [Agr94]
  - candidate itemsets are stored in a *hash-tree*
    - *leaf* node of hash-tree contains a list of itemsets and counts
    - *interior* node contains a hash table
  - subset function finds all candidates contained in a transaction
    - match transaction subsets to candidates in hash tree

Performance Issues in Apriori

- Candidate generation
  - Candidate sets may be huge
    - 2-itemset candidate generation is the most critical step
    - extracting long frequent intemsets requires generating all frequent subsets
- Multiple database scans
    - $n+1$ scans when longest frequent pattern length is n

Factors Affecting Performance

- Minimum support threshold
  - lower support threshold increases number of frequent itemsets
    - larger number of candidates
    - larger (max) length of frequent itemsets
- Dimensionality (number of items) of the data set
  - more space is needed to store support count of each item
  - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
  - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
  - transaction width increases in dense data sets
  - may increase max length of frequent itemsets and traversals of hash tree
    - number of subsets in a transaction increases with its width

Improving Apriori Efficiency

- Hash-based itemset counting [Yu95]
  - A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
- Transaction reduction [Yu95]
  - A transaction that does not contain any frequent k-itemset is useless in subsequent scans
- Partitioning [Sav96]
  - Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB

Improving Apriori Efficiency

- Sampling [Toi96]
  - mining on a subset of given data, lower support threshold + a method to determine the completeness
- Dynamic Itemset Counting [Motw98]
  - add new candidate itemsets only when all of their subsets are estimated to be frequent

FP-growth Algorithm [Han00]

- Exploits a main memory compressed representation of the database, the FP-tree
  - high compression for dense data distributions
    - less so for sparse data distributions
  - complete representation for frequent pattern mining
    - enforces support constraint
- Frequent pattern mining by means of FP-growth
  - recursive visit of FP-tree
  - applies divide-and-conquer approach
    - decomposes mining task into smaller subtasks
- Only two database scans
  - count item supports + build FP-tree

FP-tree construction

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup>1

- (1) Count item support and prune items below minsup threshold
- (2) Build Header Table by sorting items in decreasing support order

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

FP-tree construction

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup>1

- (1) Count item support and prune items below minsup threshold
- (2) Build Header Table by sorting items in decreasing support order
- (3) Create FP-tree

  For each transaction *t* in DB
  - order transaction *t* items in decreasing support order
    - same order as Header Table
  - insert transaction *t* in FP-tree
    - use existing path for common prefix
    - create new branch when path becomes different

FP-tree construction

Transaction

TID	Items
1	{A,B}

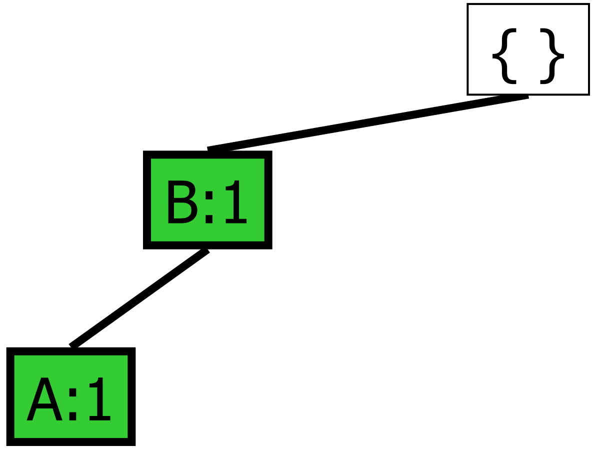
Sorted transaction

TID	Items
1	{B,A}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

FP-tree



FP-tree construction

Transaction

TID	Items
2	{B,C,D}

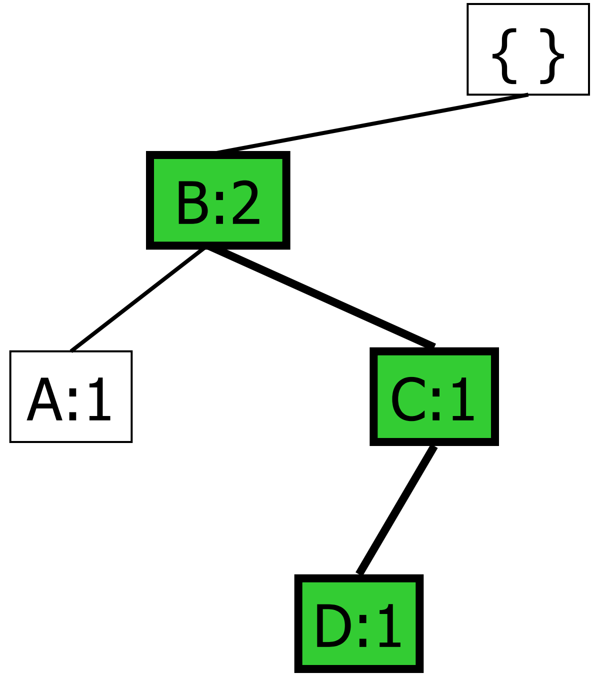
Sorted transaction

TID	Items
2	{B,C,D}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

FP-tree



FP-tree construction

Transaction

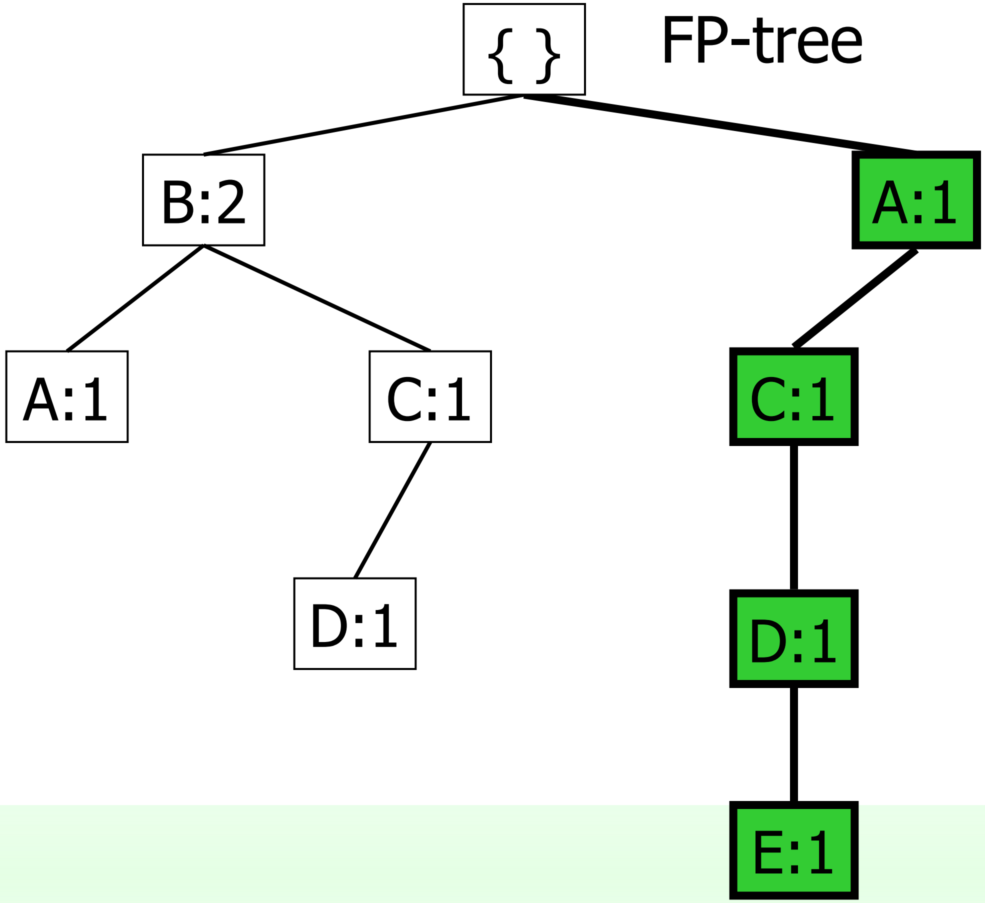
TID	Items
3	{A,C,D,E}

Sorted transaction

TID	Items
3	{A,C,D,E}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



FP-tree construction

Transaction

TID	Items
4	{A,D,E}

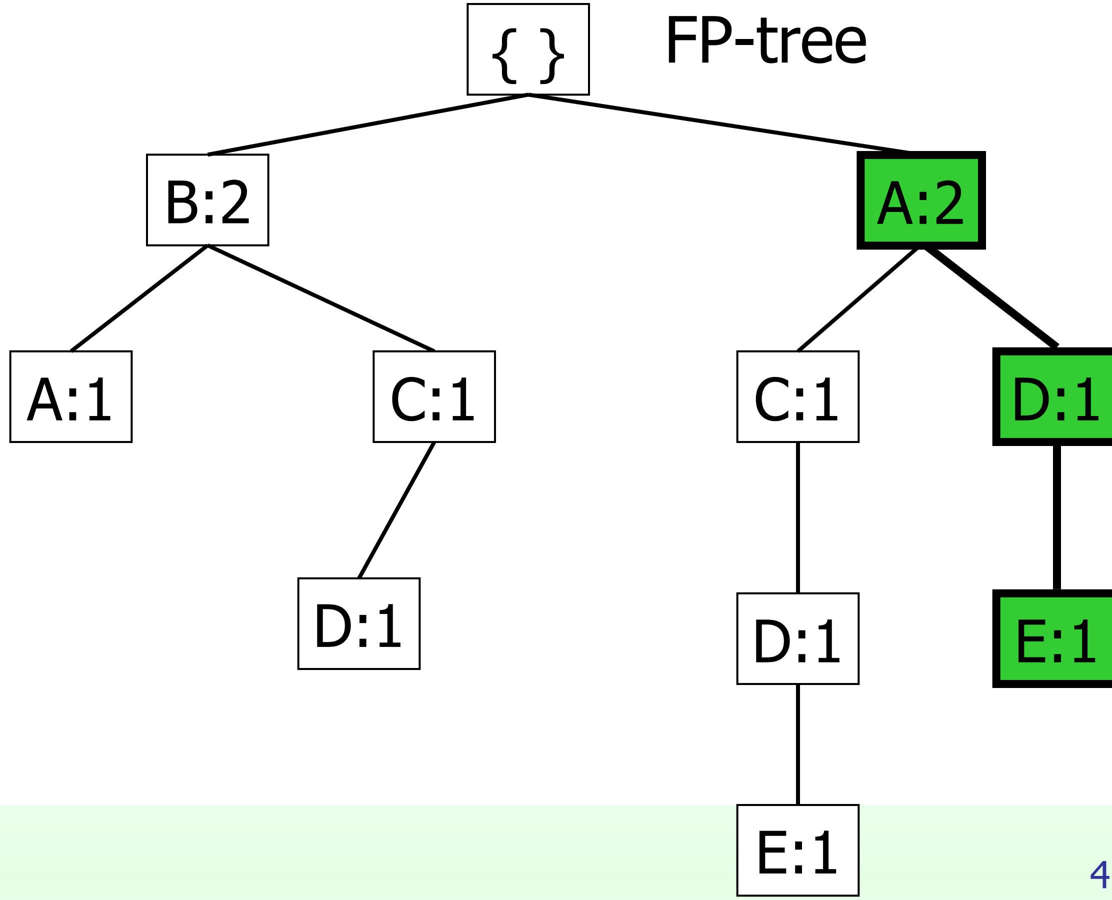
Sorted transaction

TID	Items
4	{A,D,E}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

FP-tree



FP-tree construction

Transaction

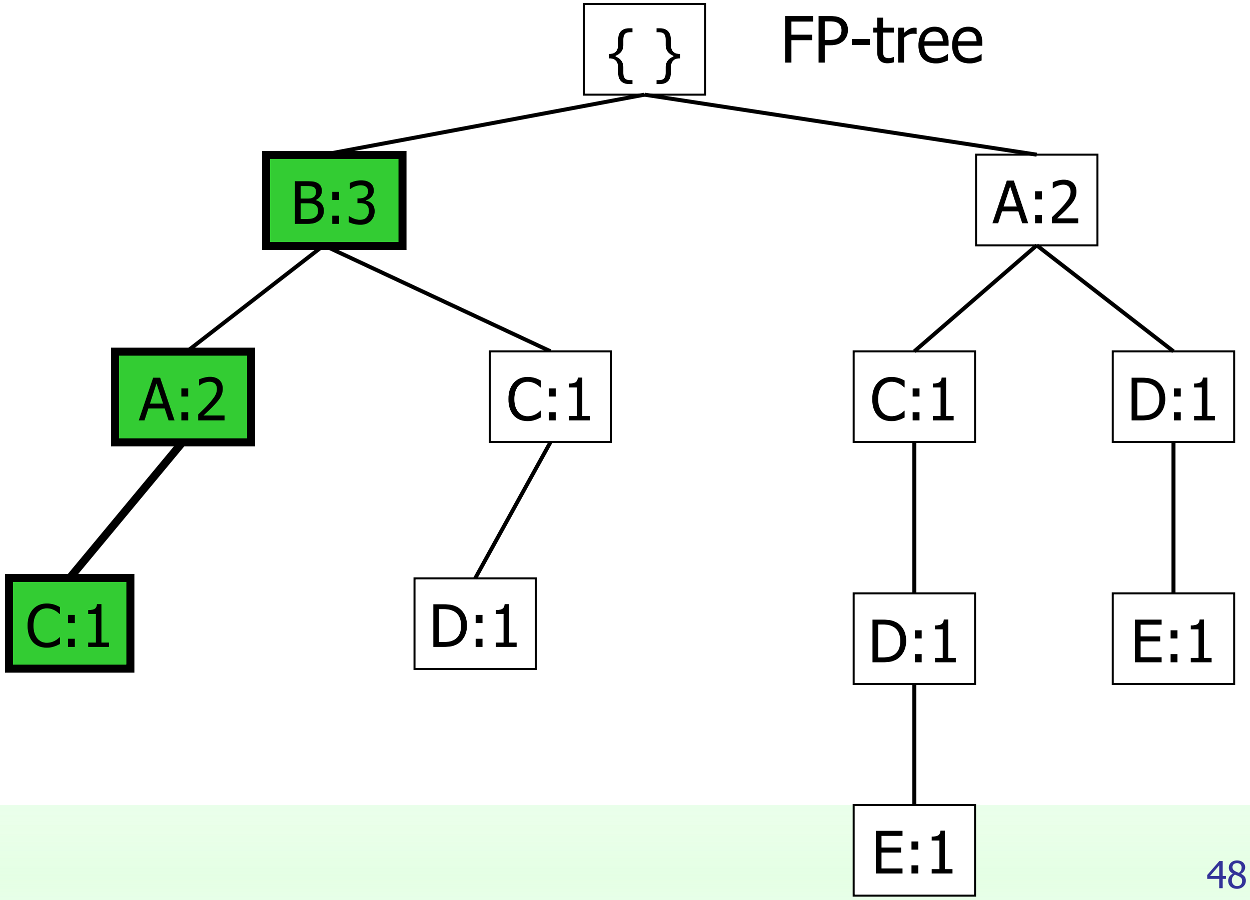
TID	Items
5	{A,B,C}

Sorted transaction

TID	Items
5	{B,A,C}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



FP-tree construction

Transaction

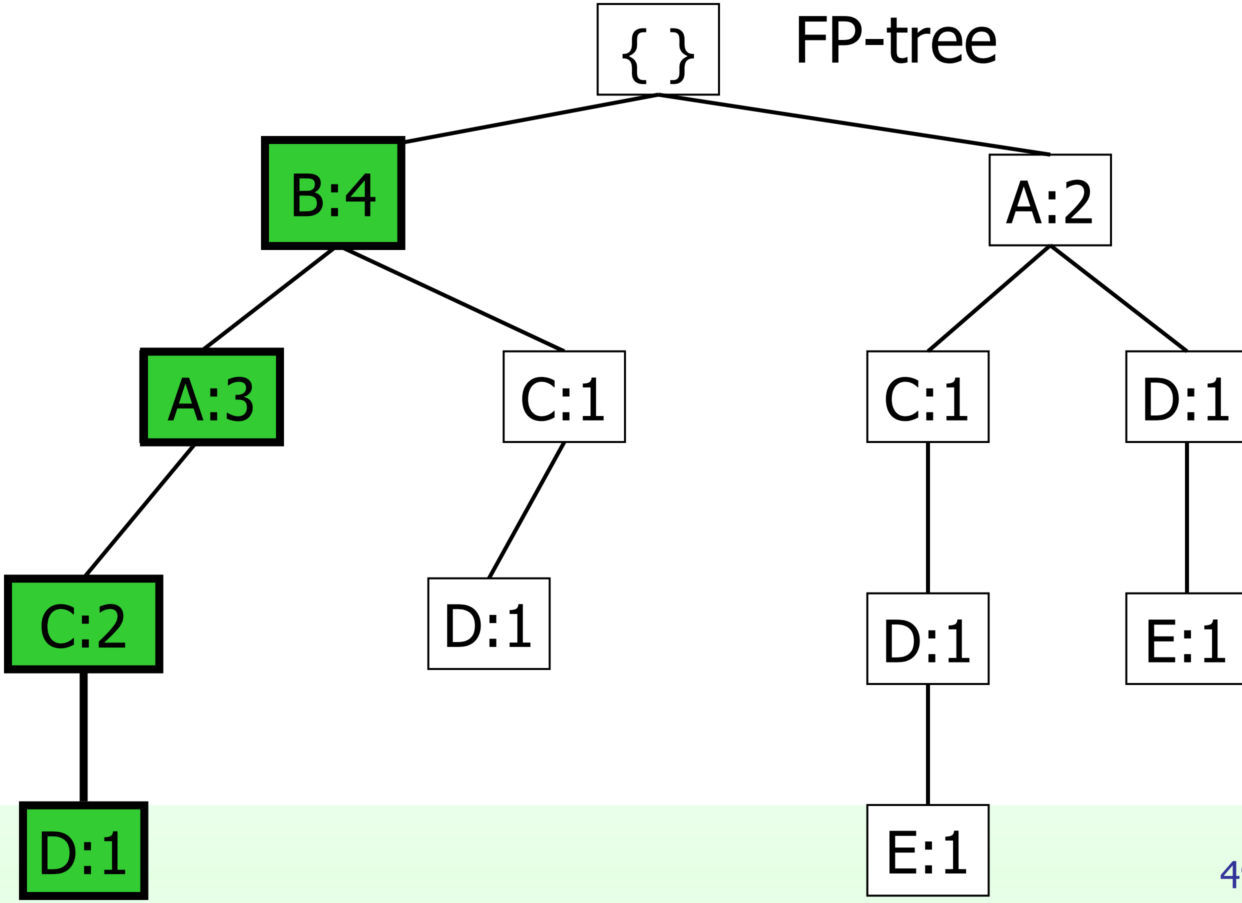
TID	Items
6	{A,B,C,D}

Sorted transaction

TID	Items
6	{B,A,C,D}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



FP-tree construction

Transaction

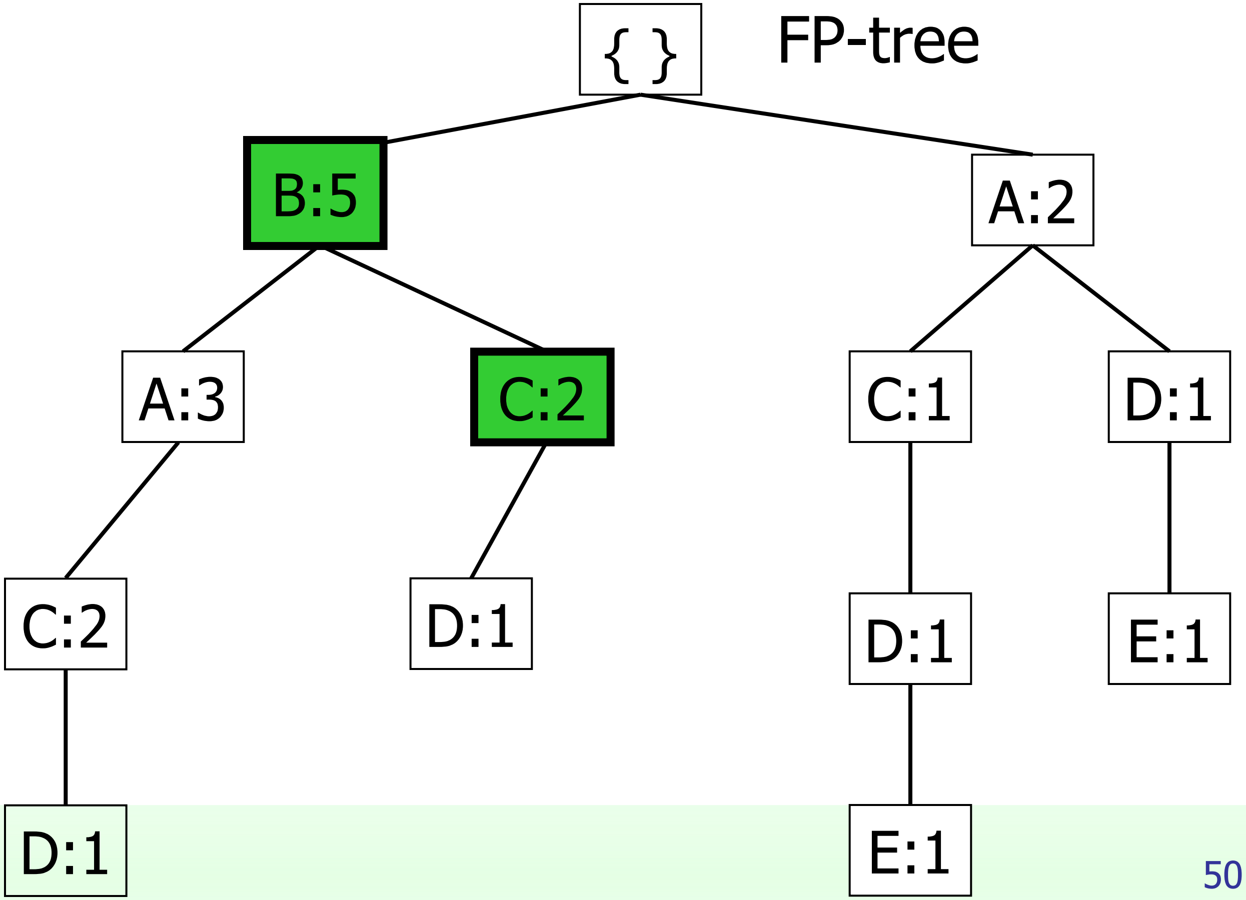
TID	Items
7	{B,C}

Sorted transaction

TID	Items
7	{B,C}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



FP-tree construction

Transaction

TID	Items
8	{A,B,C}

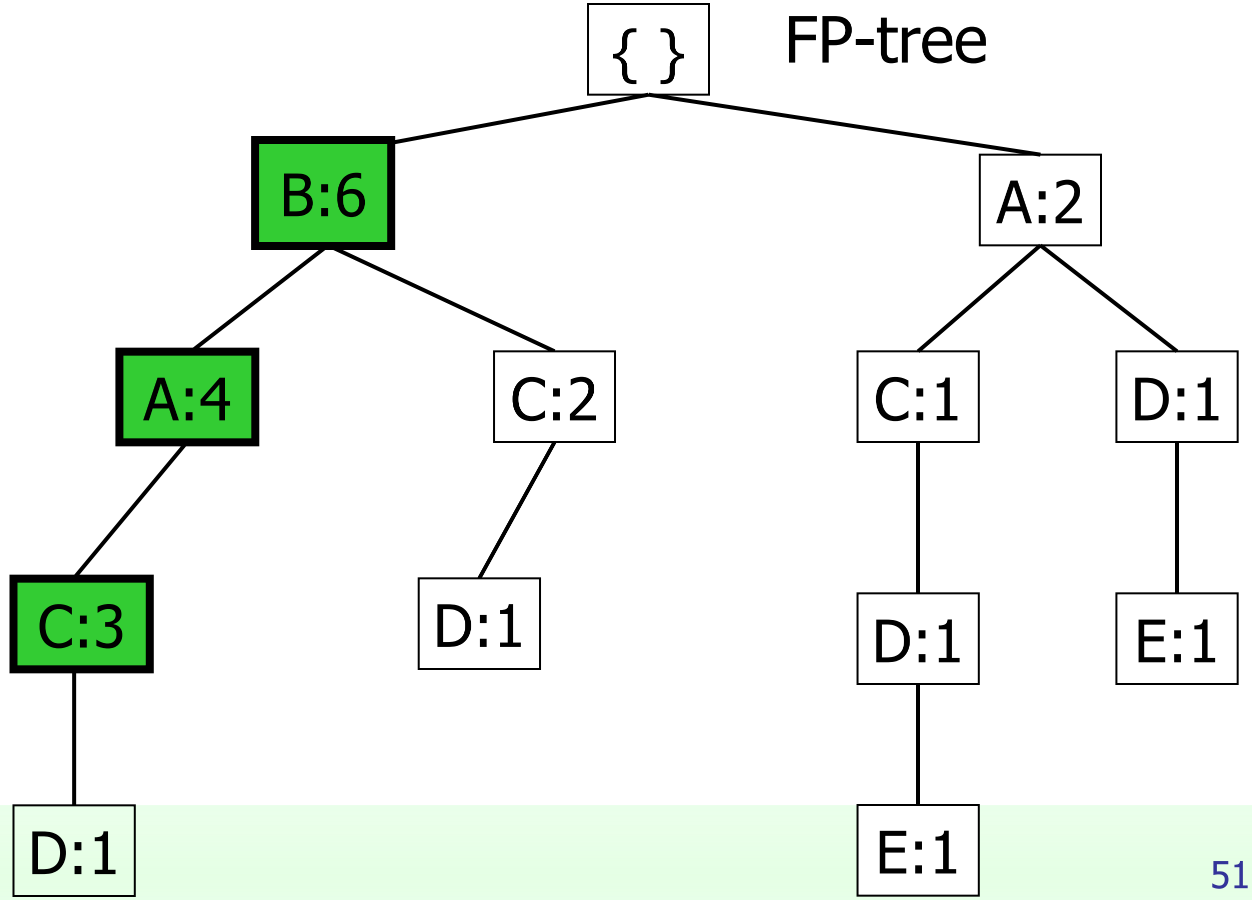
Sorted transaction

TID	Items
8	{B,A,C}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

FP-tree



FP-tree construction

Transaction

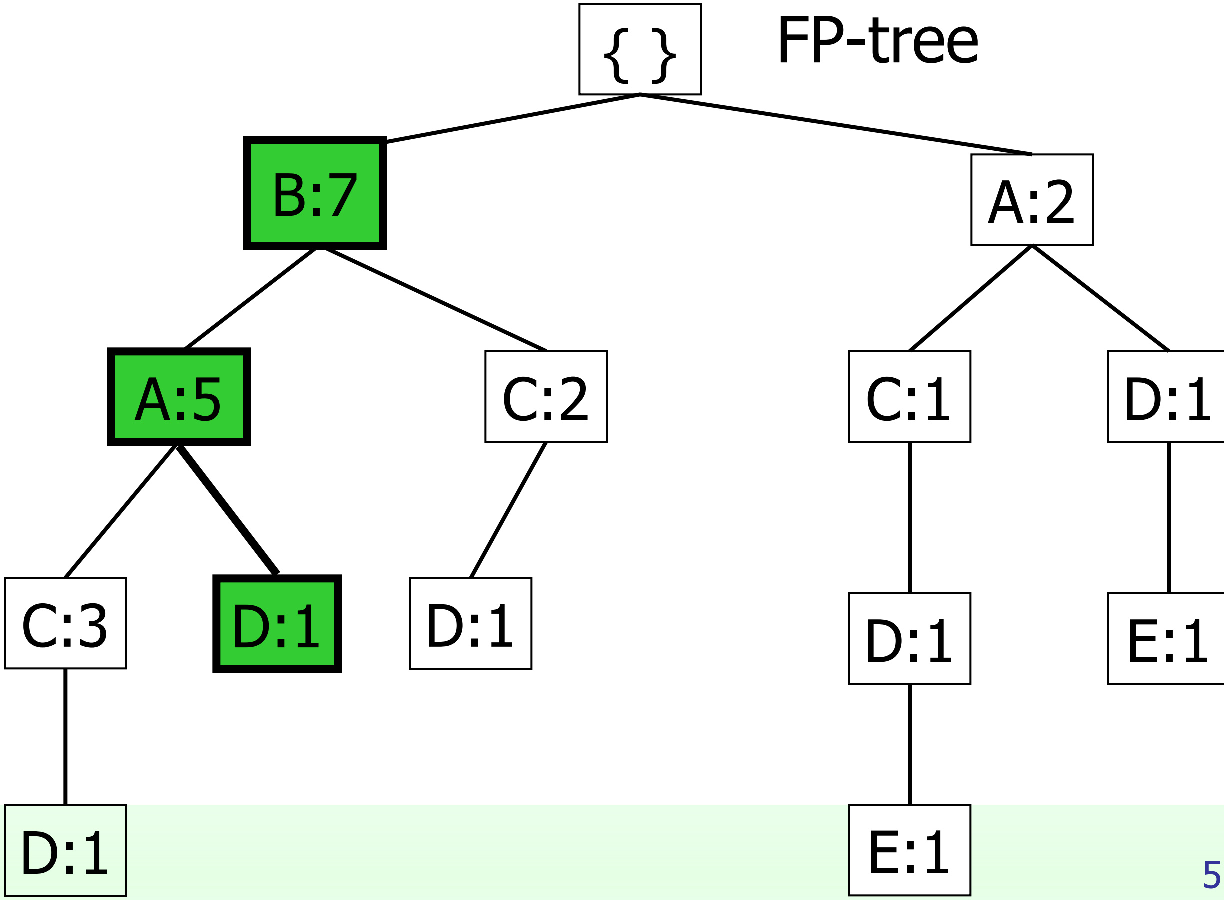
TID	Items
9	{A,B,D}

Sorted transaction

TID	Items
9	{B,A,D}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



FP-tree construction

Transaction

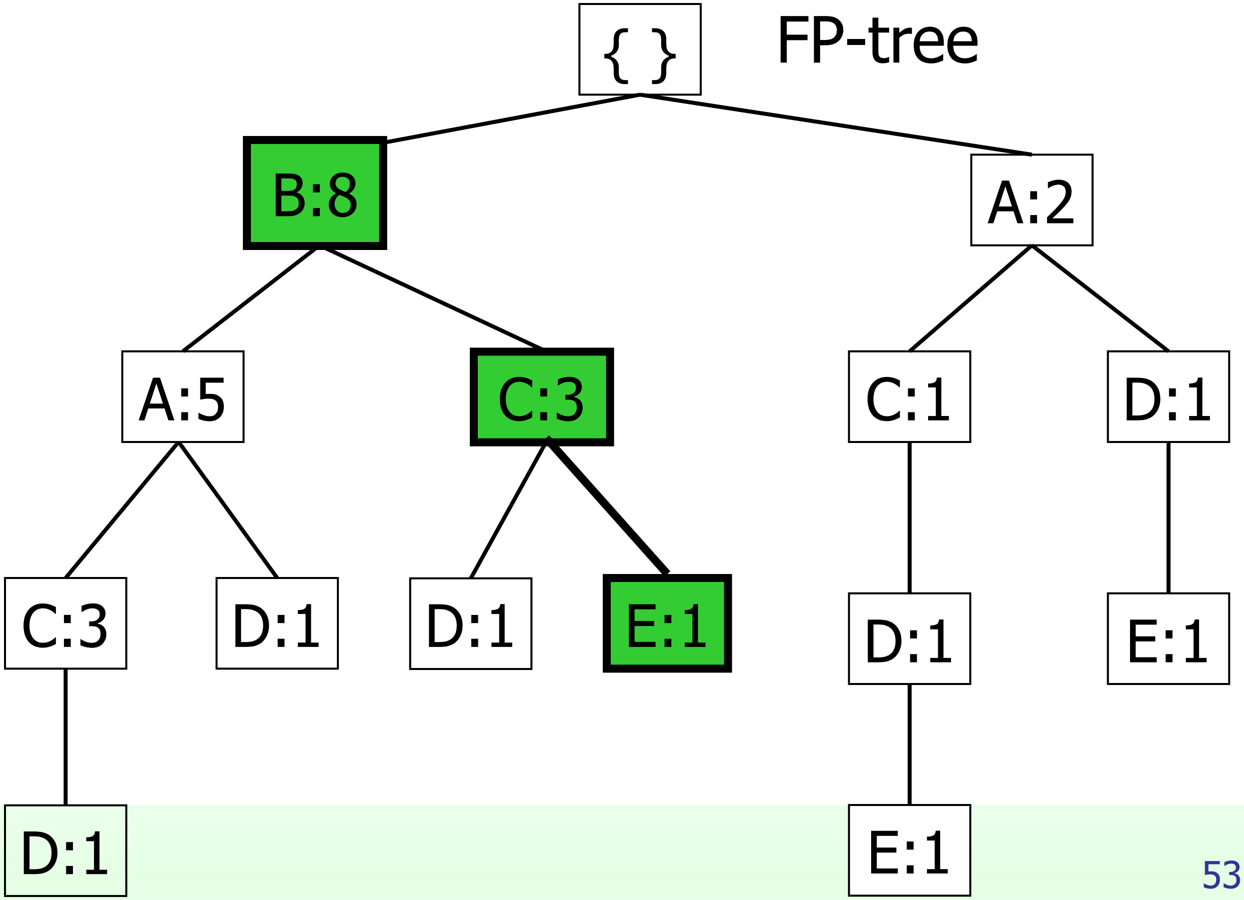
TID	Items
10	{B,C,E}

Sorted transaction

TID	Items
10	{B,C,E}

Header Table

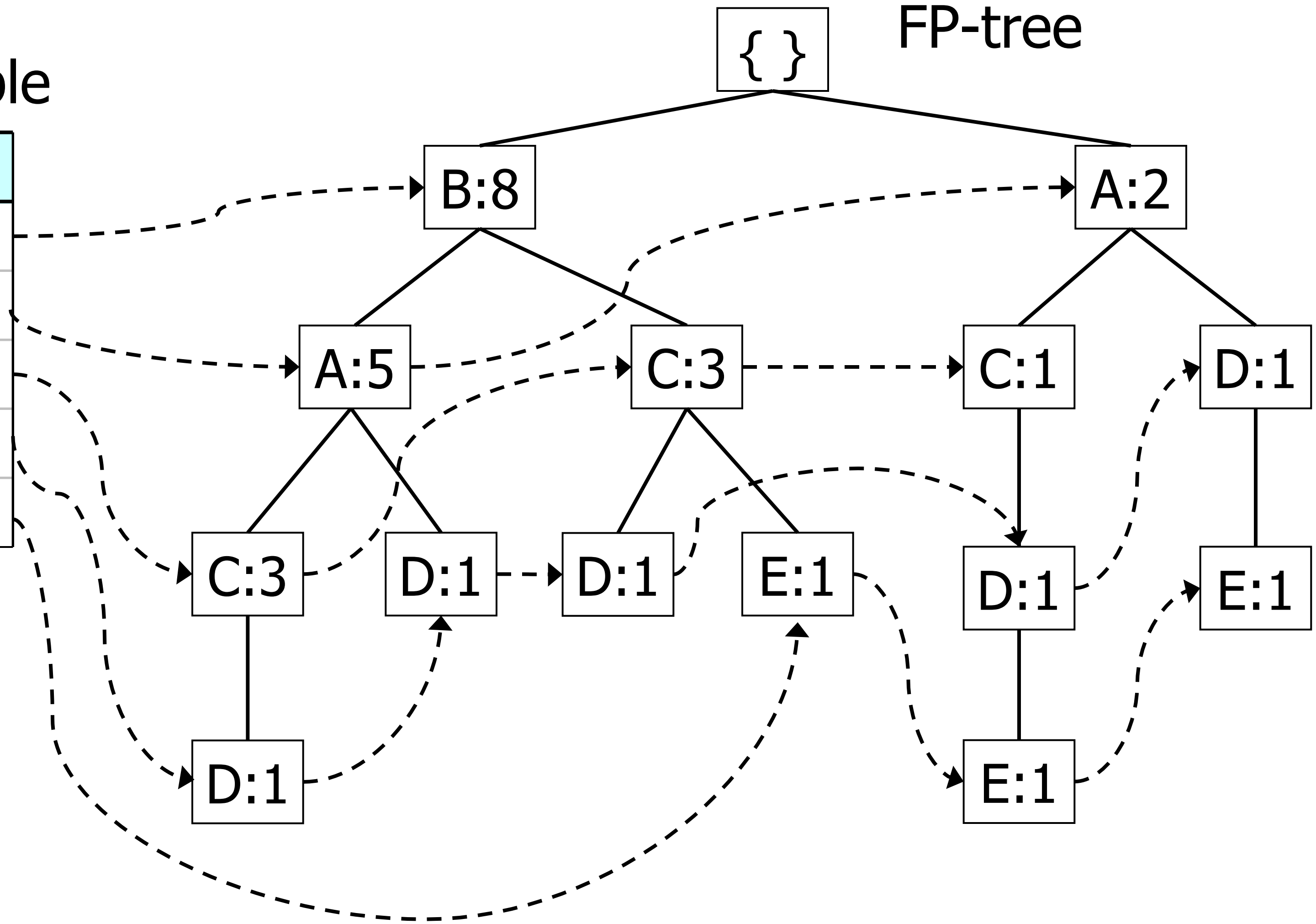
Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



Final FP-tree

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



Item pointers are used to assist frequent itemset generation

FP-growth Algorithm

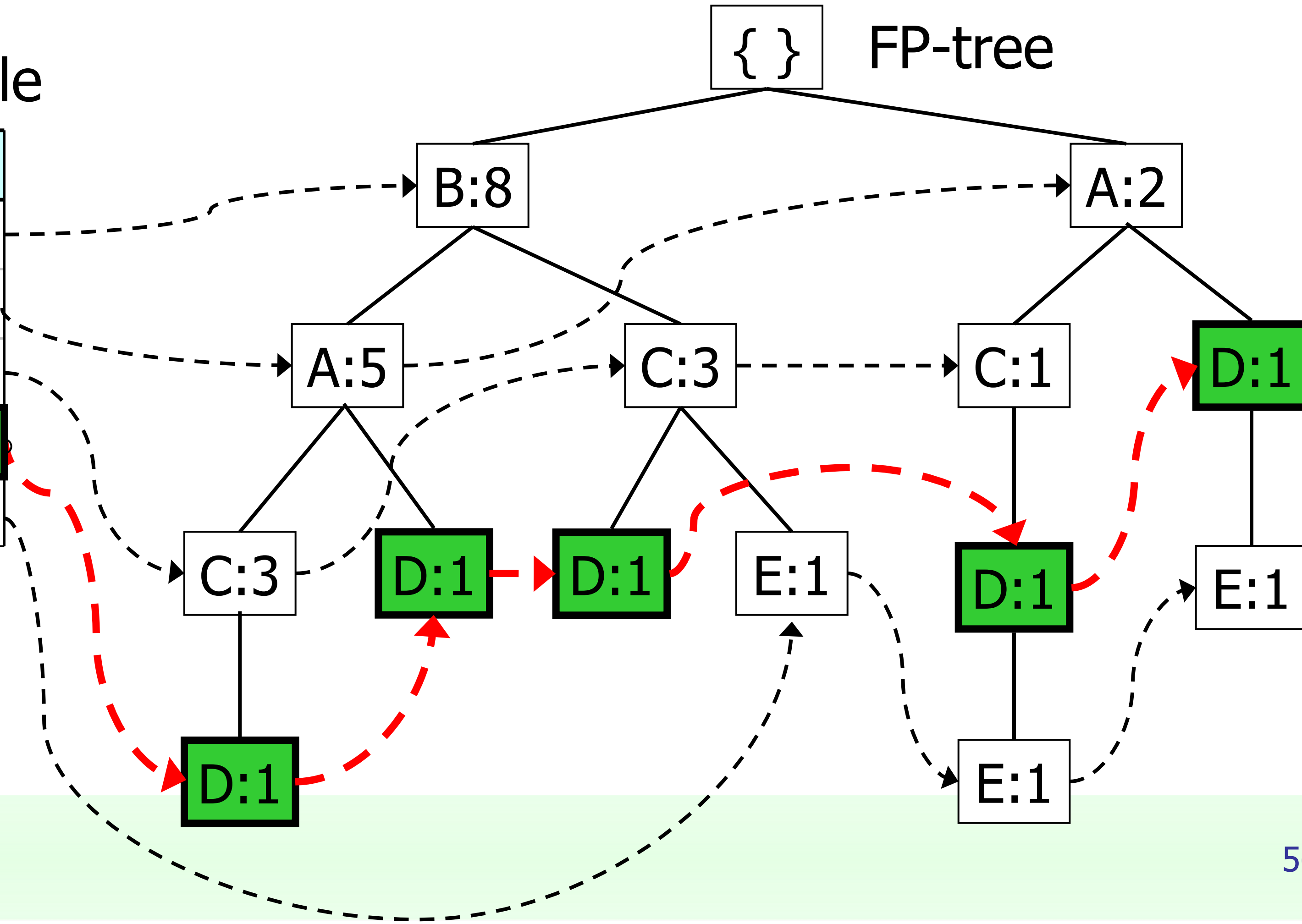
- Scan Header Table from lowest support item up
- For each item i in Header Table extract frequent itemsets including item i and items preceding it in Header Table
  - (1) build Conditional Pattern Base for item i (i-CPB)
    - Select prefix-paths of item i from FP-tree
  - (2) recursive invocation of FP-growth on i-CPB

Example

- Consider item D and extract frequent itemsets including
  - D and supported combinations of items A, B, C

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



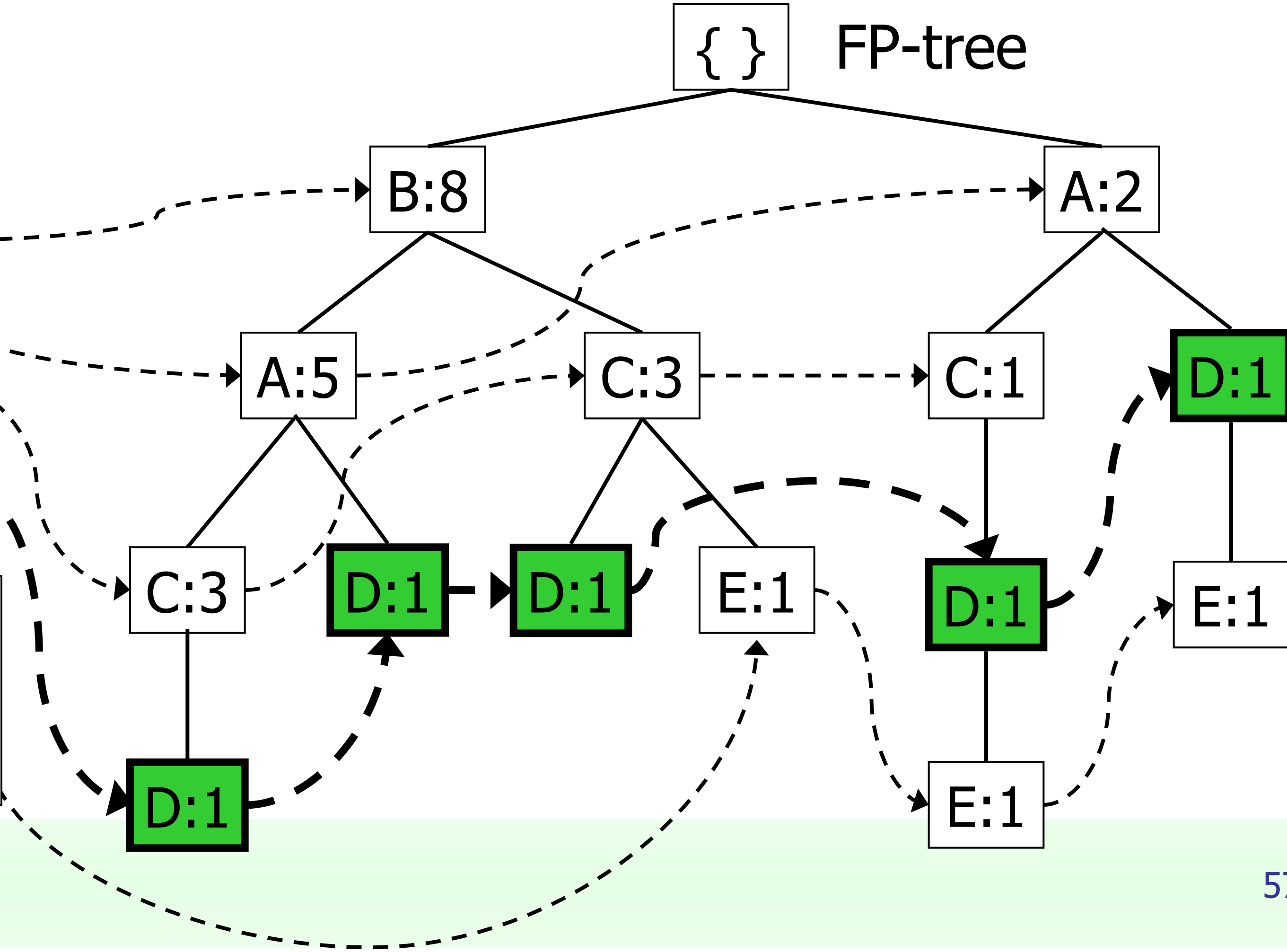
Conditional Pattern Base of D

- (1) Build D-CPB
  - Select prefix-paths of item D from FP-tree

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

Frequent itemset:
D, sup(D) = 5



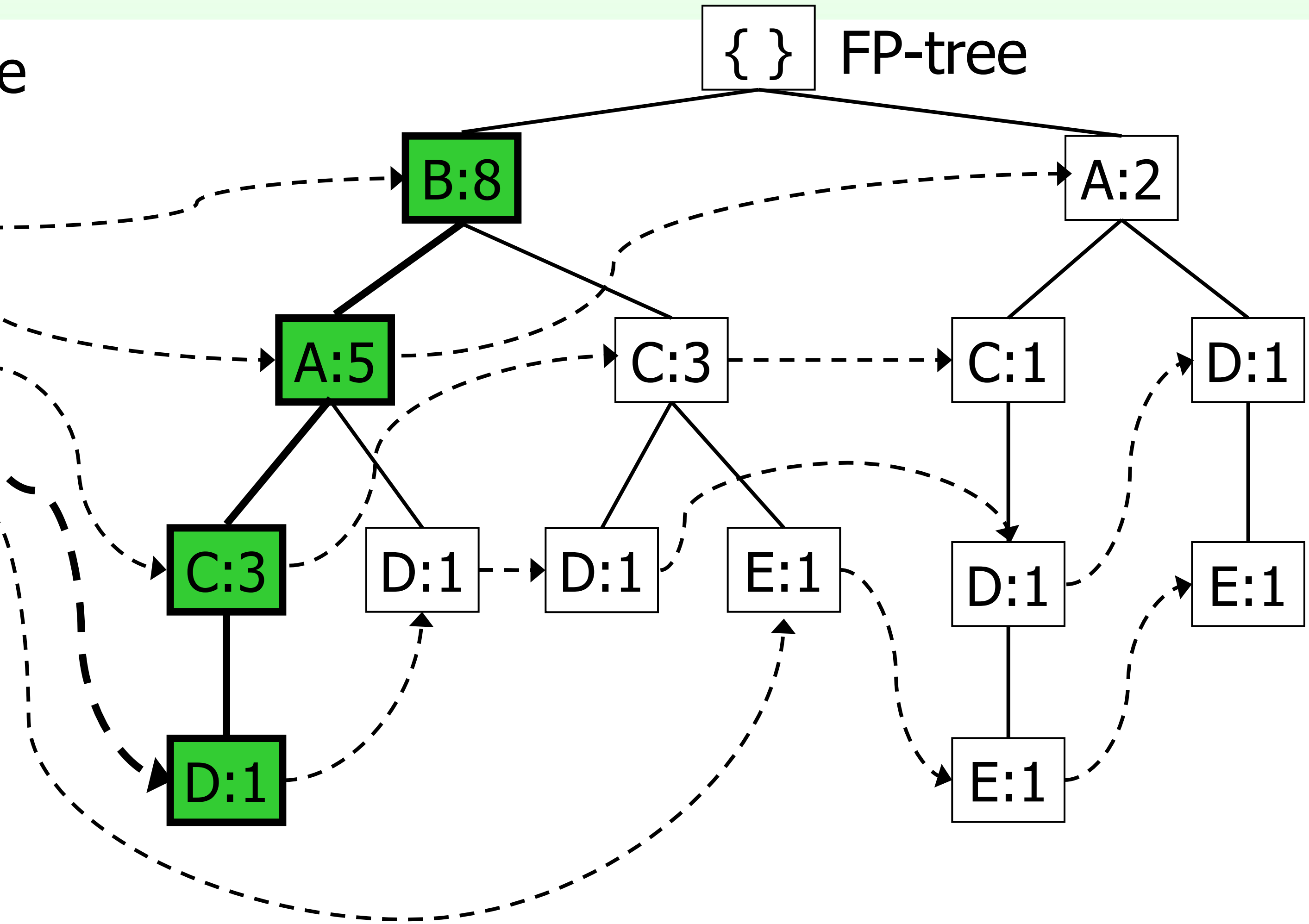
Conditional Pattern Base of D

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

D-CPB

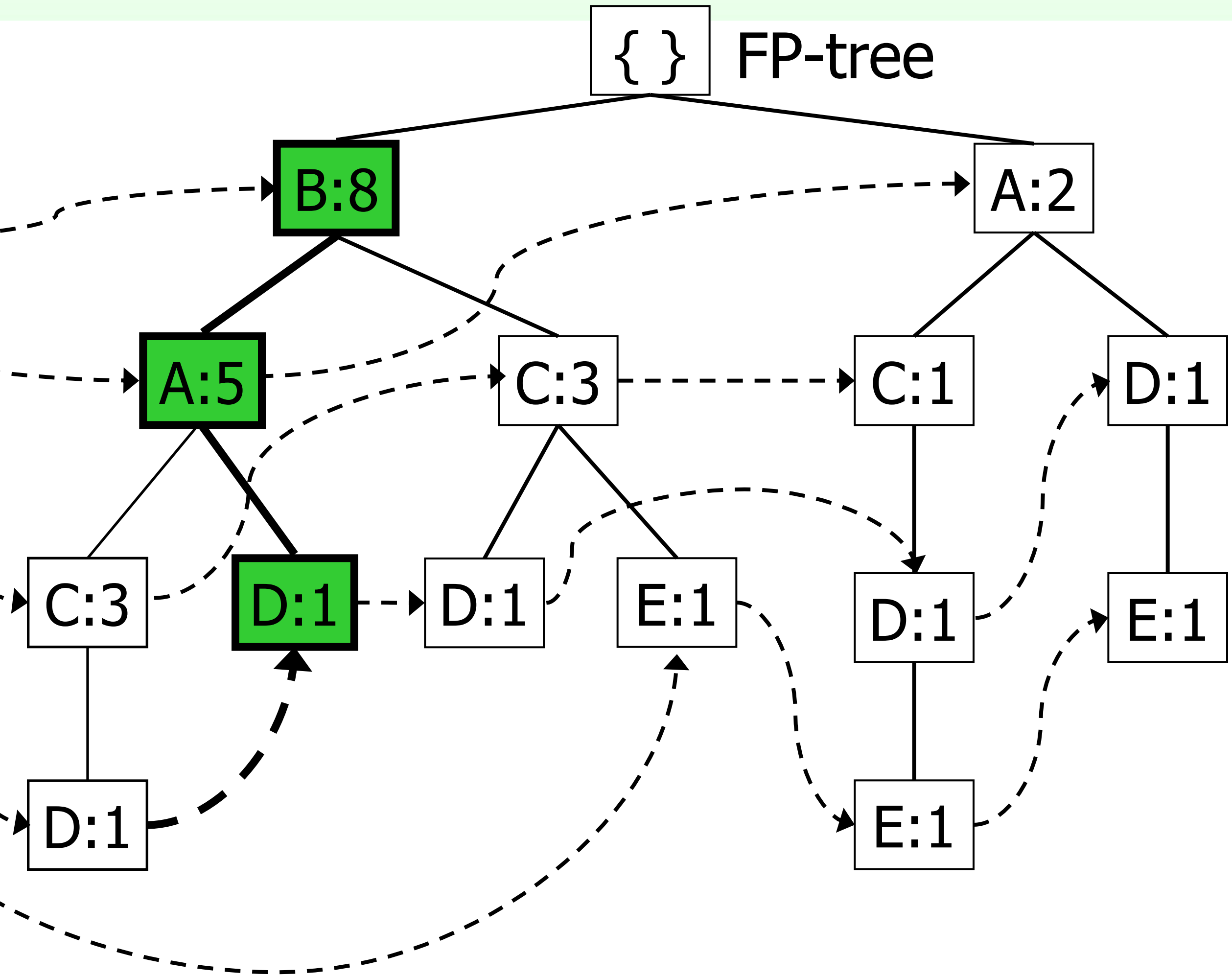
Items	sup
{B,A,C}	1



Conditional Pattern Base of D

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



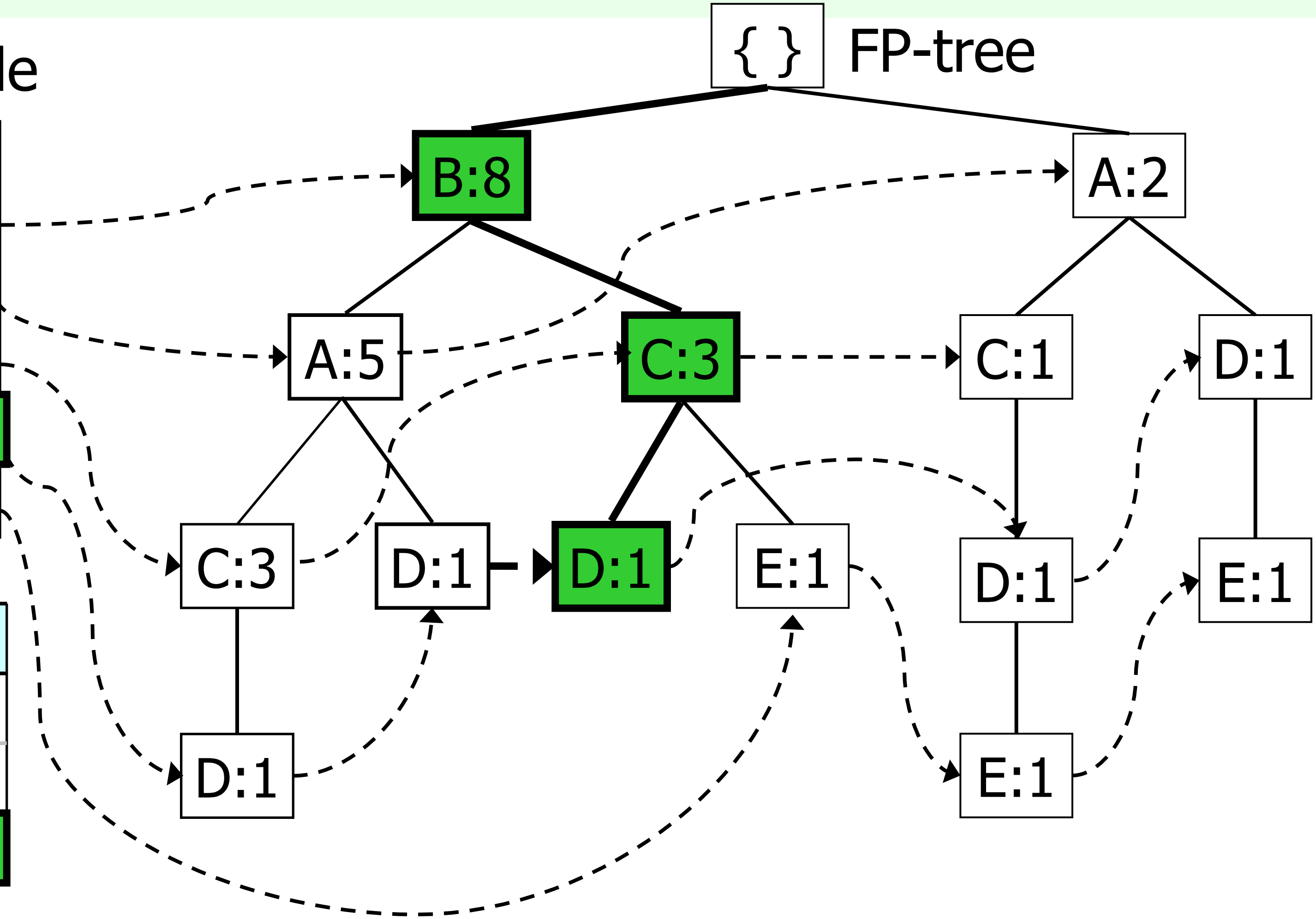
D-CPB

Items	sup
{B,A,C}	1
{B,A}	1

Conditional Pattern Base of D

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



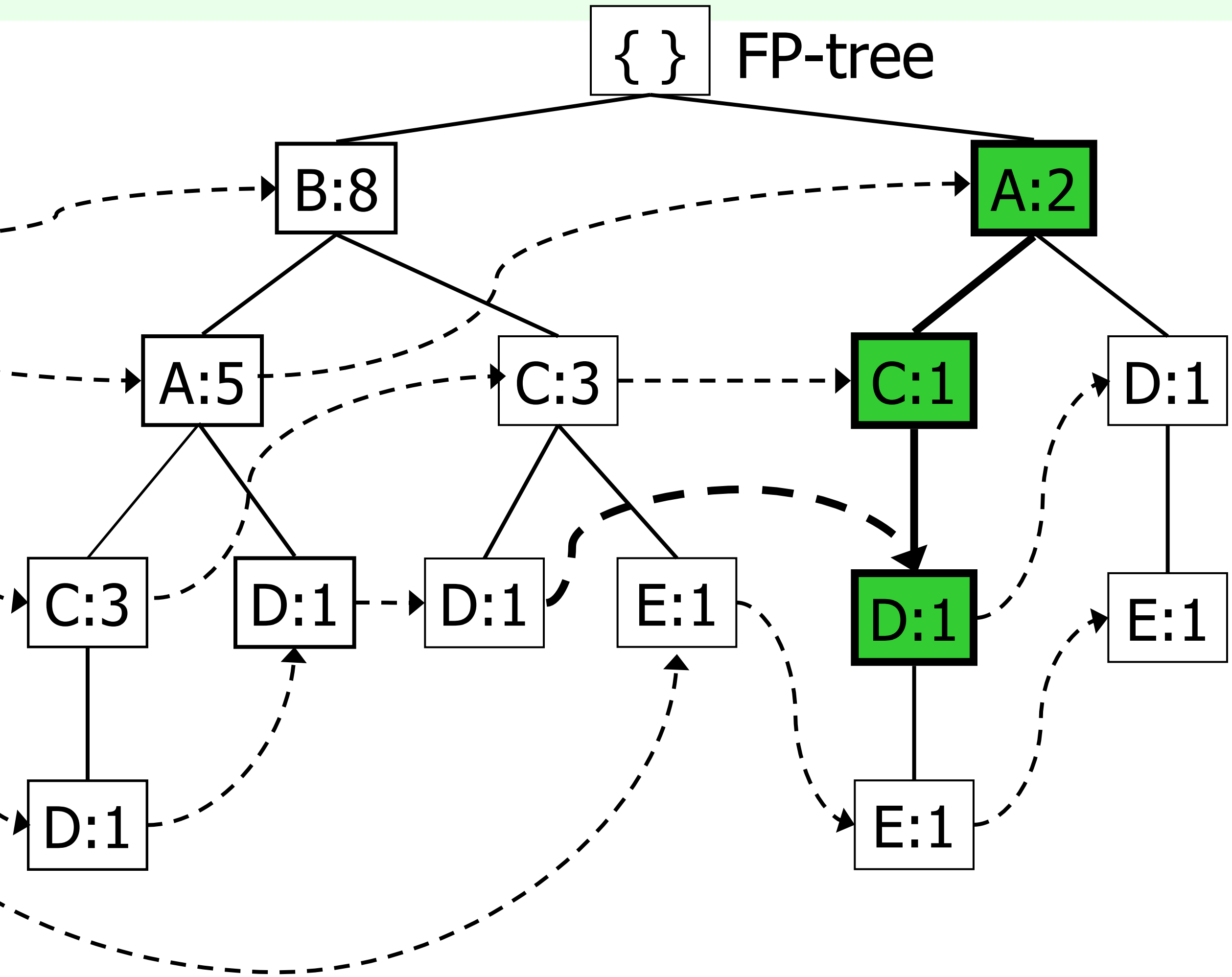
D-CPB

Items	sup
{B,A,C}	1
{B,A}	1
{B,C}	1

Conditional Pattern Base of D

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



D-CPB

Items	sup
{B,A,C}	1
{B,A}	1
{B,C}	1
{A,C}	1

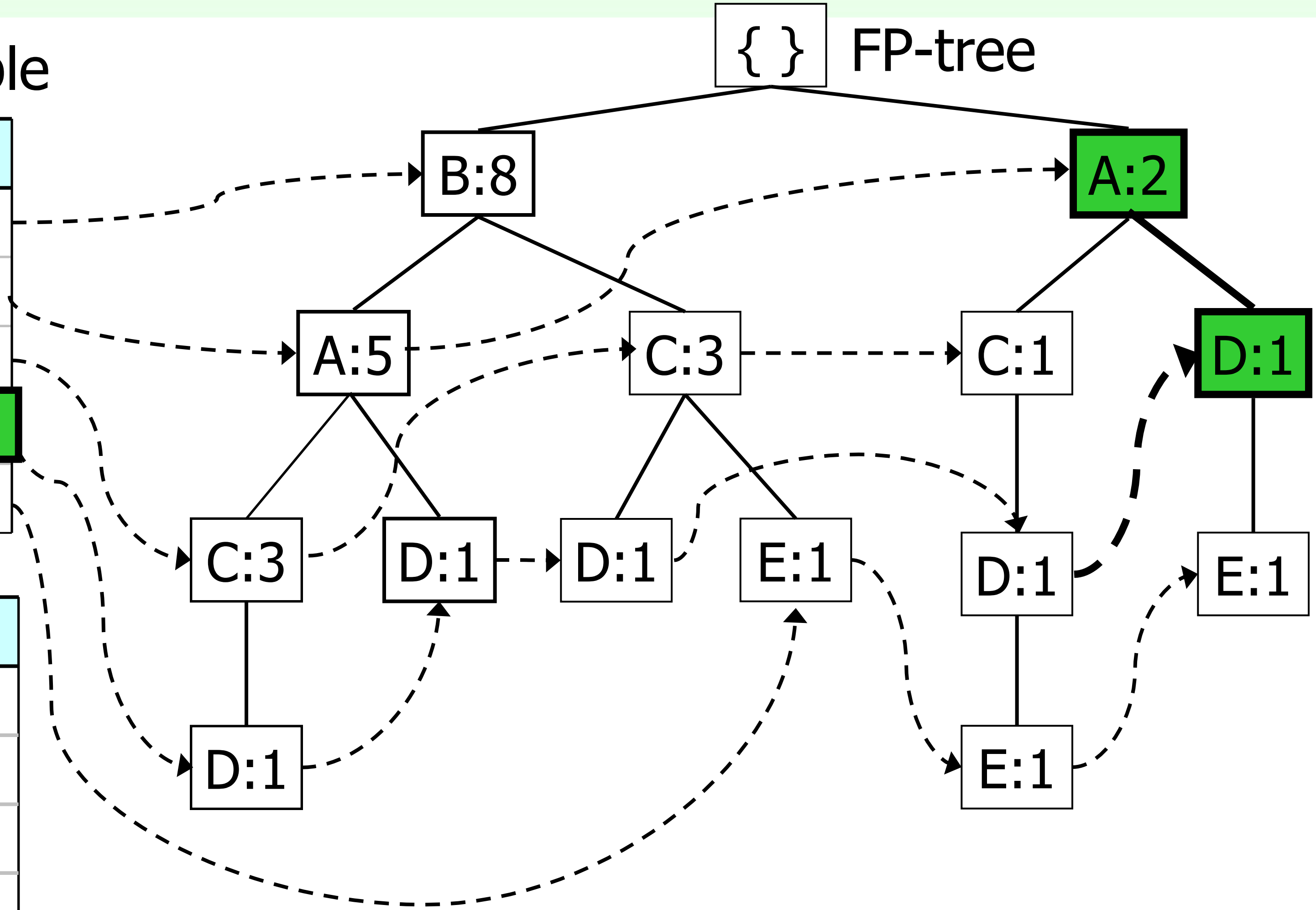
Conditional Pattern Base of D

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

D-CPB

Items	sup
{B,A,C}	1
{B,A}	1
{B,C}	1
{A,C}	1
{A}	1



Conditional Pattern Base of D

- (1) Build D-CPB
  - Select prefix-paths of item D from FP-tree

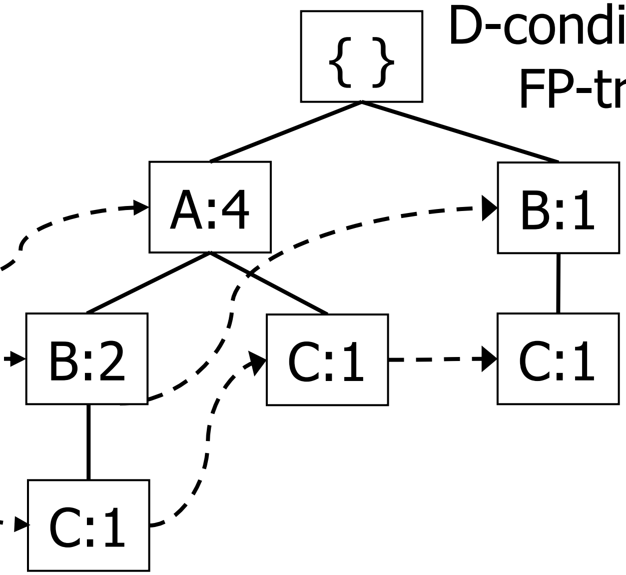
D-CPB

Items	sup
{B,A,C}	1
{B,A}	1
{B,C}	1
{A,C}	1
{A}	1

D-conditional Header Table

Item	sup
{A}	4
{B}	3
{C}	3

D-conditional FP-tree



- (2) Recursive invocation of FP-growth on D-CPB

Conditional Pattern Base of DC

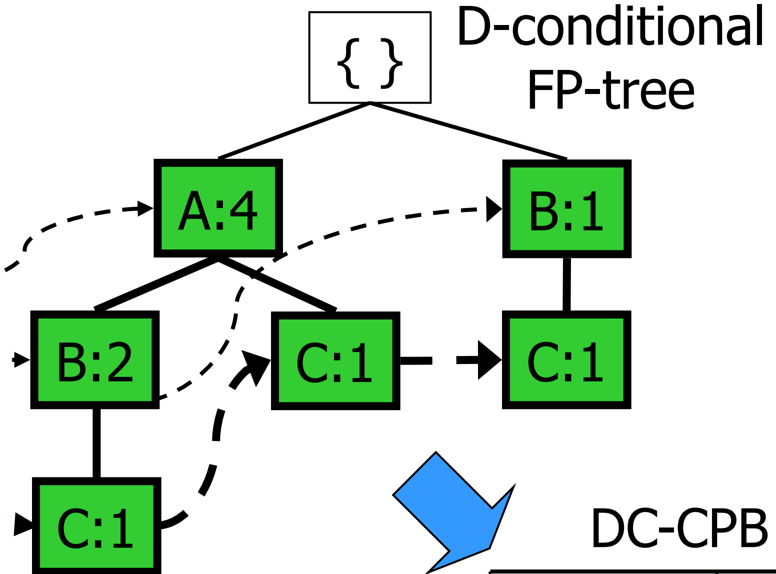
- (1) Build DC-CPB
  - Select prefix-paths of item C from D-conditional FP-tree

D-CPB

Items	sup
{B,A,C}	1
{B,A}	1
{B,C}	1
{A,C}	1
{A}	1

D-conditional Header Table

Item	sup
{A}	4
{B}	3
{C}	3



Frequent itemset:
DC, sup(DC) = 3

DC-CPB

Items	sup
{A,B}	1
{A}	1
{B}	1

Conditional Pattern Base of DC

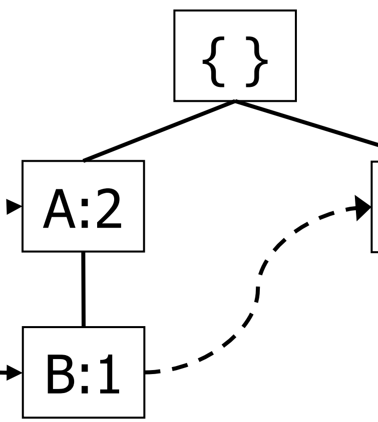
- (1) Build DC-CPB
  - Select prefix-paths of item C from D-conditional FP-tree

DC-CPB

Items	sup
{A,B}	1
{A}	1
{B}	1

DC-conditional Header Table

Item	sup
{A}	2
{B}	2



DC-conditional FP-tree

- (2) Recursive invocation of FP-growth on DC-CPB

Conditional Pattern Base of DCB

- (1) Build DCB-CPB
  - Select prefix-paths of item B from DC-conditional FP-tree

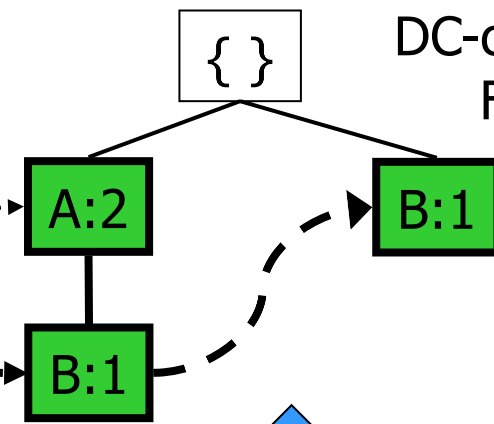
DC-CPB

Items	sup
{A,B}	1
{A}	1
{B}	1

DC-conditional Header Table

Item	sup
{A}	2
{B}	2

DC-conditional FP-tree



Frequent itemset:
DCB, sup(DCB) = 2

DCB-CPB

Items	sup
{A}	1

Conditional Pattern Base of DCB

- (1) Build DCB-CPB
  - Select prefix-paths of item B from DC-conditional FP-tree

DCB-CPB

Items	sup
{A}	1

- Item A is infrequent in DCB-CPB
  - A is removed from DCB-CPB
  - DCB-CPB is empty

- (2) The search backtracks to DC-CPB

Conditional Pattern Base of DCA

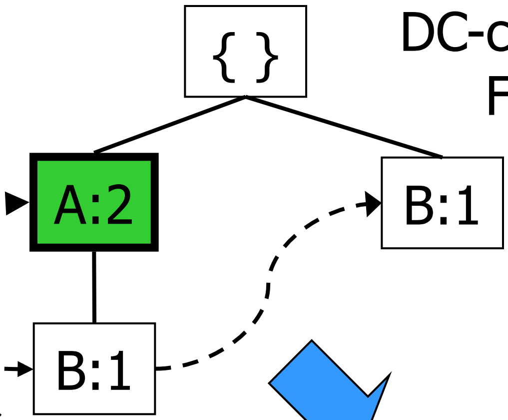
- (1) Build DCA-CPB
  - Select prefix-paths of item A from DC-conditional FP-tree

DC-CPB

Items	sup
{A,B}	1
{A}	1
{B}	1

DC-conditional Header Table

Item	sup
{A}	2
{B}	2



DC-conditional FP-tree

Frequent itemset:
DCA, sup(DCA) = 2

DCA-CPB is empty
(no transactions)

- (2) The search backtracks to D-CBP

Conditional Pattern Base of DB

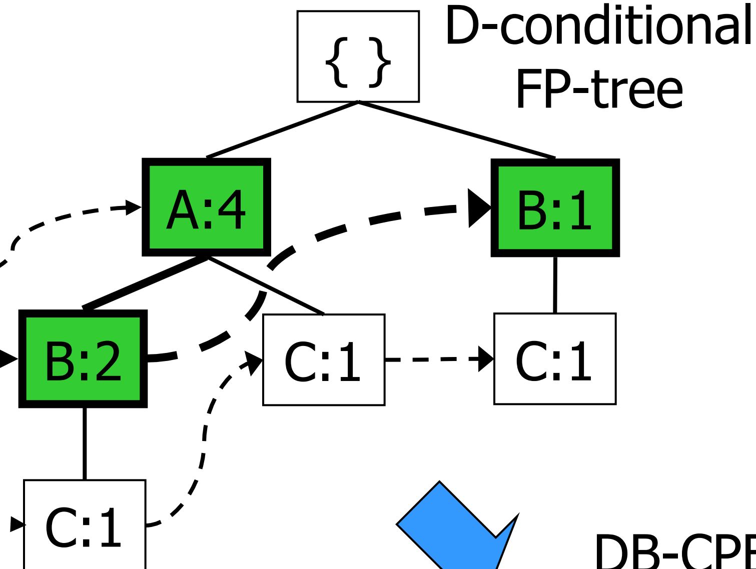
- (1) Build DB-CPB
  - Select prefix-paths of item B from D-conditional FP-tree

D-CPB

Items	sup
{B,A,C}	1
{B,A}	1
{B,C}	1
{A,C}	1
{A}	1

D-conditional Header Table

Item	sup
{A}	4
{B}	3
{C}	3



DB-CPB

Items	sup
{A}	2

Frequent itemset:
DB, sup(DB) = 3

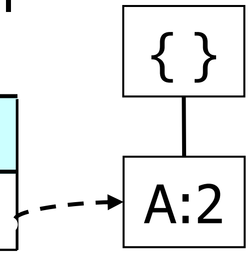
- (1) Build DB-CPB
  - Select prefix-paths of item B from D-conditional FP-tree

DB-CPB

Items	sup
{A}	2

DB-conditional Header Table

Items	sup
{A}	2



DB-conditional FP-tree

- (2) Recursive invocation of FP-growth on DB-CPB

Conditional Pattern Base of DBA

- (1) Build DBA-CPB
  - Select prefix-paths of item A from DB-conditional FP-tree

DB-CPB

Items	sup
{A}	2

DB-conditional Header Table

Items	sup
{A}	2

{ }

A:2

DB-conditional FP-tree

Frequent itemset:
DBA, sup(DBA) = 2

DBA-CPB is empty
(no transactions)

- (2) The search backtracks to D-CBP

Conditional Pattern Base of DA

- (1) Build DA-CPB
  - Select prefix-paths of item A from D-conditional FP-tree

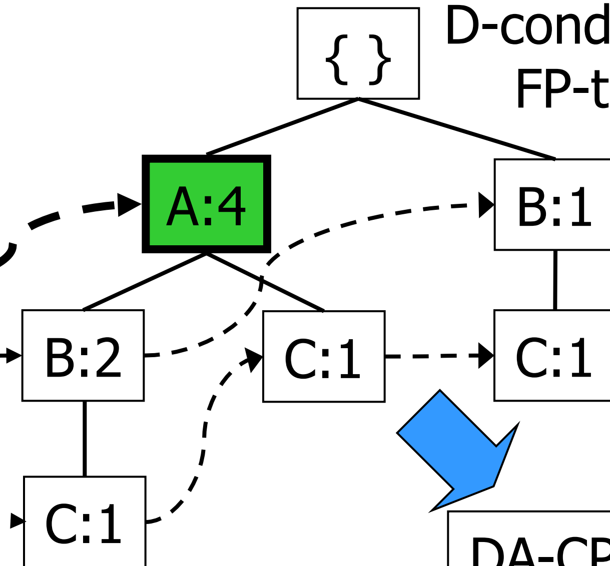
D-CPB

Items	sup
{B,A,C}	1
{B,A}	1
{B,C}	1
{A,C}	1
{A}	1

D-conditional Header Table

Item	sup
{A}	4
{B}	3
{C}	3

D-conditional FP-tree



Frequent itemset:
DA, sup(DA) = 4

DA-CPB is empty
(no transactions)

The search ends

Frequent itemsets with prefix D

- Frequent itemsets including D and supported combinations of items B,A,C

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

itemsets	sup
{D}	5
{A,D}	4
{B,D}	3
{C,D}	3
{A,B,D}	2
{A,C,D}	2
{B,C,D}	2

minsup>1

Other approaches

- Many other approaches to frequent itemset extraction
- May exploit a different database representation
  - represent the tidset of each item [Zak00]

Horizontal Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Compact Representations

- Some itemsets are redundant because they have identical support as their supersets

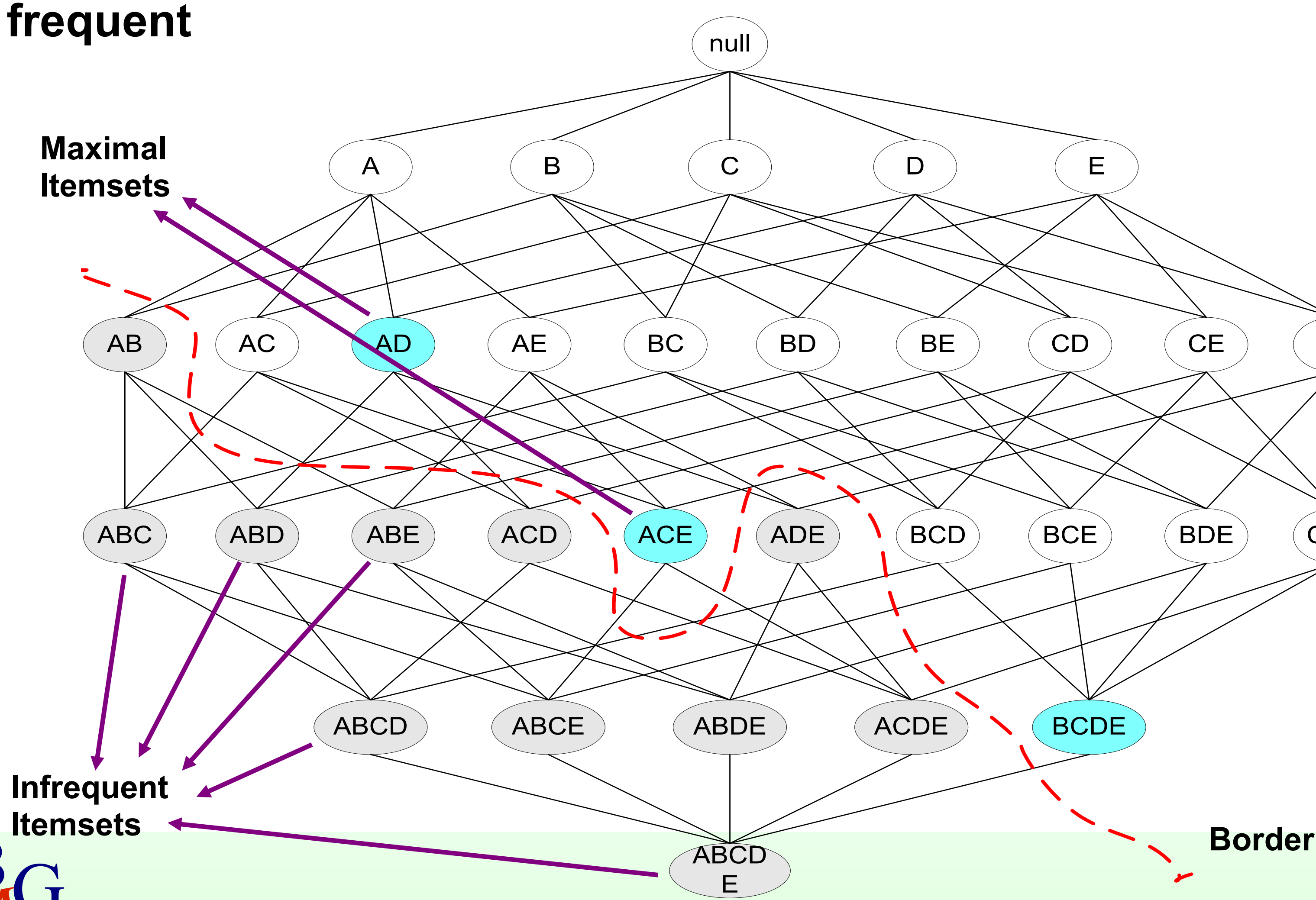
TID	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1

- Number of frequent itemsets $= 3 \times \sum_{k=1}^{10} \binom{10}{k}$

- A compact representation is needed

Maximal Frequent Itemset

An itemset is frequent maximal if none of its immediate supersets is frequent



From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Closed Itemset

- An itemset is closed if none of its immediate supersets has the same support as the itemset

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

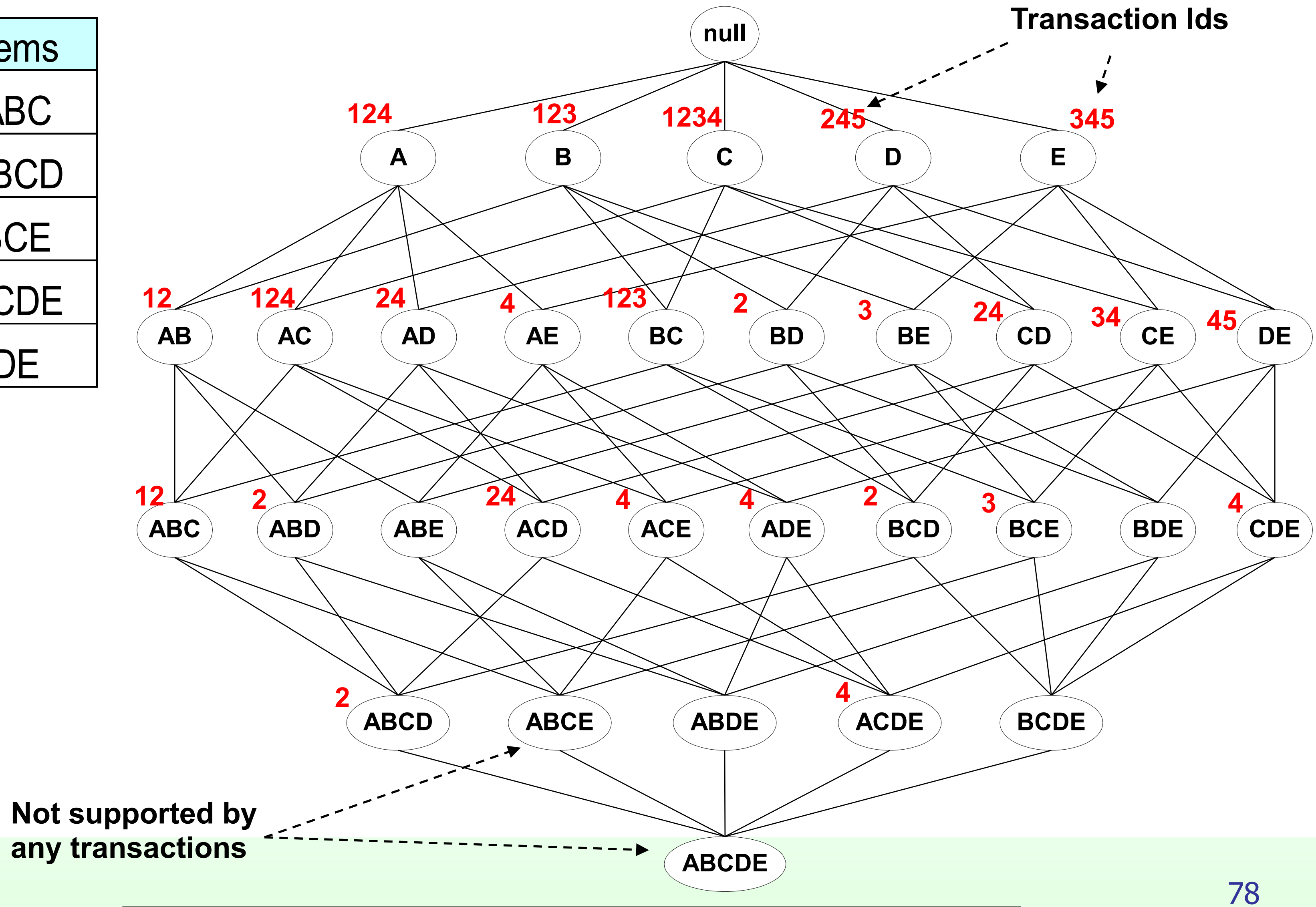
itemset	sup
{A}	4
{B}	*5*
{C}	3
{D}	4
{A,B}	*4*
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	*4*
{C,D}	3

itemset	sup
{A,B,C}	2
{A,B,D}	*3*
{A,C,D}	2
{B,C,D}	*3*
{A,B,C,D}	*2*

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

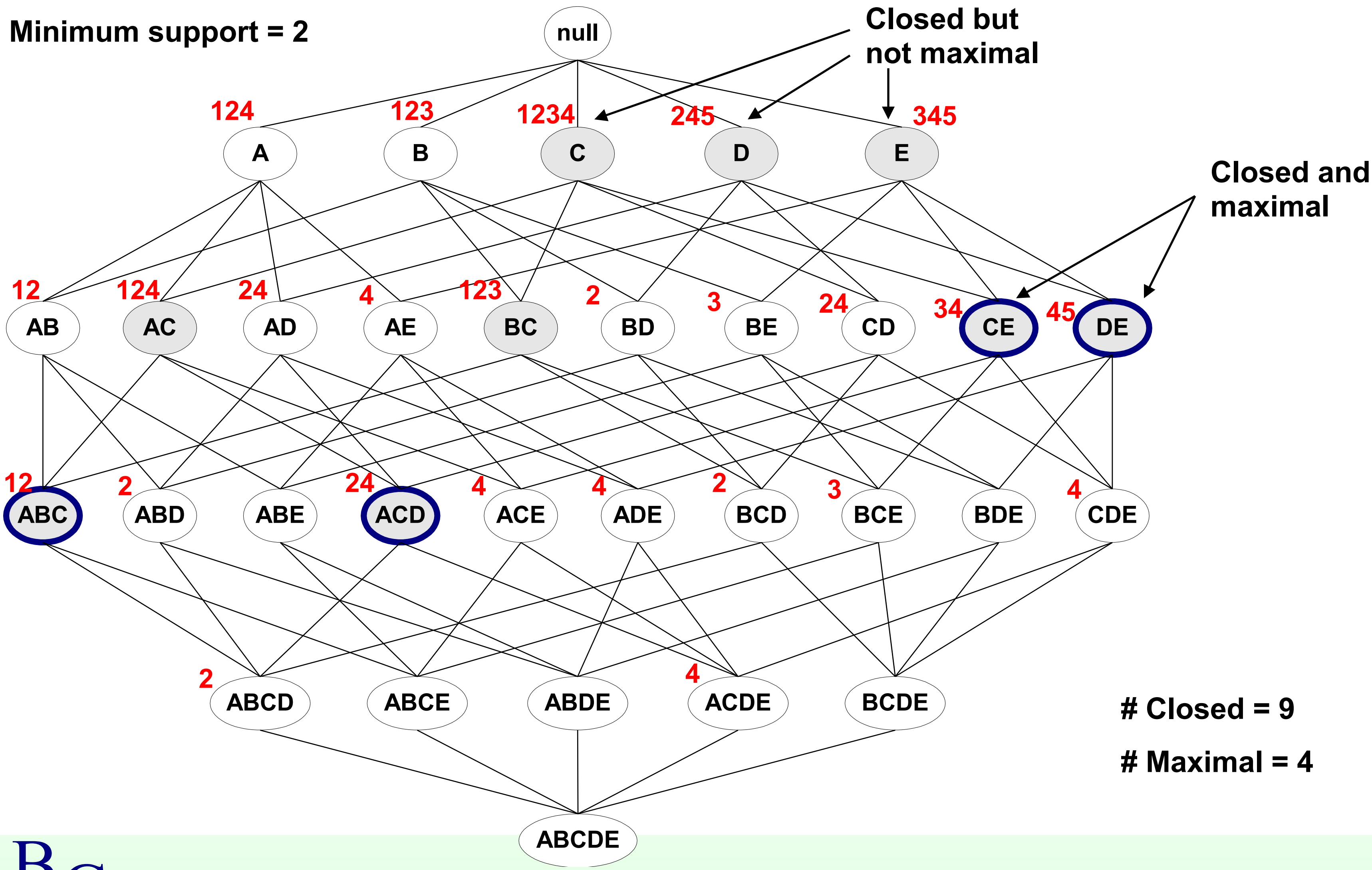
Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



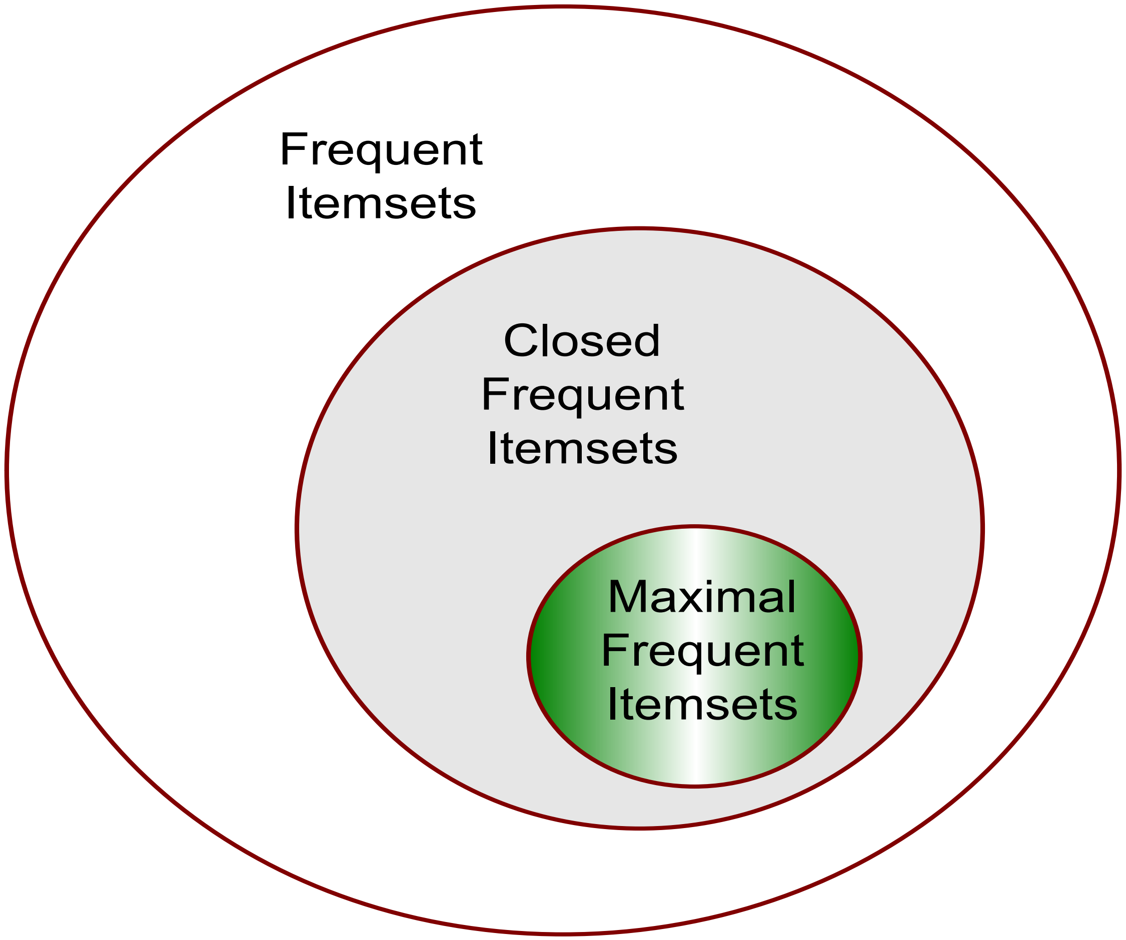
From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Maximal vs Closed Frequent Itemsets



From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Maximal vs Closed Itemsets



From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Effect of Support Threshold

- Selection of the appropriate *minsup* threshold is not obvious
  - If *minsup* is too high
    - itemsets including rare but interesting items may be lost
      - example: pieces of jewellery (or other expensive products)
  - If *minsup* is too low
    - it may become computationally *very expensive*
    - the number of frequent itemsets becomes *very large*

Interestingness Measures

- A large number of pattern may be extracted
  - rank patterns by their interestingness
- Objective measures
  - rank patterns based on statistics computed from data
  - initial framework [Agr94] only considered support and confidence
    - other statistical measures available
- Subjective measures
  - rank patterns according to user interpretation [Silb98]
    - interesting if it contradicts the expectation of a user
    - interesting if it is actionable

Confidence measure: always reliable?

- 5000 high school students are given
  - 3750 eat cereals
  - 3000 play basket
  - 2000 eat cereals and play basket
- Rule

$$\text{play basket} \Rightarrow \text{eat cereals}$$

$$\text{sup} = 40\%,\ \text{conf} = 66{,}7\%$$

is misleading because eat cereals has sup 75% (>66,7%)

- Problem caused by high frequency of rule head
  - negative correlation

	basket	not basket	total
cereals	2000	1750	3750
not cereals	1000	250	1250
total	3000	2000	5000

Correlation or lift

$$r: A \Rightarrow B$$

$$\text{Correlation} = \frac{P(A,B)}{P(A)P(B)} = \frac{\text{conf(r)}}{\text{sup(B)}}$$

- Statistical independence
  - Correlation = 1
- Positive correlation
  - Correlation > 1
- Negative correlation
  - Correlation < 1

Example

- Association rule

  play basket $\Rightarrow$ eat cereals

  has corr = 0.89

  - negative correlation

- but rule

  play basket $\Rightarrow$ not (eat cereals)

  has corr = 1,34

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i), -\sum_j P(B_j)\log P(B_j))}$
8	J-Measure (J)	$\max\Big(P(A,B)\log(\frac{P(B\|A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}),$ $P(A,B)\log(\frac{P(A\|B)}{P(A)}) + P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})\Big)$
9	Gini index (G)	$\max\Big(P(A)[P(B\|A)^2+P(\overline{B}\|A)^2]+P(\overline{A})[P(B\|\overline{A})^2+P(\overline{B}\|\overline{A})^2]$ $-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\|B)^2+P(\overline{A}\|B)^2]+P(\overline{B})[P(A\|\overline{B})^2+P(\overline{A}\|\overline{B})^2]$ $-P(A)^2-P(\overline{A})^2\Big)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\|A), P(A\|B))$
12	Laplace (L)	$\max\Big(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2}\Big)$
13	Conviction (V)	$\max\Big(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})}\Big)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B)-P(A)P(B)$
17	Certainty factor (F)	$\max\Big(\frac{P(B\|A)-P(B)}{1-P(B)}, \frac{P(A\|B)-P(A)}{1-P(A)}\Big)$
18	Added Value (AV)	$\max(P(B\|A)-P(B), P(A\|B)-P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\|A)-P(B), P(A\|B)-P(A))$

Considering weight

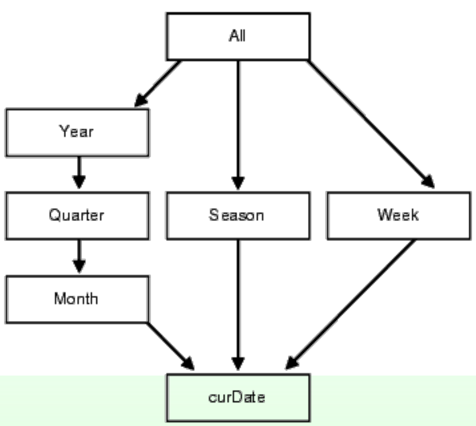
- Items may be characterized by different importance within a transaction
  - Example: product quantity or price in transactions
- Transactions may be weighted
  - Example: discount on entire market basket
- Weighted dataset
  - Each item is assigned a weight measuring its relevance in the corresponding transaction

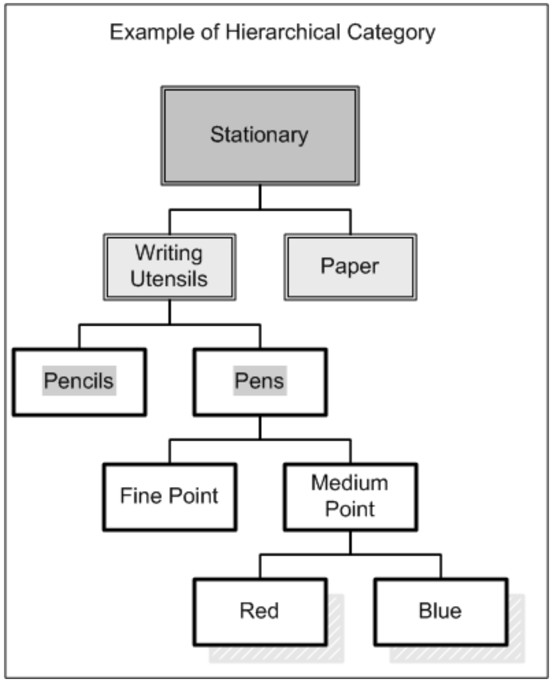
Weighted association rules

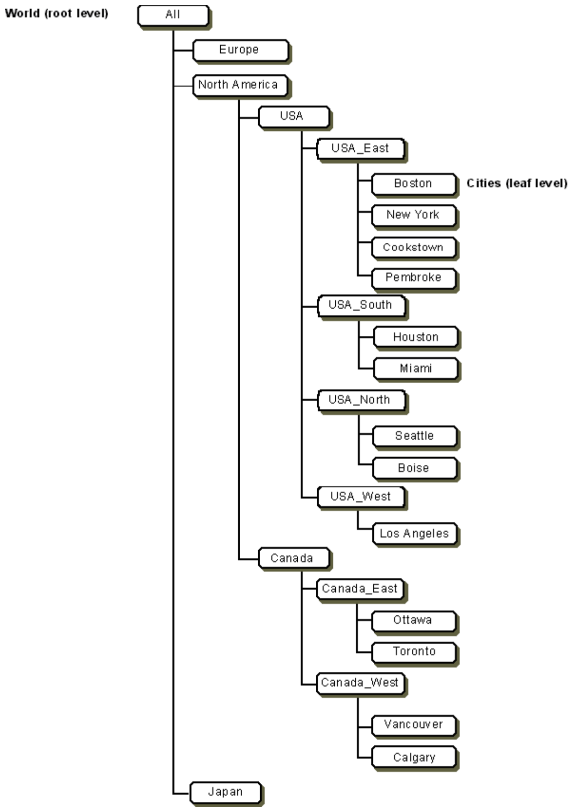
- Consider item/transaction weights during association rule extraction
- Extend rule quality measures
  - E.g., weighted support, weighted confidence
- Apply ad-hoc weight aggregation functions
  - E.g., min, max, avg

Considering hierarchies

- Generalization hierarchies
  - Aggregation over attributes in a dataset
  - Typically user provided
- Examples
  - Time hierarchy
  - Product category
  - Location hierarchy
  - …

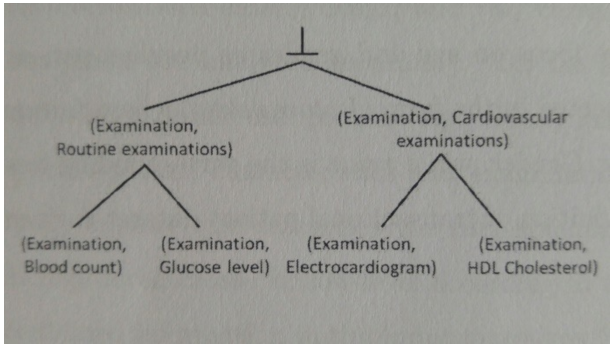






Taxonomy

- A taxonomy is a set of is-a hierarchies that aggregate data items into higher-level concepts
- Data item
  - Instance in the (transactional) dataset
  - Represents detailed concepts
- Generalized item
  - Aggregation in higher-level concepts
  - Represents abstractions on instances



Generalized itemsets

- Sets of items at different generalization levels
  - May contain data items together with generalized items defined in the taxonomy
  - Summarize knowledge represented by a set of lower-level descendants
    - Both frequent and infrequent
- A generalized itemset covers a transaction when all
  - its generalized items are ancestors of items included in the transaction
  - its data items are included in the transaction
- Generalized itemset support
  - ratio between number of covered transactions and dataset cardinality

Context-aware data analysis

- Context data provided by different, possibly heterogeneous, sources
  - Mobile devices provide information on
    - the user context (e.g., GPS coordinates)
    - the supplied services
      - temporal information
      - service description
      - duration
  - Additional information available
    - demographics of the user requesting the service

Generalized itemset example

user: **John,** ***time:*** **6.05 p.m.,** ***service:*** **Weather**
(s = 0.005%)

- A very low support
  - The itemset may be discarded
- By generalizing
  - the time attribute on a time period
  - the user on a user category

user: **employee,** ***time:*** **6 p.m. to 7 p.m.,** ***service:*** **Weather**
(s = 0.2%)

- May discover interesting properties generalizing *infrequent* items

Generalized association rules

- Extension of "classical" association rules

$$X \rightarrow Y$$

- X and Y are either generalized or not generalized itemsets
  - Extract associations among data items at multiple abstraction levels
  - Support, confidence and lift are defined accordingly

Patient data analysis

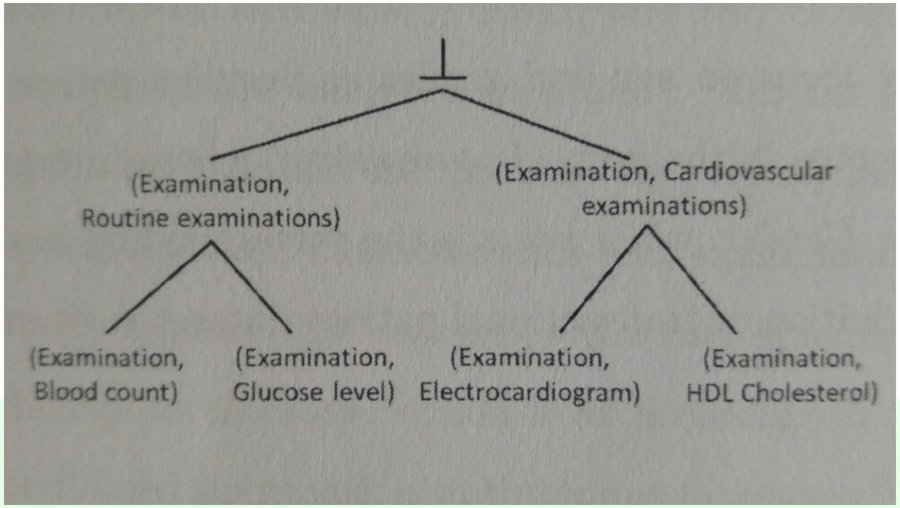
- Analysis of multiple level correlations on patient treatment historical data
  - Dataset collected by an Italian Local Health Center
    - Diabetes complications at various severity levels
    - 95K records, 3.5K patients
  - Features
    - Prescribed examinations (26 examinations, 7 categories)
    - Prescribed drugs (200 drugs, 14 categories)
    - Census patient data (gender, age discretized in age groups)
- Sparse dataset
  - Difficult setting of support threshold
    - Low: generates too many rules
    - High: interesting information at lower levels of abstraction may remain hidden

High level rules

- Only generalized itemsets
  - Represent general knowledge
    - May be too high level to perform targeted analyses
- Example

(Examination, Liver) -> (Examination, Kidney)

  - Frequently prescribed together
  - May be used for examination scheduling



Cross level rules

- Different abstraction levels (generalized items and data items)
  - Combine detailed and general information
- Example

  (Examination, Liver) -> (Examination, Uric acid)

  - Insight into specific kidney examinations correlated with liver examinations
    - Confidence: 74.8%

Low-lewel rules

- Only not generalized itemsets (only data items)
  - Very detailed knowledge
    - Covered by high and cross-level rules
  - Large rule set
    - Challenging exploration task
  - Drill down exploration based on formerly extracted high and cross-level rules