Association Rules Fundamentals

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Association rules

- Objective
  - extraction of frequent correlations or pattern from a transactional database

Tickets at a supermarket counter

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk
…	…

- Association rule
  diapers ⇒ beer
  - 2% of transactions contains both items
  - 30% of transactions containing diapers also contains beer

Association rule mining

- A collection of transactions is given
  - a transaction is a set of items
  - items in a transaction are not ordered
- Association rule
  A, B ⇒ C
  - A, B = items in the *rule body*
  - C = item in the *rule head*
- The ⇒ means co-occurrence
  - not causality
- Examples
  - cereals, cookies ⇒ milk
  - age < 40, life-insurance = yes ⇒ children = yes
  - customer, relationship ⇒ data, mining

Definitions

- *Itemset* is a set including one or more items
  - Example: {Beer, Diapers}
- *k-itemset* is an itemset that contains k items
- *Support count* (#) is the frequency of occurrence of an itemset
  - Example: #{Beer,Diapers} = 2
- *Support* is the fraction of transactions that contain an itemset
  - Example: sup({Beer, Diapers}) = 2/5
- *Frequent itemset* is an itemset whose support is greater than or equal to a *minsup* threshold

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk
…	…

Rule quality metrics

- Given the association rule
  A ⇒ B
  - A, B are itemsets
- *Support* is the fraction of transactions containing both A and B
  $$\frac{\#\{A,B\}}{|T|}$$
  - |T| is the cardinality of the transactional database
  - a priori probability of itemset AB
  - rule frequency in the database
- *Confidence* is the frequency of B in transactions containing A
  $$\frac{sup(A,B)}{sup(A)}$$
  - conditional probability of finding B having found A
  - "strength" of the "⇒"

Rule quality metrics: example

- From itemset {Milk, Diapers} the following rules may be derived
- Rule: Milk ⇒ Diapers
  - support
    sup=#{Milk,Diapers}/#trans. =3/5=60%
  - confidence
    conf=#{Milk,Diapers}/#{Milk}=3/4=75%
- Rule: Diapers ⇒ Milk
  - same support
    s=40%
  - confidence
    conf=#{Milk,Diapers}/#{Diapers}=3/3 =100%

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diapers, Milk
4	Beer, Bread, Diapers, Milk
5	Coke, Diapers, Milk
…	…

Association rule extraction

- Given a set of transactions T, association rule mining is the extraction of the rules satisfying the constraints
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold
- The result is
  - complete (*all* rules satisfying both constraints)
  - correct (*only* the rules satisfying both constraints)
- May add other more complex constraints

Association rule extraction

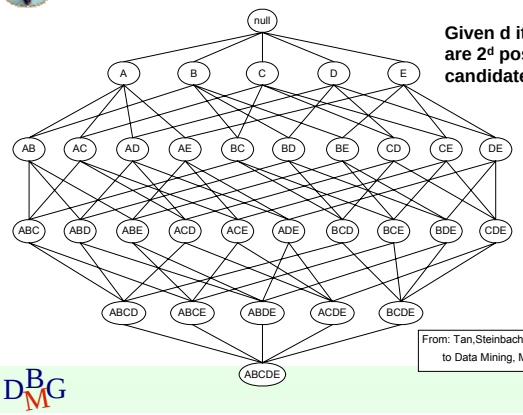
- Brute-force approach
  - enumerate all possible permutations (i.e., association rules)
  - compute support and confidence for each rule
  - prune the rules that do not satisfy the *minsup* and *minconf* constraints
- Computationally *unfeasible*
- Given an itemset, the extraction process may be split
  - first generate frequent itemsets
  - next generate rules from each frequent itemset
- Example
  - Itemset
    {Milk, Diapers} sup=60%
  - Rules
    Milk ⇒ Diapers (conf=75%)
    Diapers ⇒ Milk (conf=100%)

Association rule extraction

(1) Extraction of frequent itemsets
- many different techniques
  - level-wise approaches (Apriori, ...)
  - approaches without candidate generation (FP-growth, ...)
  - other approaches
- most computationally expensive step
  - limit extraction time by means of support threshold

(2) Extraction of association rules
- generation of all possible binary partitioning of each frequent itemset
  - possibly enforcing a confidence threshold

Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Frequent Itemset Generation

- Brute-force approach
  - each itemset in the lattice is a *candidate* frequent itemset
  - scan the database to count the support of each candidate
    - match each transaction against every candidate
  - Complexity ~ O(|T| 2^d w)
    - |T| is number of transactions
    - d is number of items
    - w is transaction length

Improving Efficiency

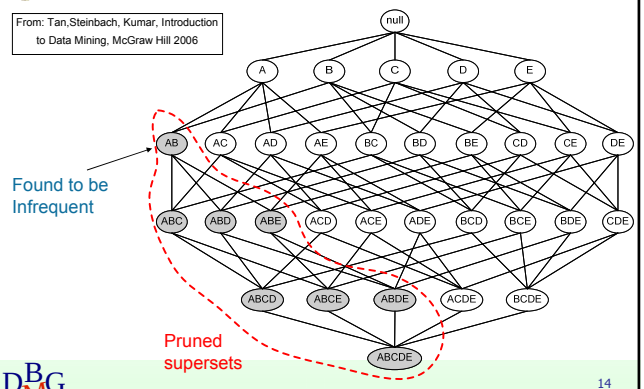
- Reduce the number of candidates
  - Prune the search space
    - complete set of candidates is 2^d
- Reduce the number of transactions
  - Prune transactions as the size of itemsets increases
    - reduce |T|
- Reduce the number of comparisons
  - Equal to |T| 2^d
  - Use efficient data structures to store the candidates or transactions

The Apriori Principle

"If an itemset is frequent, then all of its subsets must also be frequent"

- The support of an itemset can never exceed the support of any of its subsets
- It holds due to the antimonotone property of the support measure
  - Given two arbitrary itemsets A and B
    if $A \subseteq B$ then $sup(A) \geq sup(B)$
- It reduces the number of candidates

The Apriori Principle



Apriori Algorithm [Agr94]

- Level-based approach
  - at each iteration extracts itemsets of a given length k
- Two main steps for each level
  - (1) Candidate generation
    - Join Step
      - generate candidates of length k+1 by joining frequent itemsets of length k
    - Prune Step
      - apply Apriori principle: prune length k+1 candidate itemsets that contain at least one k-itemset that is not frequent
  - (2) Frequent itemset generation
    - scan DB to count support for k+1 candidates
    - prune candates below minsup

Apriori Algorithm [Agr94]

- Pseudo-code
  C_k: Candidate itemset of size k
  L_k : frequent itemset of size k

  L_1 = {frequent items};
  for (k = 1; L_k !=∅; k++) **do**
  begin
  C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
  increment the count of all candidates in C_{k+1} that are contained in t
  L_{k+1} = candidates in C_{k+1} satisfying *minsup*
  end
  return $\cup_k L_k$;

Generating Candidates

- Sort L_k candidates in lexicographical order
- For each candidate of length k
  - Self-join with each candidate sharing same L_{k-1} prefix
  - Prune candidates by applying Apriori principle
- Example: given L_3={*abc, abd, acd, ace, bcd*}
  - Self-join
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Prune by applying Apriori principle
    - *acde* is removed because *ade* is not in L_3
    - C_4={*abcd*}

Apriori Algorithm: Example

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup>1

Generate candidate 1-itemsets

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

1st DB scan

C_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

minsup>1

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Prune infrequent candidates in C_1

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

1st DB scan

C_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

$L_1 \equiv C_1$

- All itemsets in set C_1 are frequent according to minsup>1

minsup>1

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Generate candidates from L_1

L_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

C_2

itemsets
{A,B}
{A,C}
{A,D}
{A,E}
{B,C}
{B,D}
{B,E}
{C,D}
{C,E}
{D,E}

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Count support for candidates in C_2

L_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

C_2

itemsets
{A,B}
{A,C}
{A,D}
{A,E}
{B,C}
{B,D}
{B,E}
{C,D}
{C,E}
{D,E}

2nd DB scan

C_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{B,E}	1
{C,D}	3
{C,E}	2
{D,E}	2

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Prune infrequent candidates in C_2

L_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

C_2

itemsets
{A,B}
{A,C}
{A,D}
{A,E}
{B,C}
{B,D}
{B,E}
{C,D}
{C,E}
{D,E}

2nd DB scan

C_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{B,E}	1
{C,D}	3
{C,E}	2
{D,E}	2

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

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Generate candidates from L_2

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

C_3

itemsets
{A,B,C}
{A,B,D}
{A,B,E}
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

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Apply Apriori principle on C_3

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

C_3

itemsets
{A,B,C}
{A,B,D}
~~{A,B,E}~~
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

- Prune {A,B,E}
  - Its subset {B,E} is infrequent ({B,E} is not in L_2)

Count support for candidates in C_3

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

C_3

itemsets
{A,B,C}
{A,B,D}
~~{A,B,E}~~
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

3rd DB scan

C_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,C,E}	1
{A,D,E}	2
{B,C,D}	2
{C,D,E}	1

Prune infrequent candidates in C_3

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

C_3

itemsets
{A,B,C}
{A,B,D}
~~{A,B,E}~~
{A,C,D}
{A,C,E}
{A,D,E}
{B,C,D}
{C,D,E}

3rd DB scan

C_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
~~{A,C,E}~~	~~1~~
{A,D,E}	2
{B,C,D}	2
~~{C,D,E}~~	~~1~~

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

- {A,C,E} and {C,D,E} are actually infrequent
  - They are discarded from C_3

Generate candidates from L_3

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

C_4

itemsets
{A,B,C,D}

Apply Apriori principle on C_4

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

C_4

itemsets
{A,B,C,D}

- Check if {A,C,D} and {B,C,D} belong to L_3
  - L_3 contains all 3-itemset subsets of {A,B,C,D}
  - {A,B,C,D} is potentially frequent

Count support for candidates in C_4

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

C_4

itemsets
{A,B,C,D}

4th DB scan

C_4

itemsets	sup
{A,B,C,D}	1

Prune infrequent candidates in C_4

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

C_4

itemsets
{A,B,C,D}

4th DB scan

C_4

itemsets	sup
~~{A,B,C,D}~~	~~1~~

$L_4 = \emptyset$

- {A,B,C,D} is actually infrequent
  - {A,B,C,D} is discarded from C_4

Final set of frequent itemsets

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup>1

L_1

itemsets	sup
{A}	7
{B}	8
{C}	7
{D}	5
{E}	3

L_2

itemsets	sup
{A,B}	5
{A,C}	4
{A,D}	4
{A,E}	2
{B,C}	6
{B,D}	3
{C,D}	3
{C,E}	2
{D,E}	2

L_3

itemsets	sup
{A,B,C}	3
{A,B,D}	2
{A,C,D}	2
{A,D,E}	2
{B,C,D}	2

Counting Support of Candidates

- Scan transaction database to count support of each itemset
  - total number of candidates may be large
  - one transaction may contain many candidates
- Approach [Agr94]
  - candidate itemsets are stored in a *hash-tree*
    - *leaf* node of hash-tree contains a list of itemsets and counts
    - *interior* node contains a hash table
  - subset function finds all candidates contained in a transaction
    - match transaction subsets to candidates in hash tree

Performance Issues in Apriori

- Candidate generation
  - Candidate sets may be huge
    - 2-itemset candidate generation is the most critical step
    - extracting long frequent intemsets requires generating all frequent subsets
- Multiple database scans
  - $n+1$ scans when longest frequent pattern length is n

Factors Affecting Performance

- Minimum support threshold
  - lower support threshold increases number of frequent itemsets
    - larger number of candidates
    - larger (max) length of frequent itemsets
- Dimensionality (number of items) of the data set
  - more space is needed to store support count of each item
  - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
  - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
  - transaction width increases in dense data sets
  - may increase max length of frequent itemsets and traversals of hash tree
    - number of subsets in a transaction increases with its width

Improving Apriori Efficiency

- Hash-based itemset counting [Yu95]
  - A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
- Transaction reduction [Yu95]
  - A transaction that does not contain any frequent k-itemset is useless in subsequent scans
- Partitioning [Sav96]
  - Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB

Improving Apriori Efficiency

- Sampling [Toi96]
  - mining on a subset of given data, lower support threshold + a method to determine the completeness
- Dynamic Itemset Counting [Motw98]
  - add new candidate itemsets only when all of their subsets are estimated to be frequent

FP-growth Algorithm [Han00]

- Exploits a main memory compressed rappresentation of the database, the FP-tree
  - high compression for dense data distributions
    - less so for sparse data distributions
  - complete representation for frequent pattern mining
    - enforces support constraint
- Frequent pattern mining by means of FP-growth
  - recursive visit of FP-tree
  - applies divide-and-conquer approach
    - decomposes mining task into smaller subtasks
- Only two database scans
  - count item supports + build FP-tree

FP-tree construction

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup>1

- (1) Count item support and prune items below minsup threshold
- (2) Build Header Table by sorting items in decreasing support order

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

FP-tree construction

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

minsup>1

- (1) Count item support and prune items below minsup threshold
- (2) Build Header Table by sorting items in decreasing support order
- (3) Create FP-tree

  For each transaction t in DB
  - order transaction t items in decreasing support order
    - same order as Header Table
  - insert transaction t in FP-tree
    - use existing path for common prefix
    - create new branch when path becomes different

FP-tree construction

Transaction

TID	Items
1	{A,B}

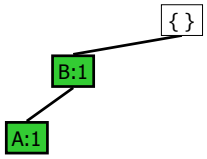
Sorted transaction

TID	Items
1	{B,A}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

FP-tree



FP-tree construction

Transaction

TID	Items
2	{B,C,D}

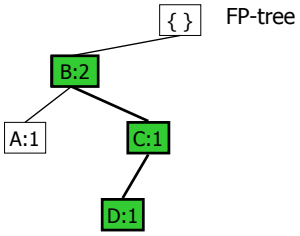
Sorted transaction

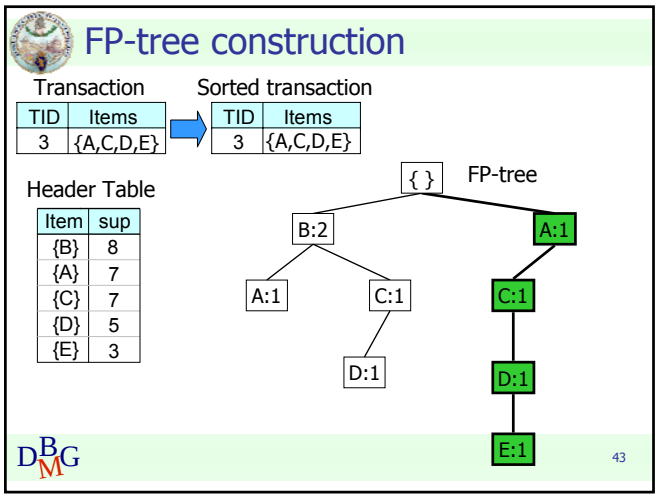
TID	Items
2	{B,C,D}

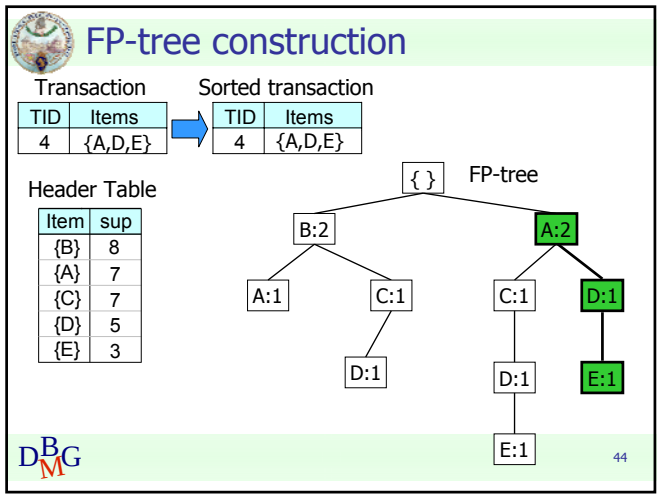
Header Table

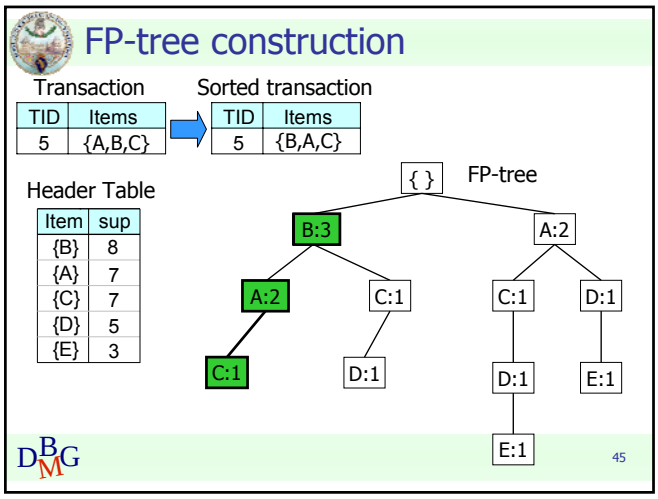
Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

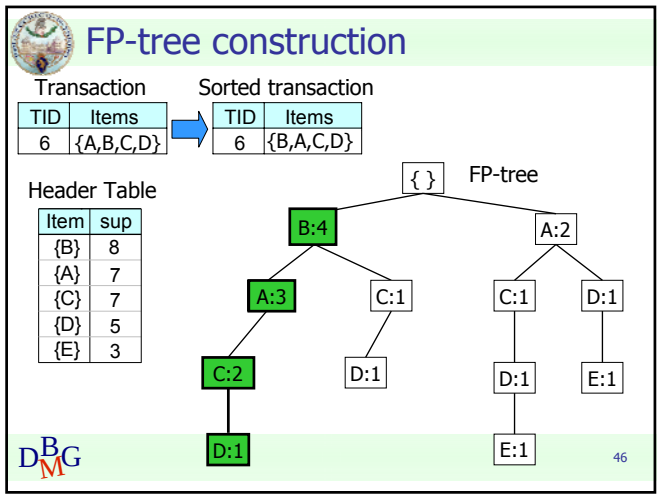
FP-tree

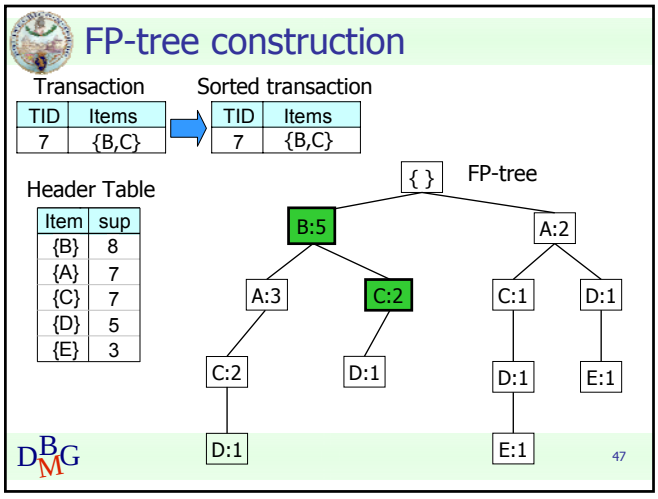


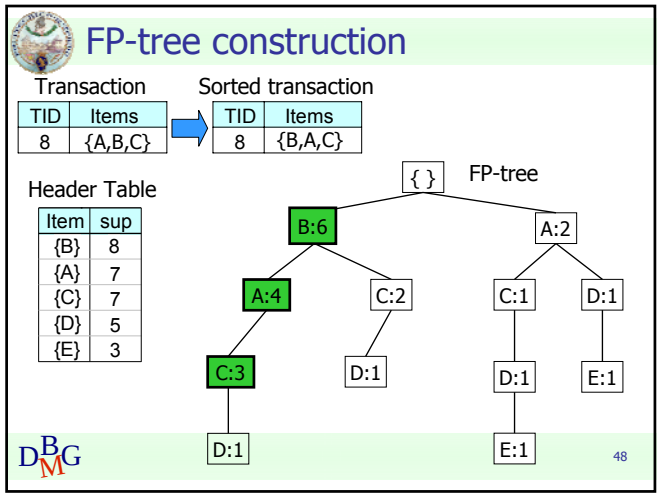

FP-tree construction
Transaction
Sorted transaction
TID Items
3 {A,C,D,E}
TID Items
3 {A,C,D,E}
Header Table
Item sup
{B} 8
{A} 7
{C} 7
{D} 5
{E} 3
FP-tree
{ }
B:2
A:1
A:1
C:1
C:1
D:1
D:1
E:1
43


FP-tree construction
Transaction
Sorted transaction
TID Items
4 {A,D,E}
TID Items
4 {A,D,E}
Header Table
Item sup
{B} 8
{A} 7
{C} 7
{D} 5
{E} 3
FP-tree
{ }
B:2
A:2
A:1
C:1
C:1
D:1
D:1
D:1
E:1
E:1
44


FP-tree construction
Transaction
Sorted transaction
TID Items
5 {A,B,C}
TID Items
5 {B,A,C}
Header Table
Item sup
{B} 8
{A} 7
{C} 7
{D} 5
{E} 3
FP-tree
{ }
B:3
A:2
A:2
C:1
C:1
D:1
C:1
D:1
D:1
E:1
E:1
45


FP-tree construction
Transaction
Sorted transaction
TID Items
6 {A,B,C,D}
TID Items
6 {B,A,C,D}
Header Table
Item sup
{B} 8
{A} 7
{C} 7
{D} 5
{E} 3
FP-tree
{ }
B:4
A:2
A:3
C:1
C:1
D:1
C:2
D:1
D:1
E:1
D:1
E:1
46


FP-tree construction
Transaction
Sorted transaction
TID Items
7 {B,C}
TID Items
7 {B,C}
Header Table
Item sup
{B} 8
{A} 7
{C} 7
{D} 5
{E} 3
FP-tree
{ }
B:5
A:2
A:3
C:2
C:1
D:1
C:2
D:1
D:1
E:1
D:1
E:1
47


FP-tree construction
Transaction
Sorted transaction
TID Items
8 {A,B,C}
TID Items
8 {B,A,C}
Header Table
Item sup
{B} 8
{A} 7
{C} 7
{D} 5
{E} 3
FP-tree
{ }
B:6
A:2
A:4
C:2
C:1
D:1
C:3
D:1
D:1
E:1
D:1
E:1
48

FP-tree construction

Transaction

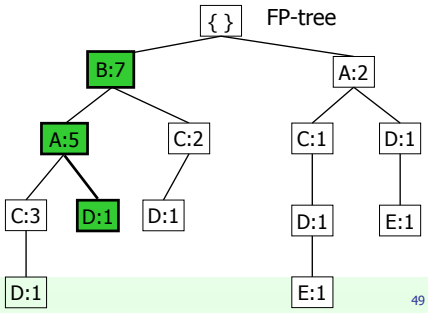
TID	Items
9	{A,B,D}

Sorted transaction

TID	Items
9	{B,A,D}

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



FP-tree construction

Transaction

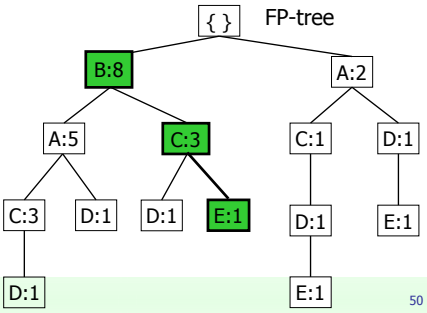
TID	Items
10	{B,C,E}

Sorted transaction

TID	Items
10	{B,C,E}

Header Table

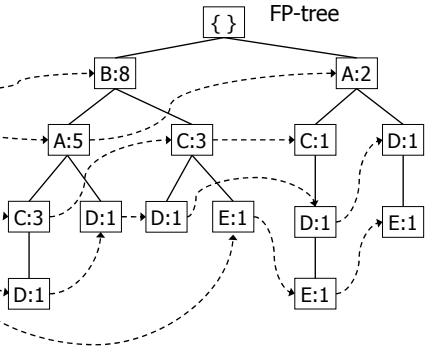
Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



Final FP-tree

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3



Item pointers are used to assist frequent itemset generation

FP-growth Algorithm

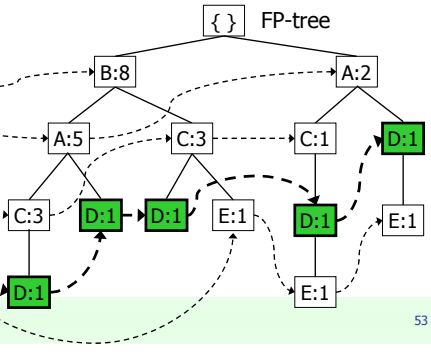
- Scan Header Table from lowest support item up
- For each item i in Header Table extract frequent itemsets including item i and items preceding it in Header Table
  - (1) build Conditional Pattern Base for item i (i-CPB)
    - Select prefix-paths of item i from FP-tree
  - (2) recursive invocation of FP-growth on i-CPB

Example

- Consider item D and extract frequent itemsets including
  - D and supported combinations of items A, B, C

Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

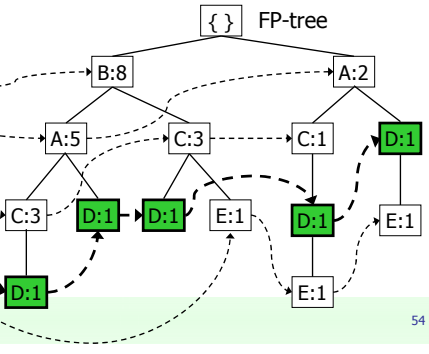


Conditional Pattern Base of D

- (1) Build D-CPB
  - Select prefix-paths of item D from FP-tree

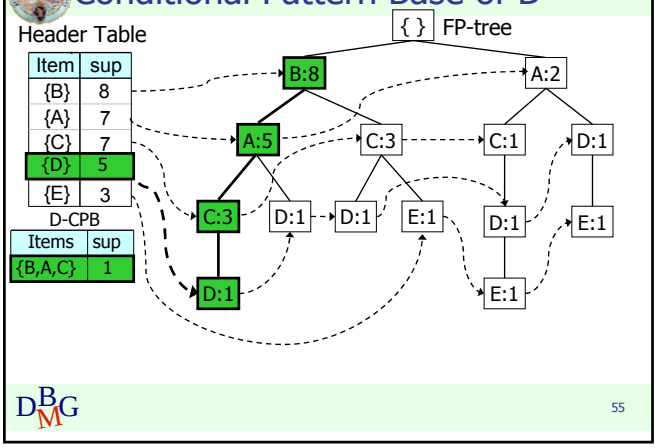
Header Table

Item	sup
{B}	8
{A}	7
{C}	7
{D}	5
{E}	3

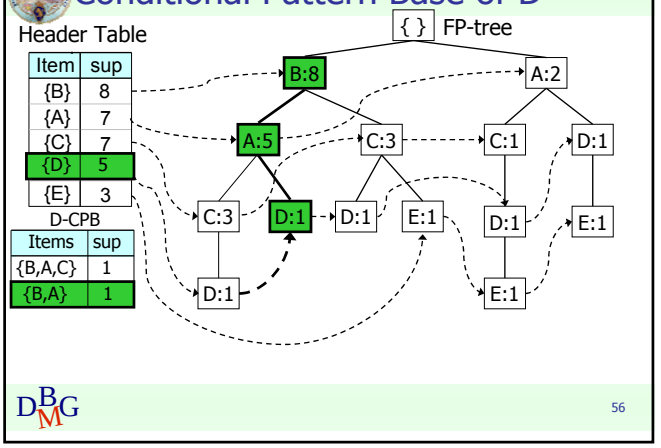


Frequent itemset:
D, sup(D) = 5

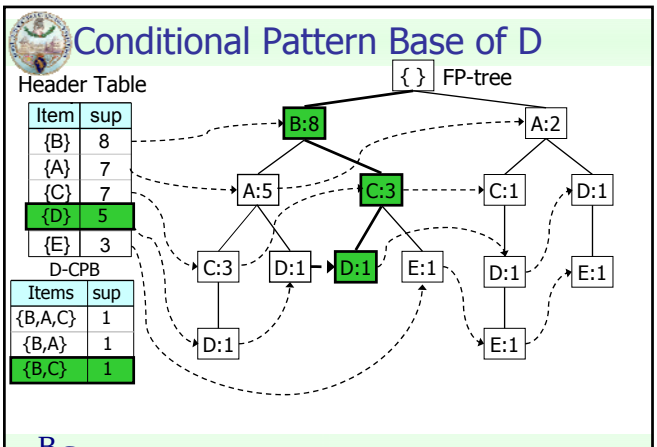
Conditional Pattern Base of D



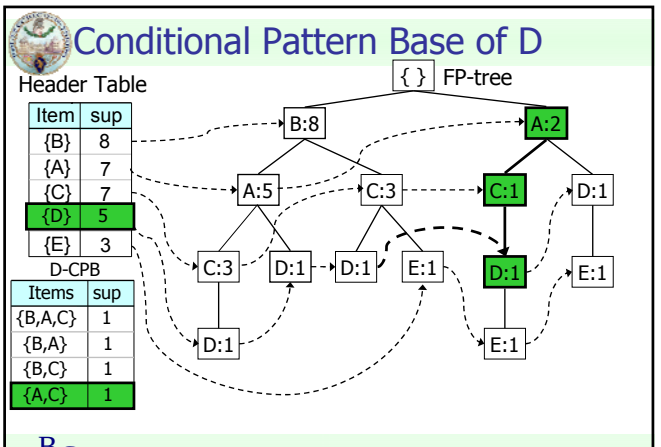
Conditional Pattern Base of D



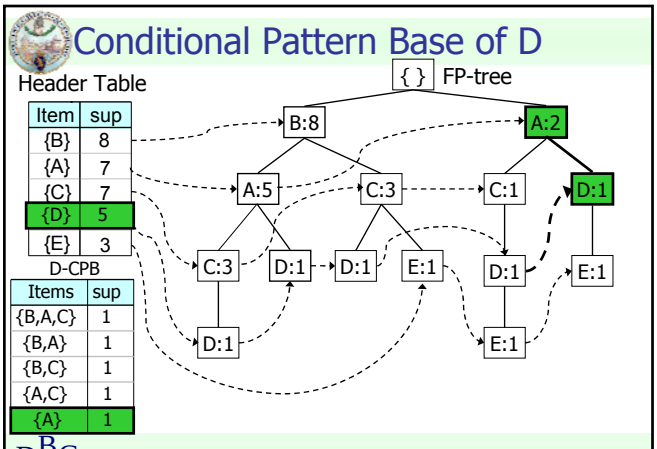
Conditional Pattern Base of D



Conditional Pattern Base of D

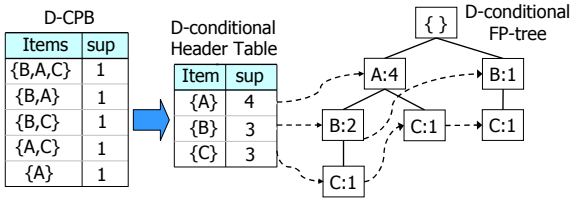


Conditional Pattern Base of D



Conditional Pattern Base of D

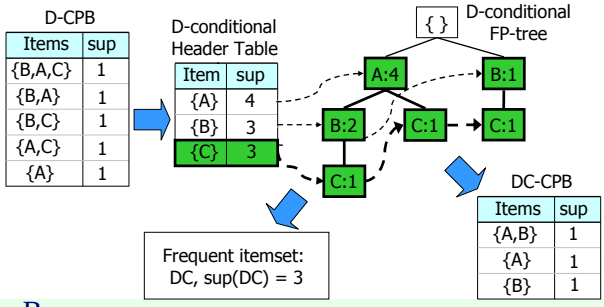
- (1) Build D-CPB
  - Select prefix-paths of item D from FP-tree



- (2) Recursive invocation of FP-growth on D-CPB

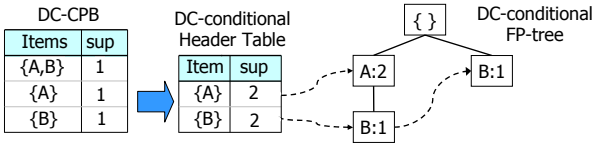
Conditional Pattern Base of DC

- (1) Build DC-CPB
  - Select prefix-paths of item C from D-conditional FP-tree



Conditional Pattern Base of DC

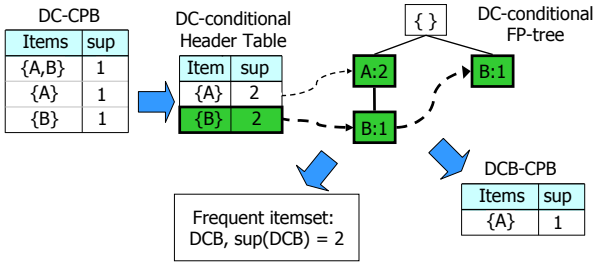
- (1) Build DC-CPB
  - Select prefix-paths of item C from D-conditional FP-tree



- (2) Recursive invocation of FP-growth on DC-CPB

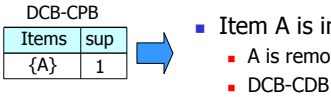
Conditional Pattern Base of DCB

- (1) Build DCB-CPB
  - Select prefix-paths of item B from DC-conditional FP-tree



Conditional Pattern Base of DCB

- (1) Build DCB-CPB
  - Select prefix-paths of item B from DC-conditional FP-tree

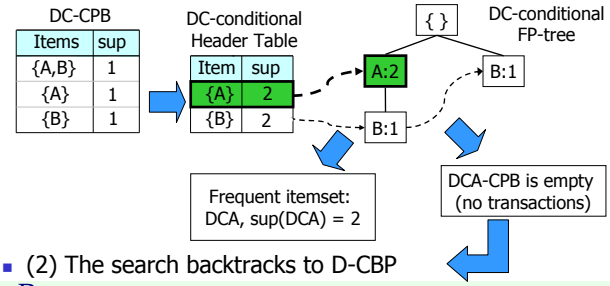


- Item A is infrequent in DCB-CPB
  - A is removed from DCB-CBP
  - DCB-CDB is empty

- (2) The search backtracks to DC-CBP

Conditional Pattern Base of DCA

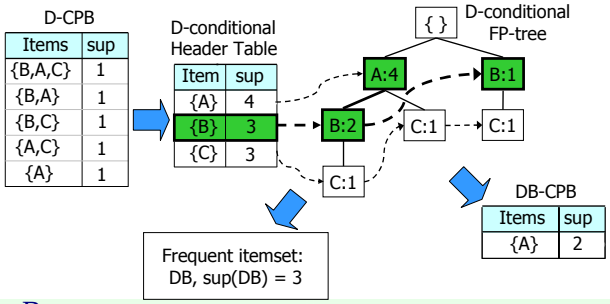
- (1) Build DCA-CPB
  - Select prefix-paths of item A from DC-conditional FP-tree



- (2) The search backtracks to D-CBP

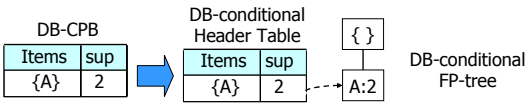
Conditional Pattern Base of DB

- (1) Build DB-CPB
  - Select prefix-paths of item B from D-conditional FP-tree



Conditional Pattern Base of DB

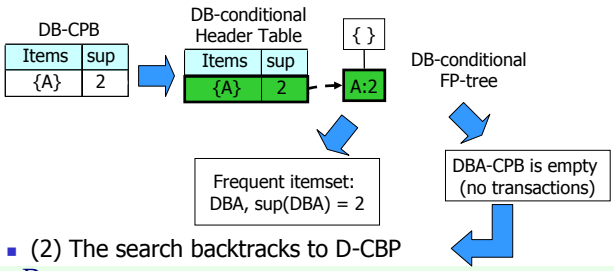
- (1) Build DB-CPB
  - Select prefix-paths of item B from D-conditional FP-tree



- (2) Recursive invocation of FP-growth on DB-CPB

Conditional Pattern Base of DBA

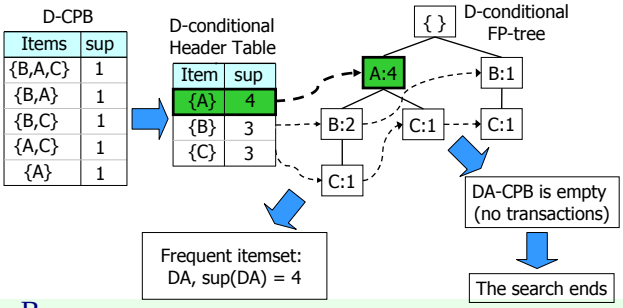
- (1) Build DBA-CPB
  - Select prefix-paths of item A from DB-conditional FP-tree



- (2) The search backtracks to D-CBP

Conditional Pattern Base of DA

- (1) Build DA-CPB
  - Select prefix-paths of item A from D-conditional FP-tree



Frequent itemsets with prefix D

- Frequent itemsets including D and supported combinations of items B,A,C

Example DB

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

itemsets	sup
{D}	5
{A,D}	4
{B,D}	3
{C,D}	3
{A,B,D}	2
{A,C,D}	2
{B,C,D}	2

minsup>1

Other approaches

- Many other approaches to frequent itemset extraction
  - some covered later
- May exploit a different database representation
  - represent the tidset of each item [Zak00]

Horizontal Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

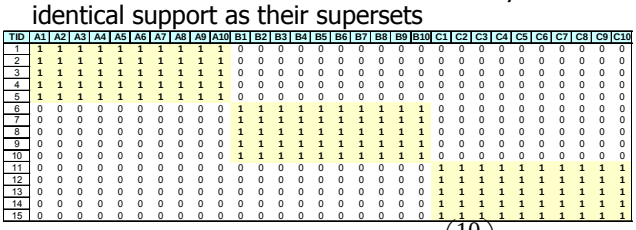
Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Compact Representations

- Some itemsets are redundant because they have identical support as their supersets

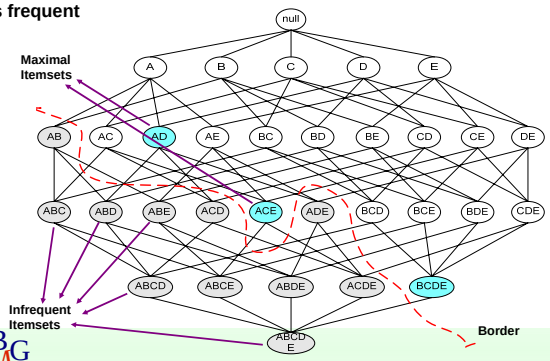


- Number of frequent itemsets $= 3 \times \sum_{k=1}^{10} \binom{10}{k}$
- A compact representation is needed

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Maximal Frequent Itemset

An itemset is maximal frequent if none of its immediate supersets is frequent



From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Closed Itemset

- An itemset is closed if none of its immediate supersets has the same support as the itemset

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

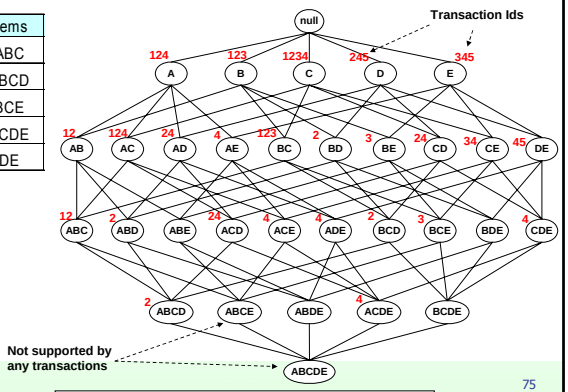
itemset	sup
{A}	4
{B}	*5*
{C}	3
{D}	4
{A,B}	*4*
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	*4*
{C,D}	3

itemset	sup
{A,B,C}	2
{A,B,D}	*3*
{A,C,D}	2
{B,C,D}	*3*
{A,B,C,D}	*2*

From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

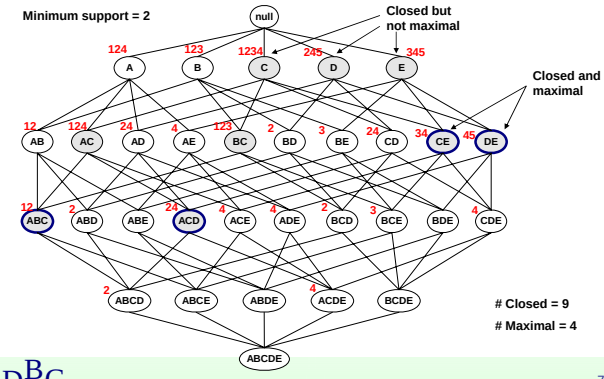
Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



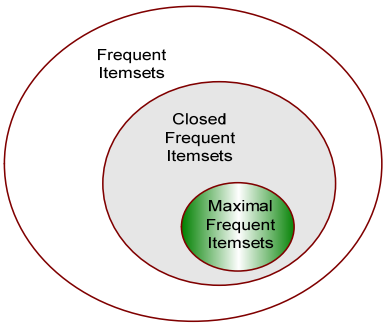
From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Maximal vs Closed Frequent Itemsets



From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Maximal vs Closed Itemsets



From: Tan,Steinbach, Kumar, Introduction to Data Mining, McGraw Hill 2006

Effect of Support Threshold

- Selection of the appropriate *minsup* threshold is not obvious
  - If *minsup* is too high
    - itemsets including rare but interesting items may be lost
      - example: pieces of jewellery (or other expensive products)
  - If *minsup* is too low
    - it may become computationally *very expensive*
    - the number of frequent itemsets becomes *very large*

Interestingness Measures

- A large number of pattern may be extracted
  - rank patterns by their interestingness
- Objective measures
  - rank patterns based on statistics computed from data
  - initial framework [Agr94] only considered support and confidence
    - other statistical measures available
- Subjective measures
  - rank patterns according to user interpretation [Silb98]
    - interesting if it contradicts the expectation of a user
    - interesting if it is actionable

Confidence measure: always reliable?

- 5000 high school students are given
  - 3750 eat cereals
  - 3000 play basket
  - 2000 eat cereals and play basket
- Rule

play basket ⇒ eat cereals
sup = 40%, conf = 66,7%

is misleading because eat cereals has sup 75% (>66,7%)

- Problem caused by high frequency of rule head
  - negative correlation

	basket	not basket	total
cereals	2000	1750	3750
not cereals	1000	250	1250
total	3000	2000	5000

Correlation or lift

r: A ⇒ B

$$\text{Correlation} = \frac{P(A,B)}{P(A)P(B)} = \frac{\text{conf(r)}}{\text{sup(B)}}$$

- Statistical independence
  - Correlation = 1
- Positive correlation
  - Correlation > 1
- Negative correlation
  - Correlation < 1

Example

- Association rule

play basket ⇒ eat cereals

has corr = 0.89
  - negative correlation
- but rule

play basket ⇒ not (eat cereals)

has corr = 1,34

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{A}\overline{B})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{A}\overline{B})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{A}\overline{B})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{A}\overline{B})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i), -\sum_j P(B_j)\log P(B_j))}$
8	J-Measure (J)	$\max\left(P(A,B)\log\left(\frac{P(B\|A)}{P(B)}\right)+P(A\overline{B})\log\left(\frac{P(\overline{B}\|A)}{P(\overline{B})}\right), P(A,B)\log\left(\frac{P(A\|B)}{P(A)}\right)+P(\overline{A}B)\log\left(\frac{P(\overline{A}\|B)}{P(\overline{A})}\right)\right)$
9	Gini index (G)	$\max\left(P(A)[P(B\|A)^2+P(\overline{B}\|A)^2]+P(\overline{A})[P(B\|\overline{A})^2+P(\overline{B}\|\overline{A})^2]-P(B)^2-P(\overline{B})^2, P(B)[P(A\|B)^2+P(\overline{A}\|B)^2]+P(\overline{B})[P(A\|\overline{B})^2+P(\overline{A}\|\overline{B})^2]-P(A)^2-P(\overline{A})^2\right)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\|A), P(A\|B))$
12	Laplace (L)	$\max\left(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2}\right)$
13	Conviction (V)	$\max\left(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})}\right)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B)-P(A)P(B)$
17	Certainty factor (F)	$\max\left(\frac{P(B\|A)-P(B)}{1-P(B)}, \frac{P(A\|B)-P(A)}{1-P(A)}\right)$
18	Added Value (AV)	$\max(P(B\|A)-P(B), P(A\|B)-P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{A}\overline{B})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{A}\overline{B})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\|A)-P(B), P(A\|B)-P(A))$