

# Visual perception

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## Data Management and Visualization



**SoftEng**  
<http://softeng.polito.it>

Version 2.1.1  
© Marco Torchiano, 2020








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# GRAPHICAL INTEGRITY

- **Proportionality**

- ◆ Representation as physical quantities should be proportional to the represented numbers

- **Utility**

- ◆ Graphical element should convey useful information

- **Clarity**

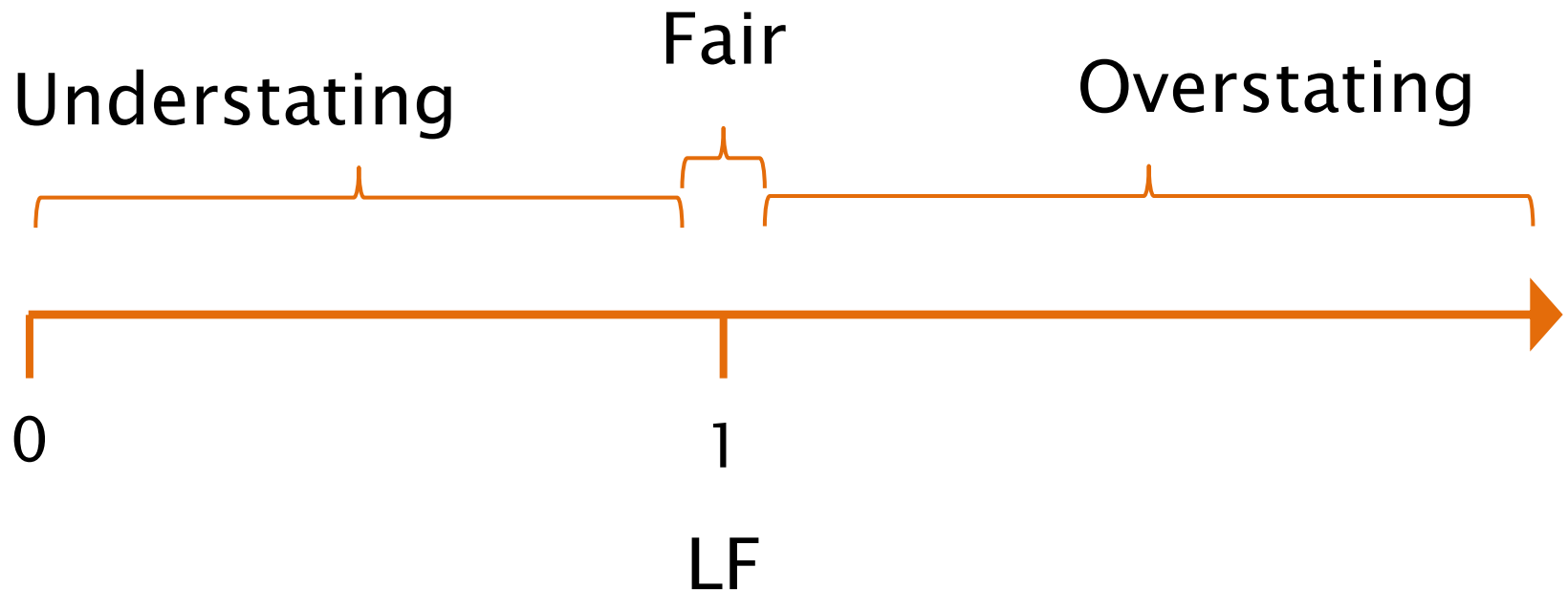
- ◆ Labeling should counter graphical distortion and ambiguity

- The magnitude of visual attributes should represent faithfully the magnitude of measures
- They should allow
  - ◆ Discrimination: are they different?
  - ◆ Comparison: which is larger?
  - ◆ Magnitude Assessment: how much larger?

$$LF = \frac{\text{size of effect shown in graphic}}{\text{size of effect in data}}$$

- Overstating
  - ◆  $LF > 1 \Leftrightarrow \text{Log}(LF) > 0$
- Understating
  - ◆  $LF < 1 \Leftrightarrow \text{Log}(LF) < 0$
- Fair
  - $LF = 1 \Leftrightarrow \text{Log}(LF) = 0$

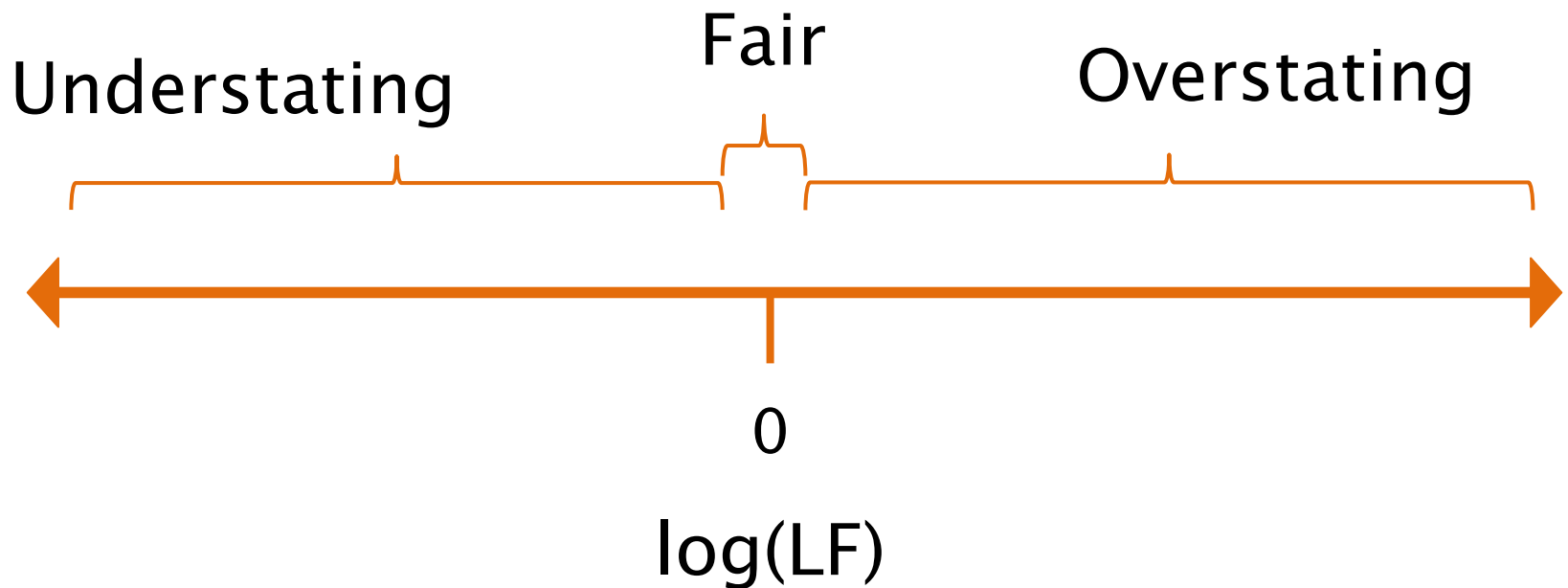
$$LF = \frac{\text{size of effect shown in graphic}}{\text{size of effect in data}}$$



# Lie Factor

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$$LF = \frac{\text{size of effect shown in graphic}}{\text{size of effect in data}}$$





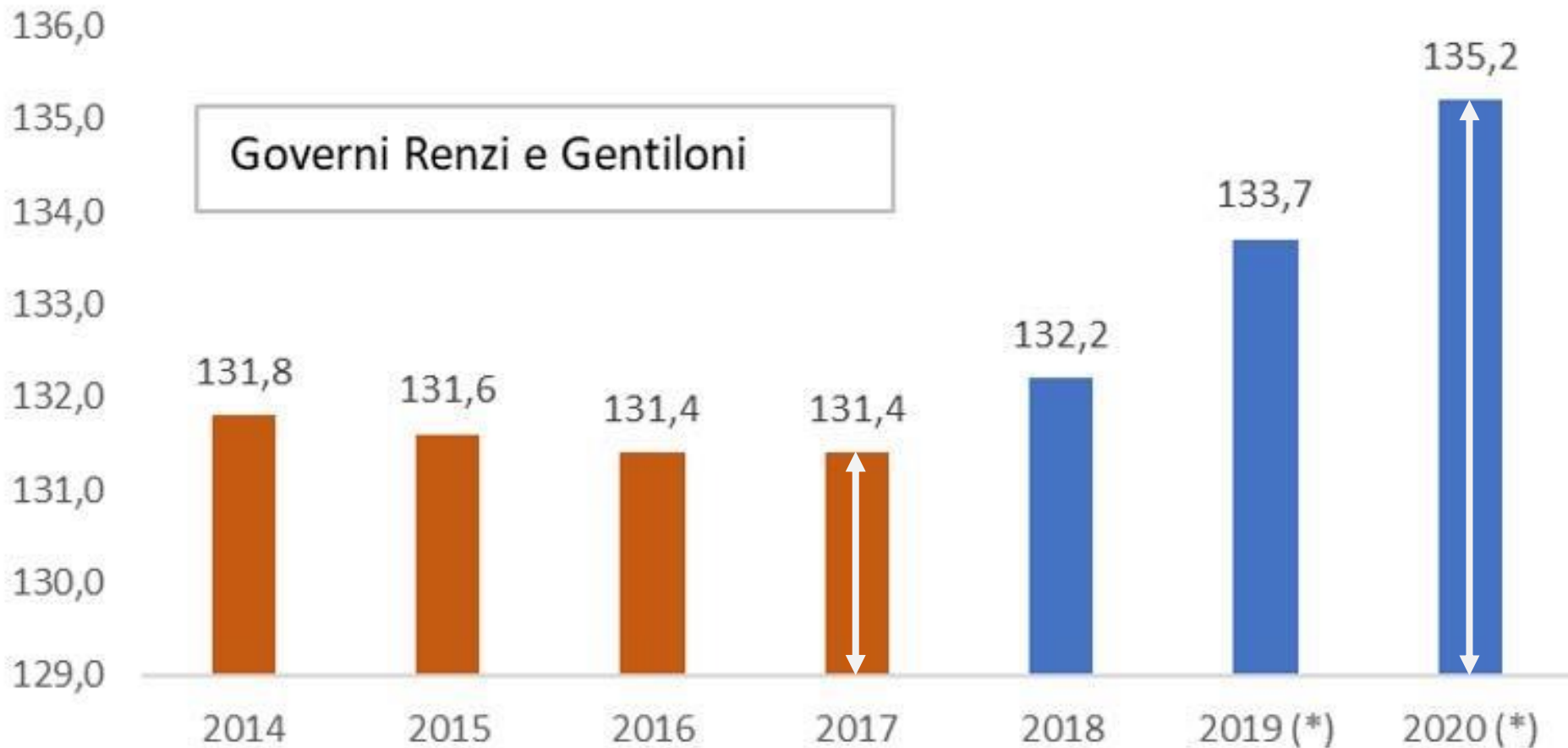
# Example

## Debito pubblico (% PIL)

(\*) previsioni Commissione UE

Governo Conte

Governi Renzi e Gentiloni

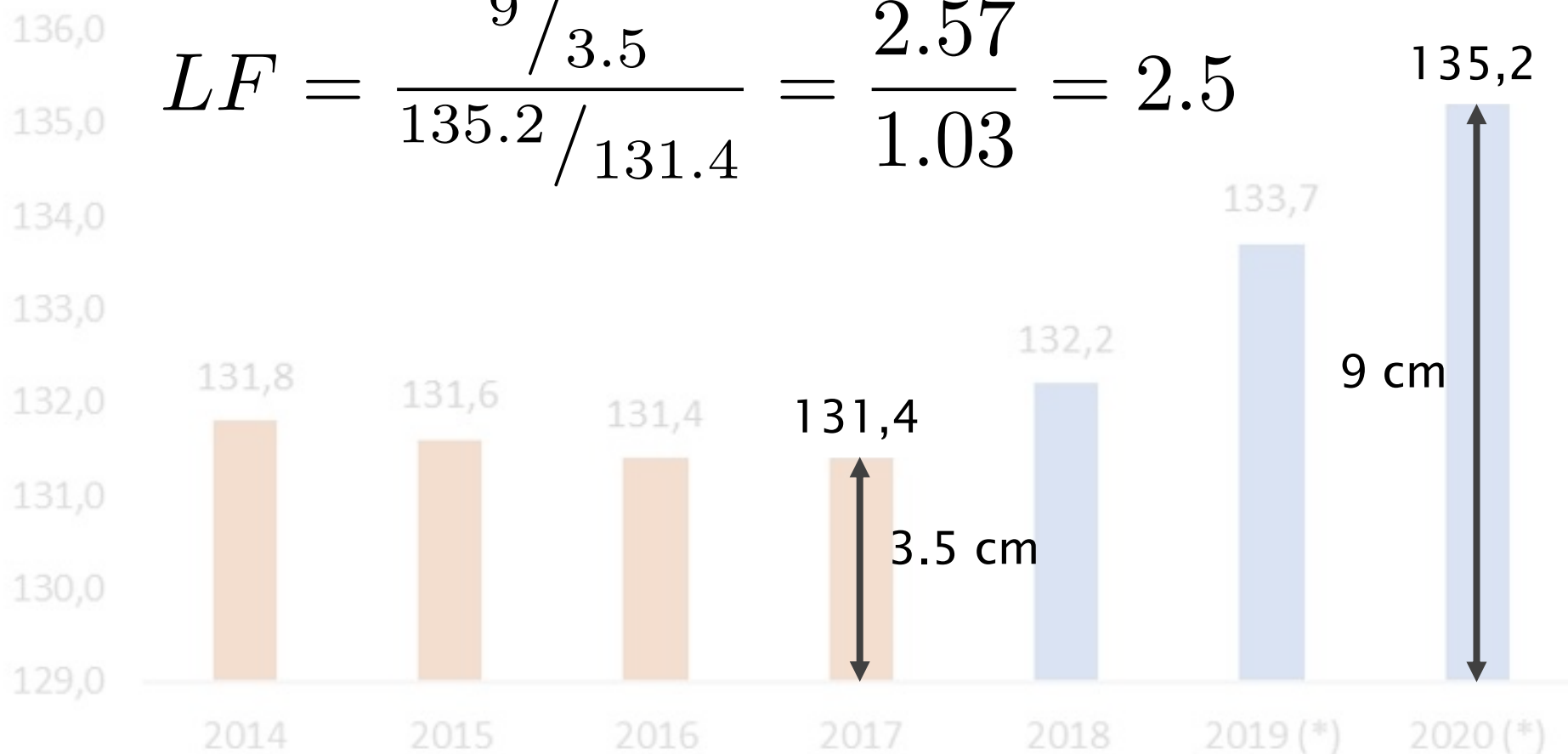


# Example – Lie Factor

Debito pubblico (% PIL)

(\*) previsioni Commissione UE

$$LF = \frac{9 / 3.5}{135.2 / 131.4} = \frac{2.57}{1.03} = 2.5$$

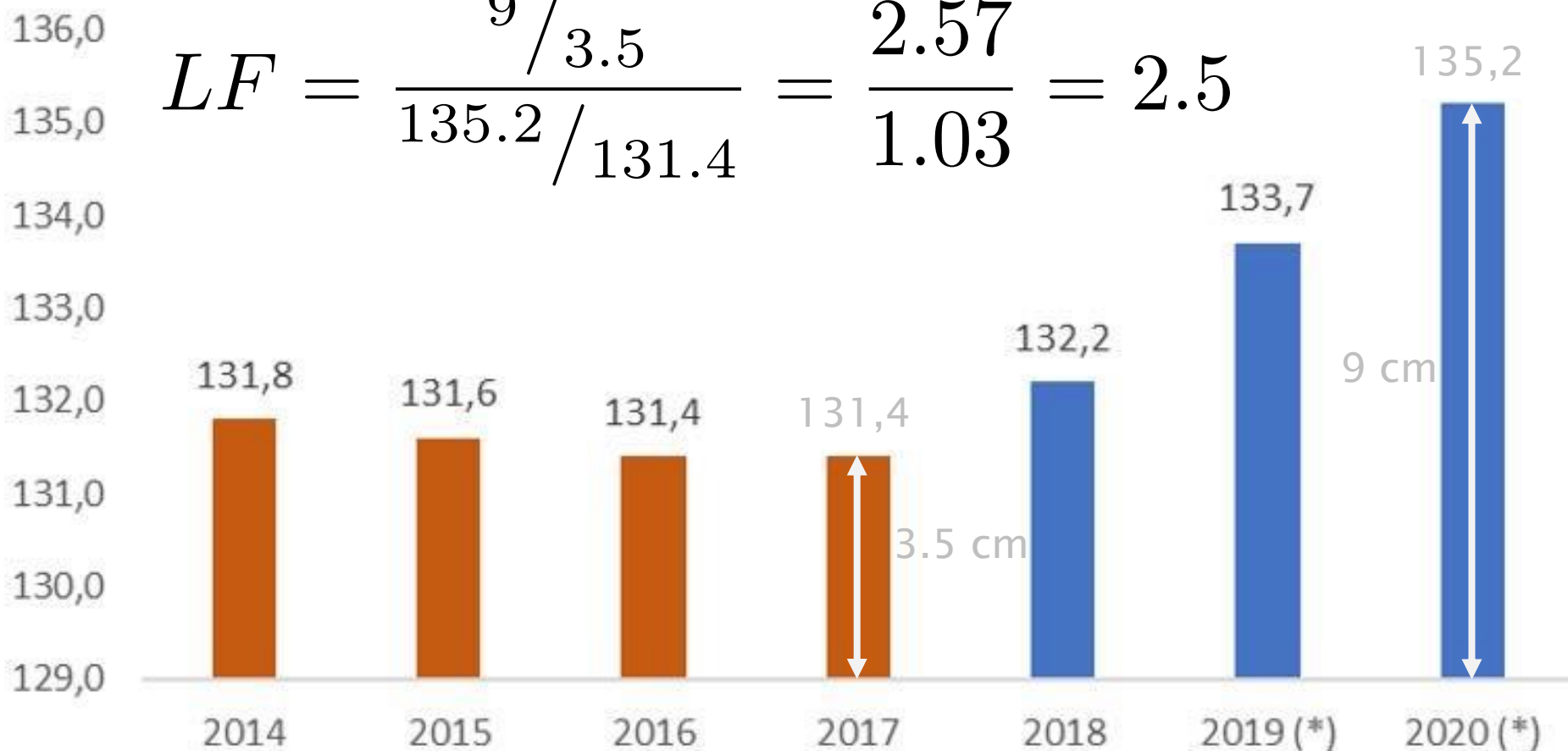


# Example – Lie Factor

Debito pubblico (% PIL)

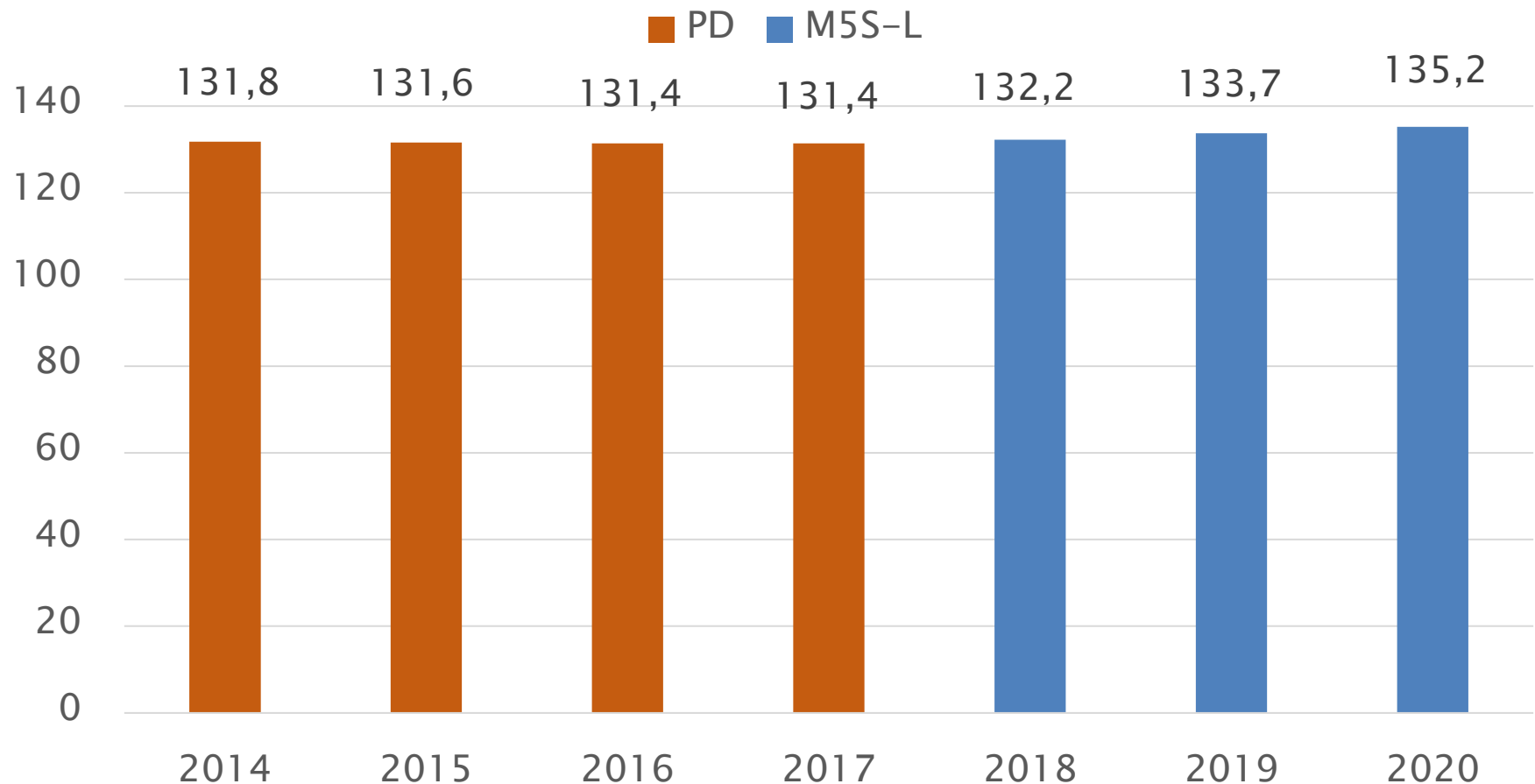
(\*) previsioni Commissione UE

$$LF = \frac{9 / 3.5}{135.2 / 131.4} = \frac{2.57}{1.03} = 2.5$$



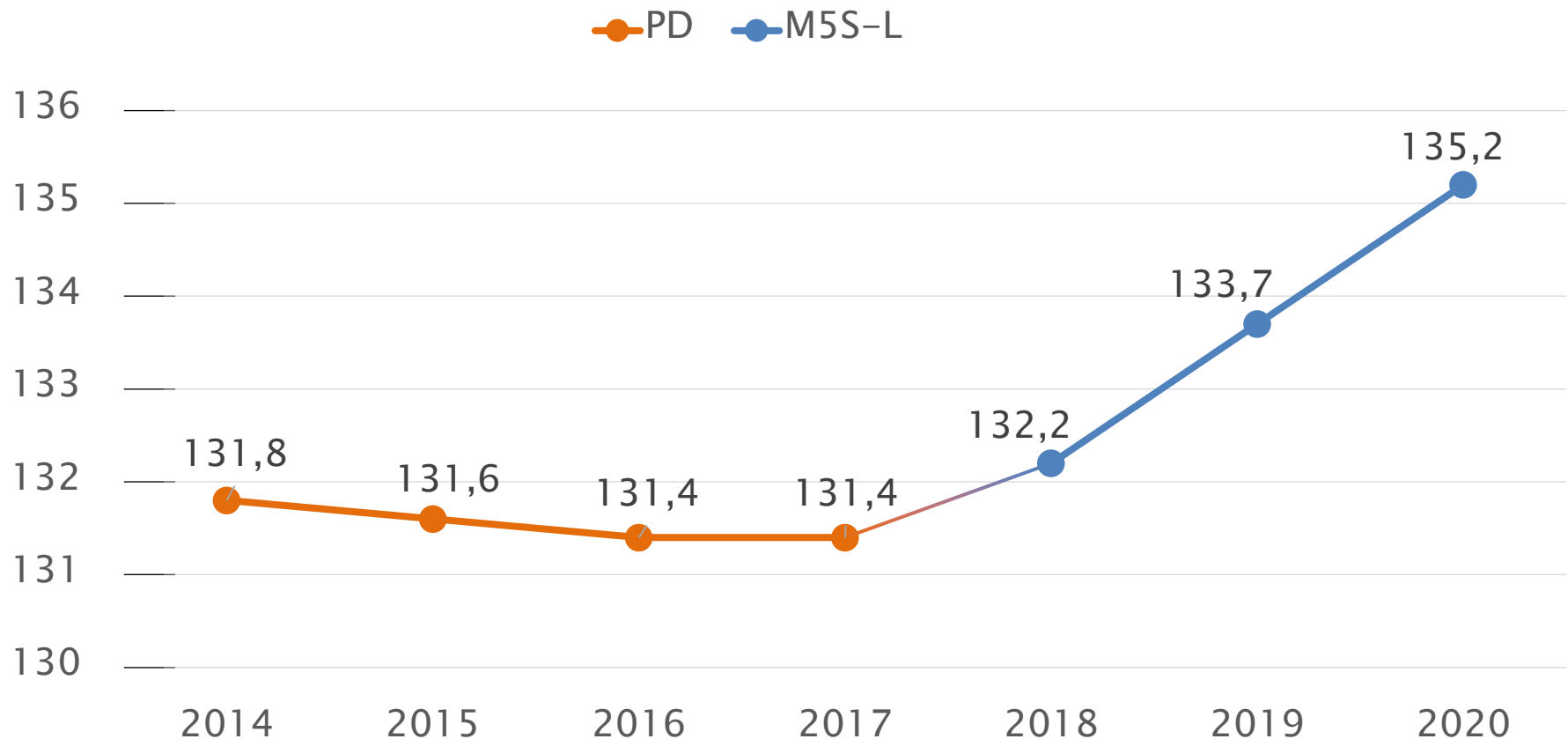
# Example – Redesign

## Debito Pubblico (% PIL)

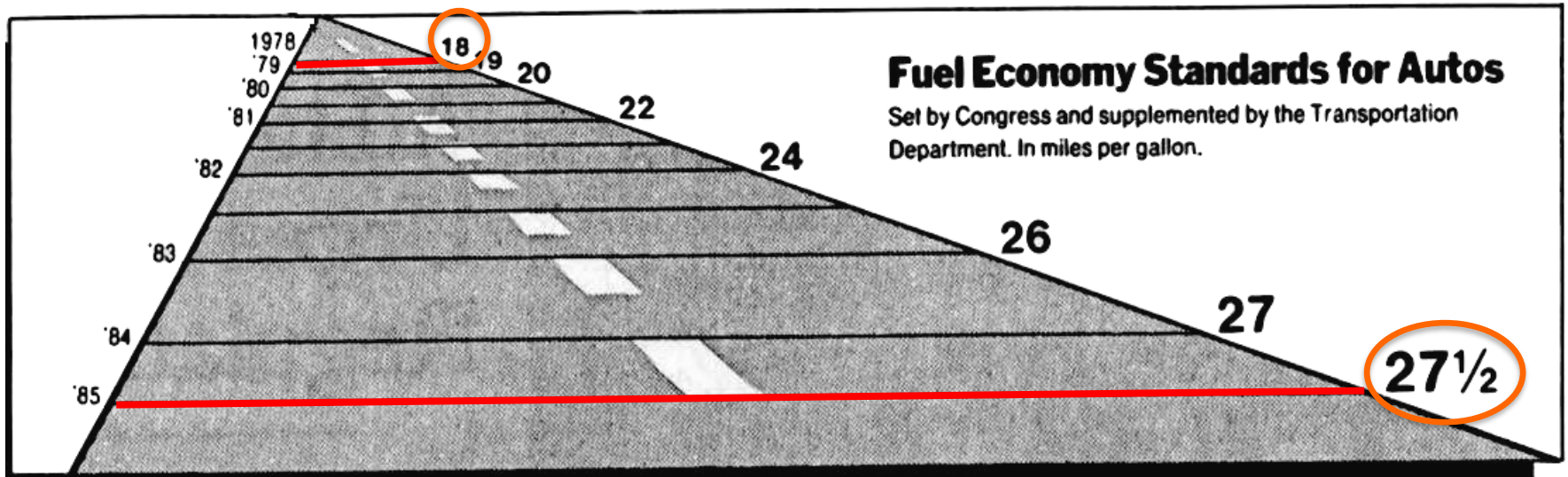


# Example – Redesign

## Debito Pubblico (% PIL)



# Lie Factor



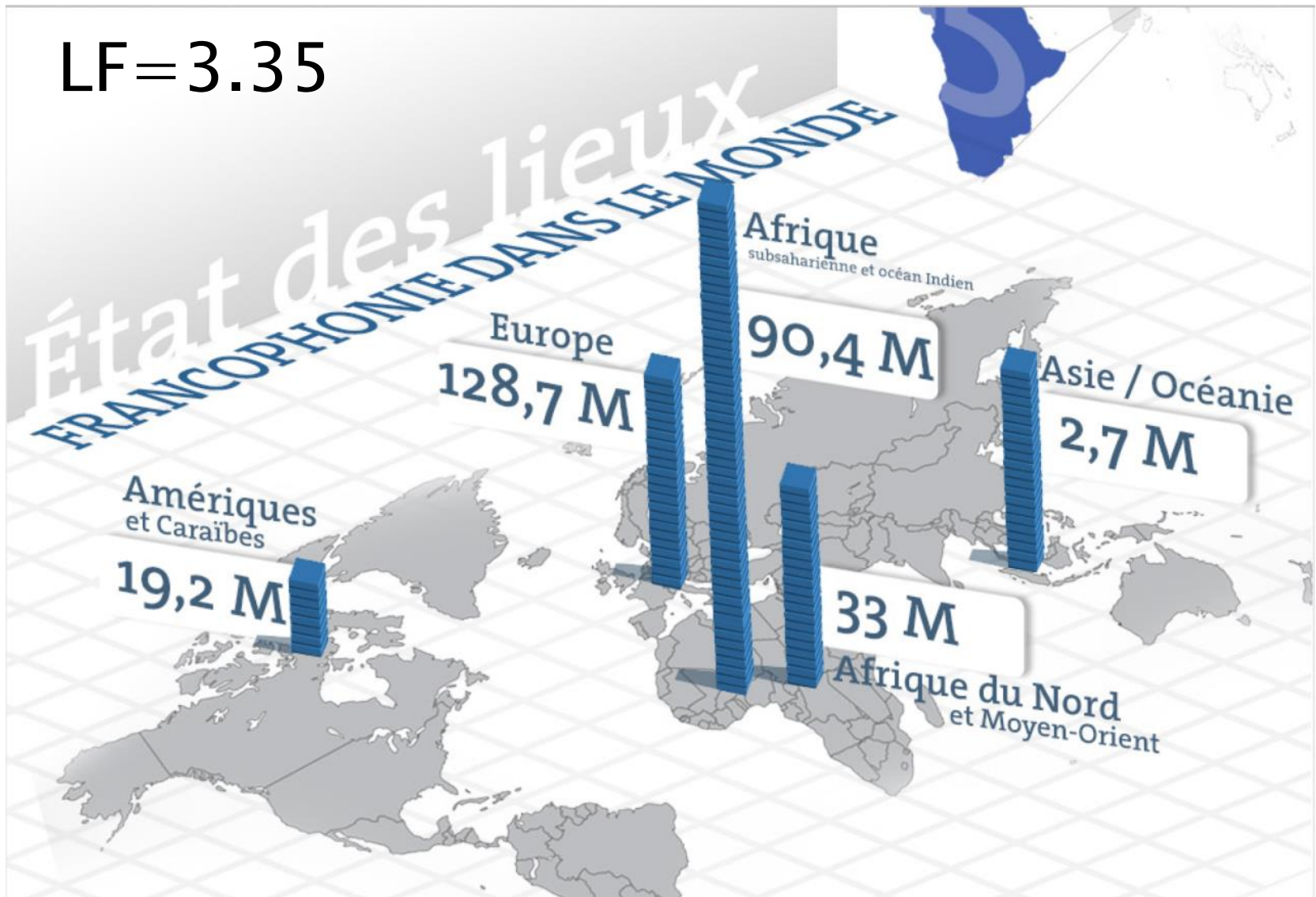
$$\frac{18.7}{2.2} = 8.5 \text{ on graphic}$$

$$\frac{27.5}{18} = 1.52 \text{ in data}$$

$$LF = 8.5 / 1.52 = 5.59$$

# Lie Factor

LF=3.35



# Principle of design

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- Keep the physical Lie Factor = 0
- Limit the perceptual Lie Factor as much as possible
  - ◆ Per Steven's law, avoid area comparisons



- Every element should convey useful information
- Unnecessary visual objects or attributes distract from the message
  - ◆ Different attributes trigger a search for a rationale (e.g. random colors)

$$\text{Data-ink ratio} = \frac{\text{data ink}}{\text{total ink used to print the graphic}}$$

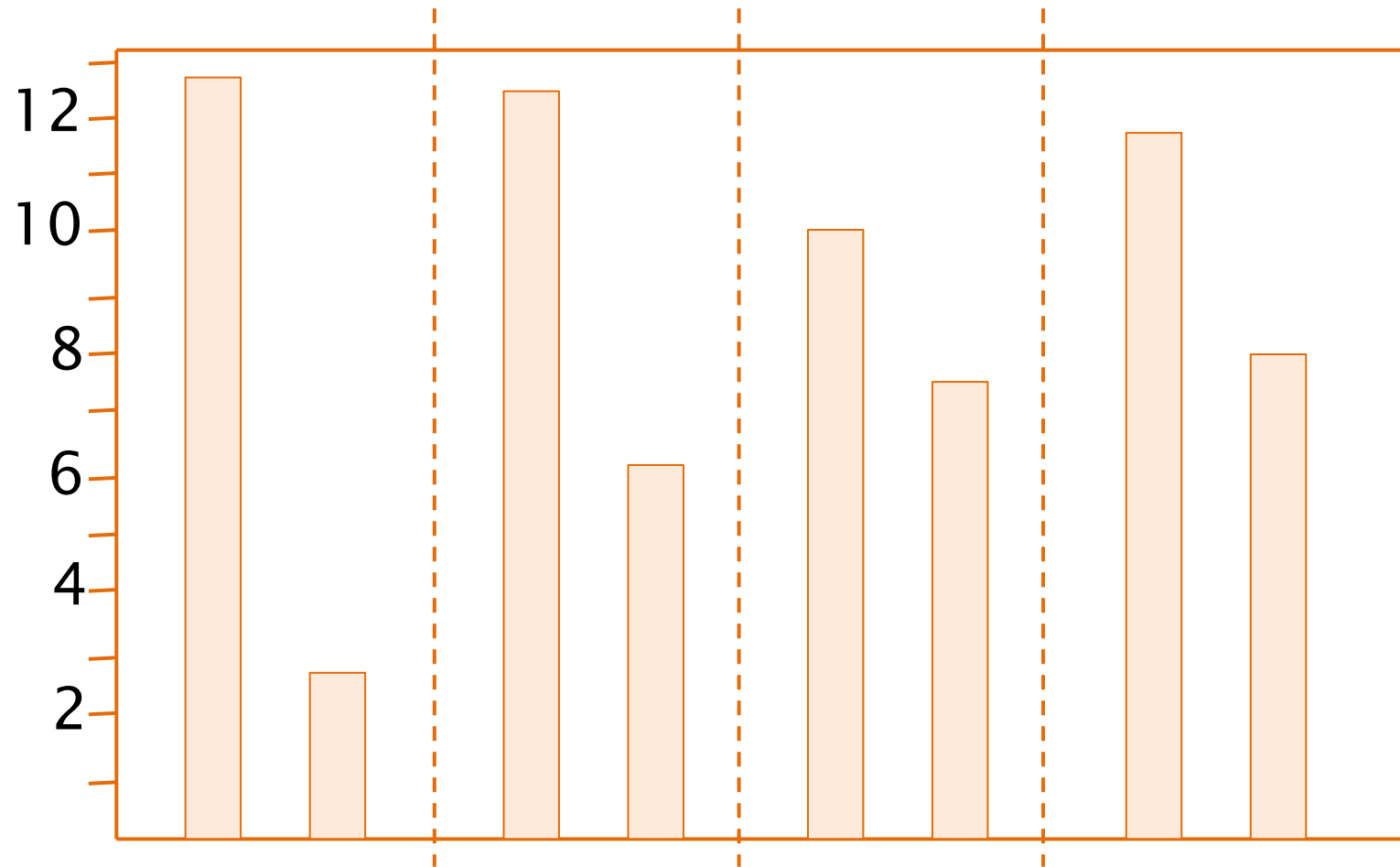
- Proportion of a graphic's ink devoted to the non-redundant display of data information

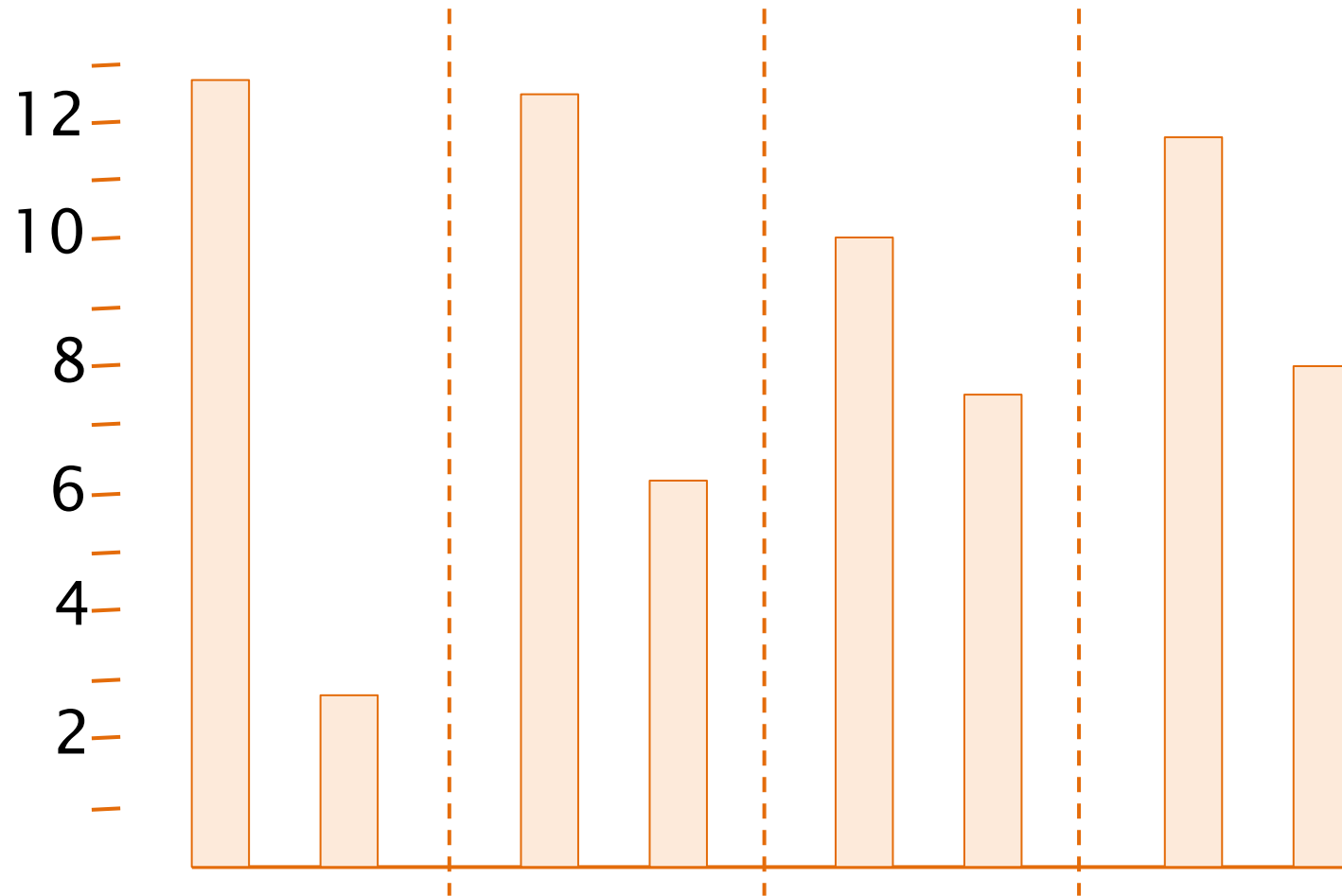
◆ Or:

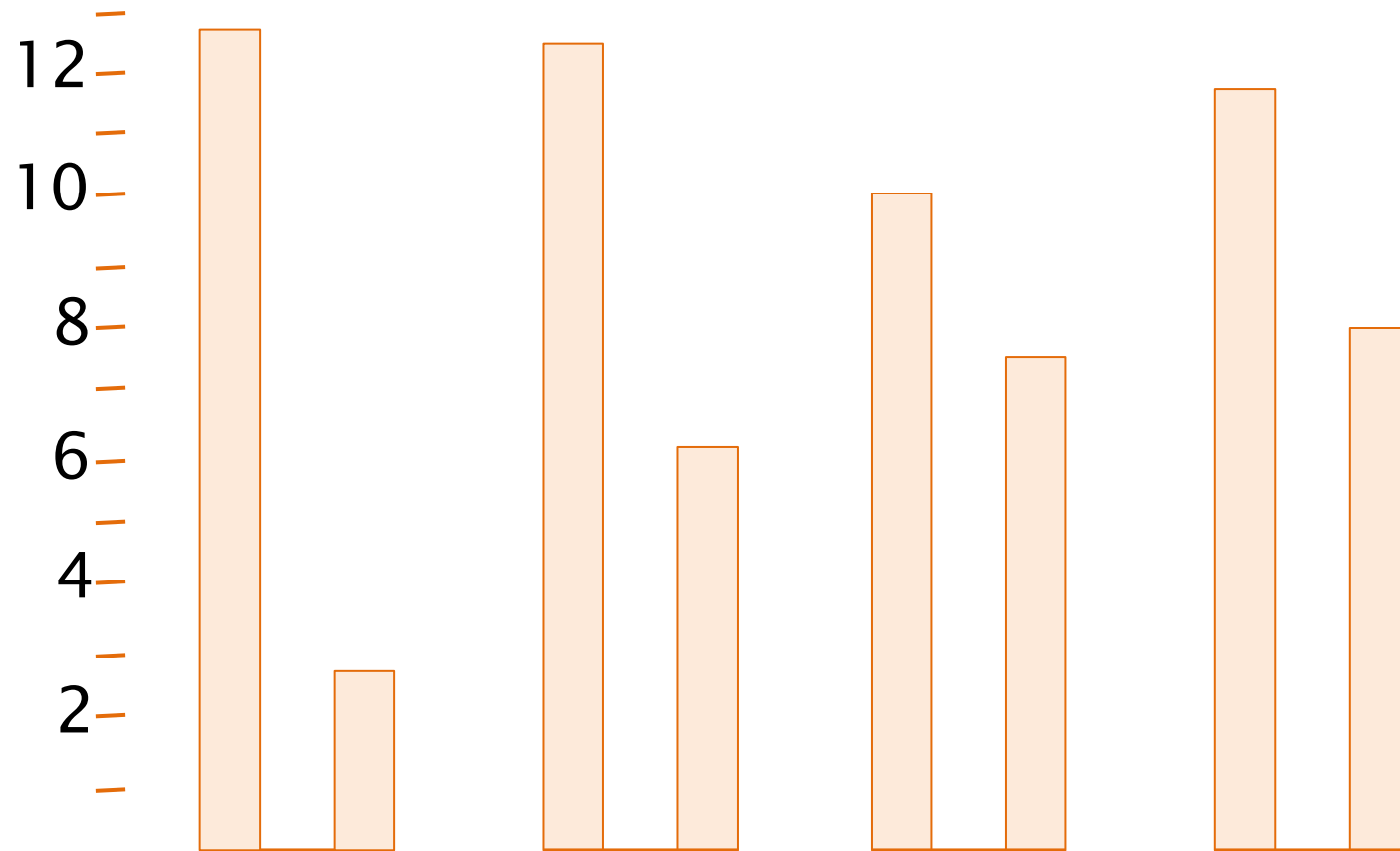
$$1 - \frac{\text{ink that can be erased without loss of information}}{\text{total ink used to print the graphic}}$$

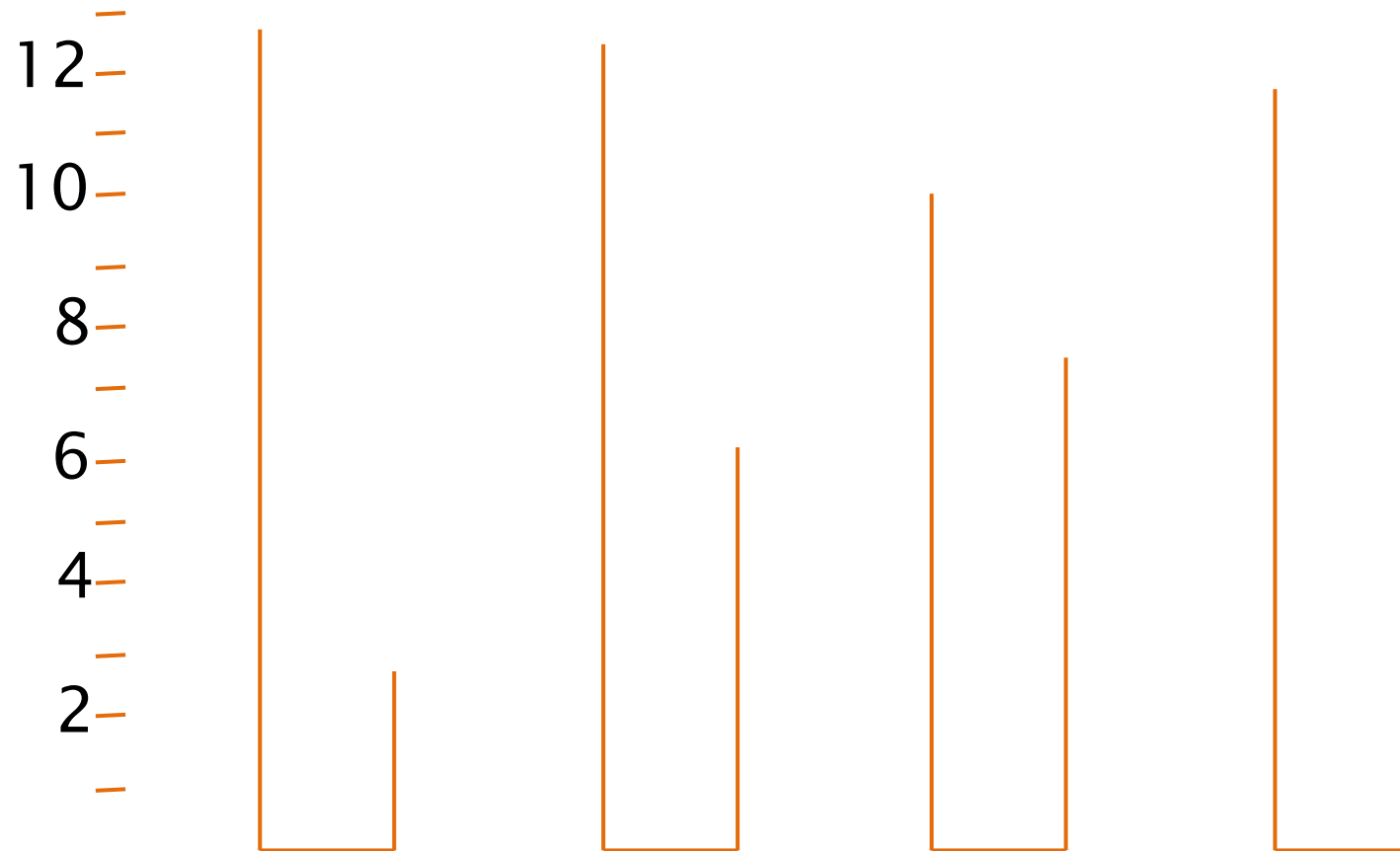
- Maximize data–ink ratio
  - ◆ Erase non–data–ink
  - ◆ Erase redundant data–ink
  
- “Within reason”

*Above all else show the data*  
*E. Tufte*









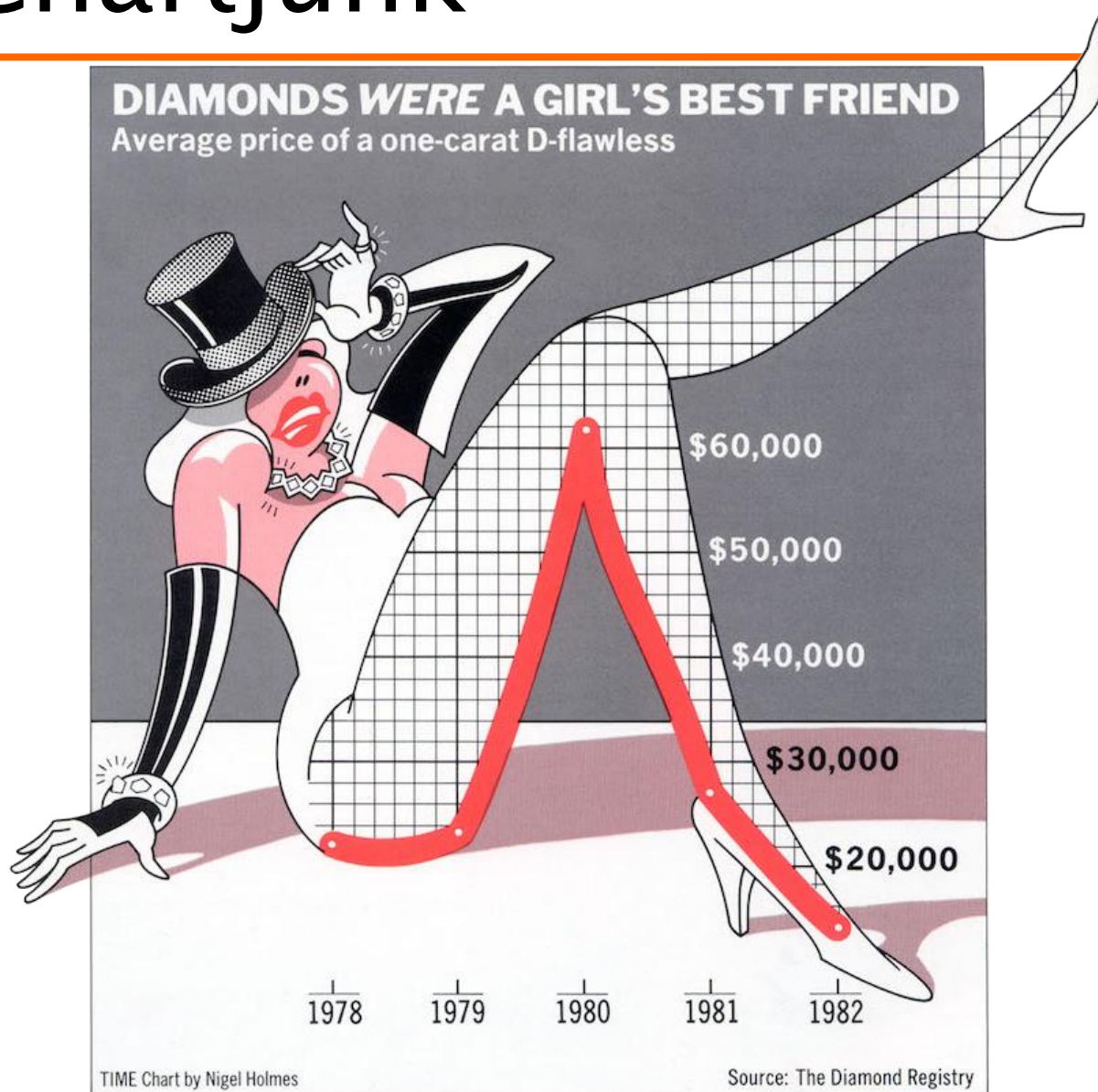
Tufte's proposed redesign

- Include differences corresponding to actual differences
- Effective when one item is different in a context of other items that are the same
  - ◆ Bright saturated color among mid colors



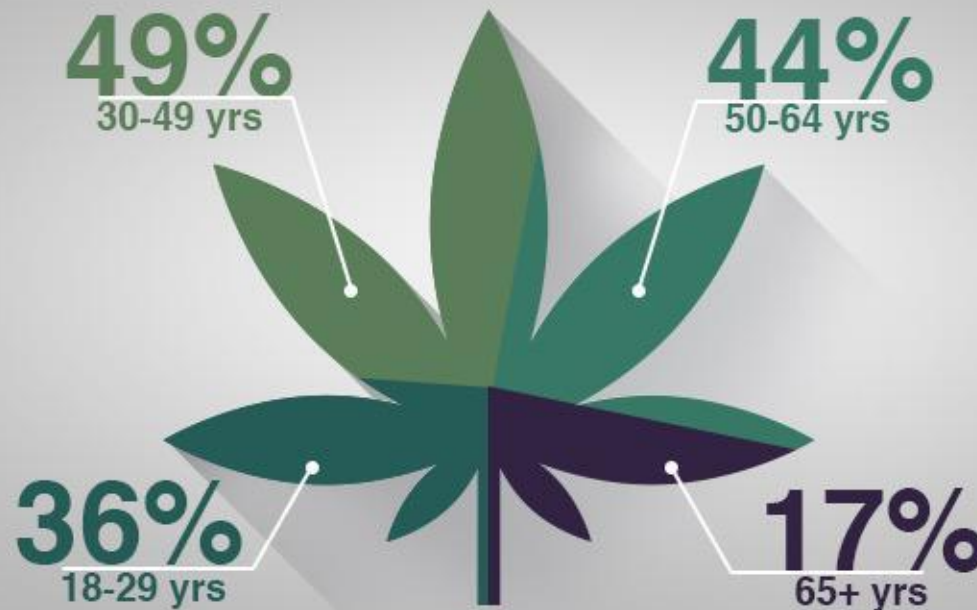
- The presence of unnecessary elements that distract or hide the message conveyed by the diagram

# Chartjunk



Nigel Holmes:  
<http://nigelholmes.com>

## AMERICANS WHO HAVE TRIED WEED



Source: Gallup



SUNDAYS  
10<sup>P</sup>  
ET/PT

#highprofits

- Visual encoding and layout should make perception tasks easy and effortless
- Textual and support elements should provide effective support to understanding the information
- Any variation in the graph should represent useful information otherwise it is noise obfuscating the message

- **Textual** elements should provide effective support to understanding
  - ◆ Hierarchical
    - Size and position reflects importance
  - ◆ Readable
    - Large enough
  - ◆ Horizontal
  - ◆ Close to data (avoid legends)
- Always label the axes

- Get it right in black and white
- Use medium hues or pastels
  - ◆ Bright colors distract and tire out
- Use color only when needed to serve a particular communication goal

Efficiency and efficacy of perception tasks is affected by:

- **Detection**

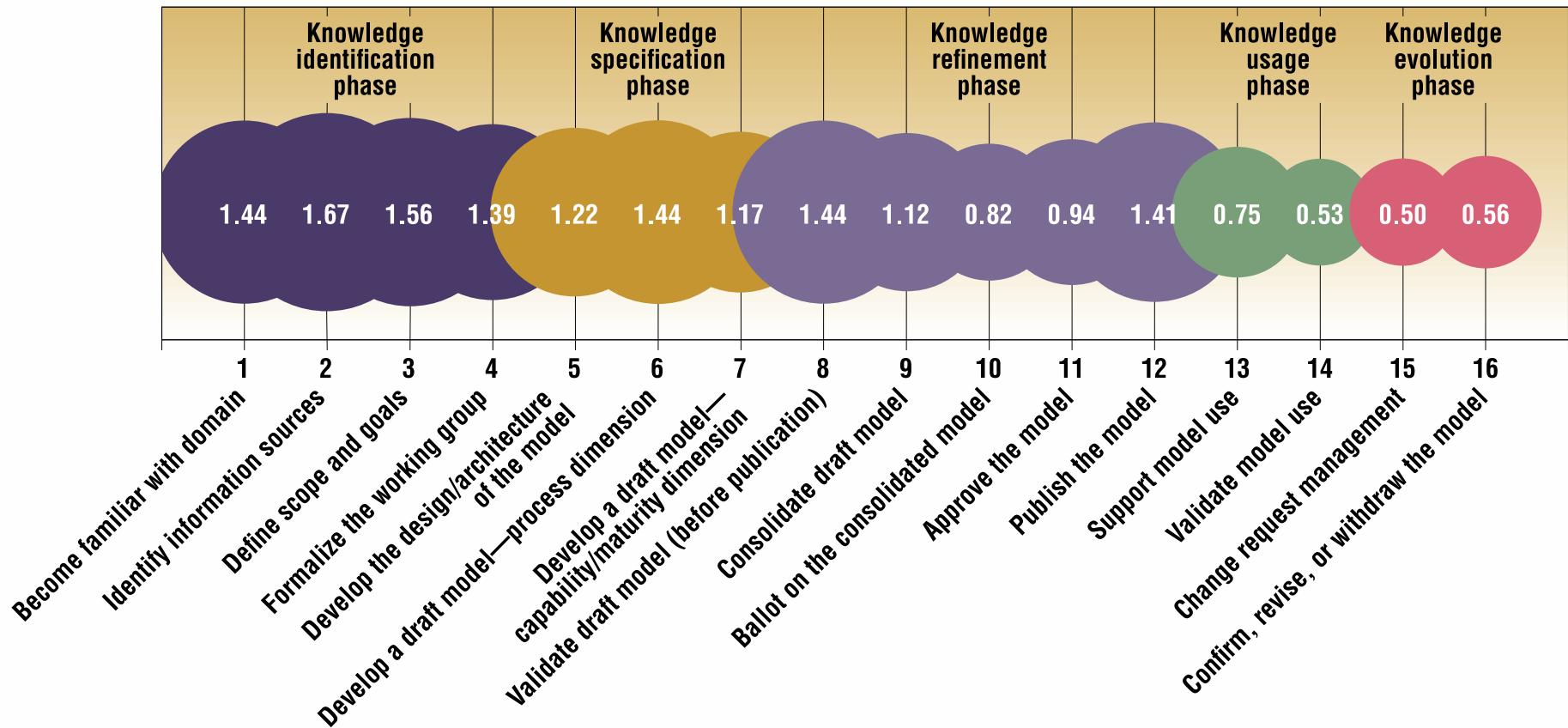
The capability to visually identify the objects that represent the data to be compared

- **Separation**

The distance between the objects to be compared

- affects negatively the accuracy

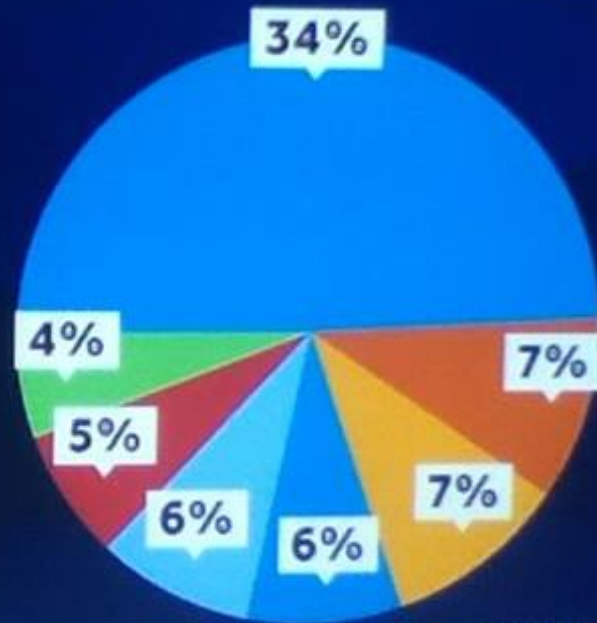
# Clarity





# Case study

## WHICH NFL TEAM IS YOUR FAVORITE?



SOURCE: PUBLIC  
POLICY POLLING

WXII  
12

# Assessment

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- Question:
  - ◆ Is there one (or more) question addressed by the visualization?
- Data:
  - ◆ Is the data quality appropriate?
- Visual Integrity:
  - ◆ Are the visual features appropriate?

# Visual Integrity

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- **Proportionality:**
  - ◆ Are the values encoded in a uniformly proportional way?
- **Utility:**
  - ◆ All the elements in the graph convey useful information?
- **Clarity:**
  - ◆ Are the data in the graph identifiable and understandable (properly described)?

# Question

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- What are the most popular/favorite NFL teams in our audience?
- ...

# Data

WXII-TV is an NBC-affiliated television station serving North Carolina: home of Panthers

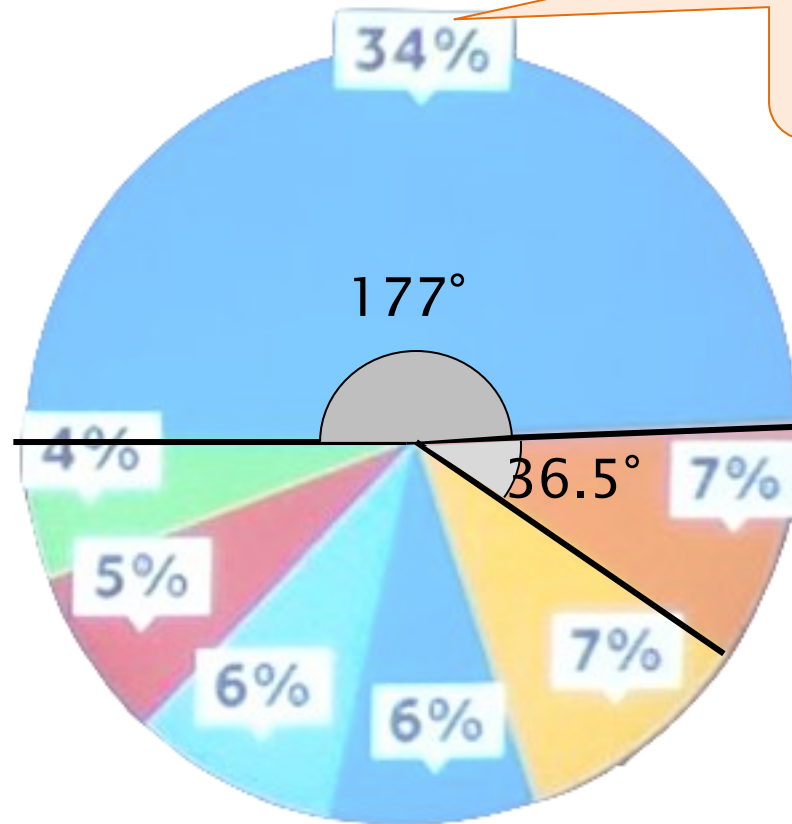
Team	Preferences
Panthers	34%
Cowboys	7%
Packers	7%
Patriots	6%
Steelers	6%
Redskins	5%
Broncos	4%
Total:	69%

# Full data

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Team	Preferences
Panthers	34%
Cowboys	7%
Packers	7%
Patriots	6%
Steelers	6%
Redskins	5%
Broncos	4%
<i>Other</i>	31%
Total:	100%

# Integrity – Proportionality



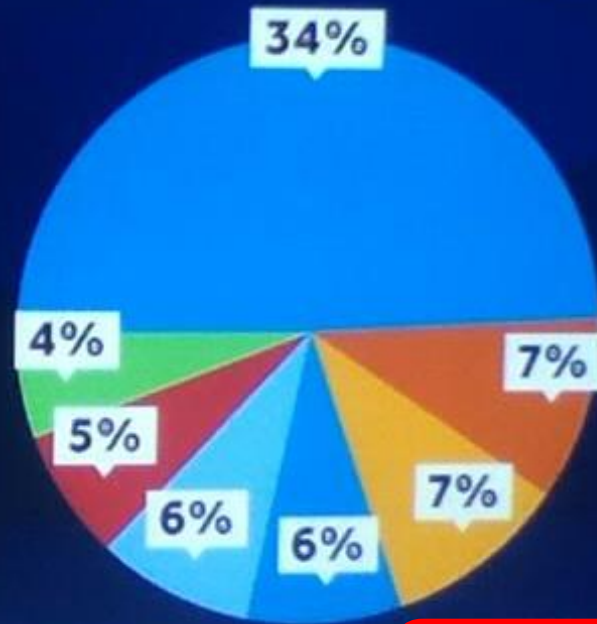
34% corresponds to 50% of the pie!

$$\begin{aligned} 177/36.5 &= 4.8 \\ 34/7 &= 4.8 \end{aligned}$$

# Utility

WHICH NFL TEAM IS YOUR FAVORITE?

- PANTHERS
- COWBOYS
- PACKERS
- PATRIOTS
- STEELERS
- REDSKINS
- BRONCOS



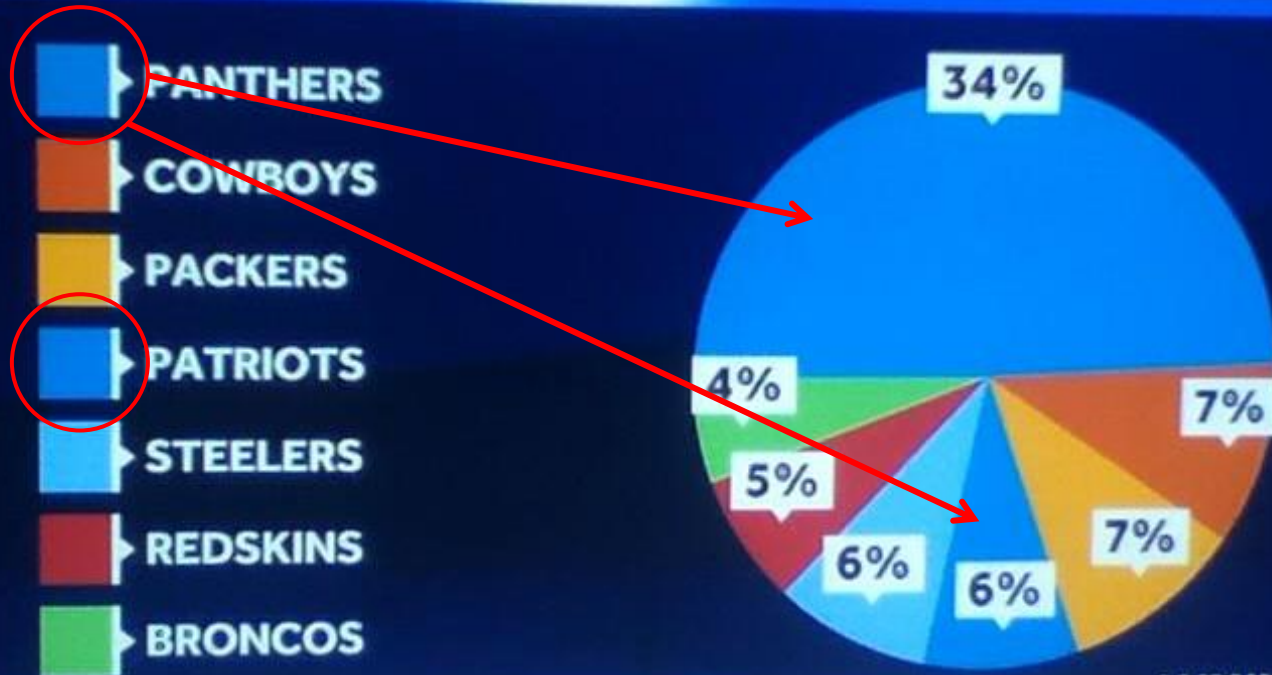
SOURCE: PUBLIC  
POLICY POLLING

WXII  
12



# Clarity

## WHICH NFL TEAM IS YOUR FAVORITE?



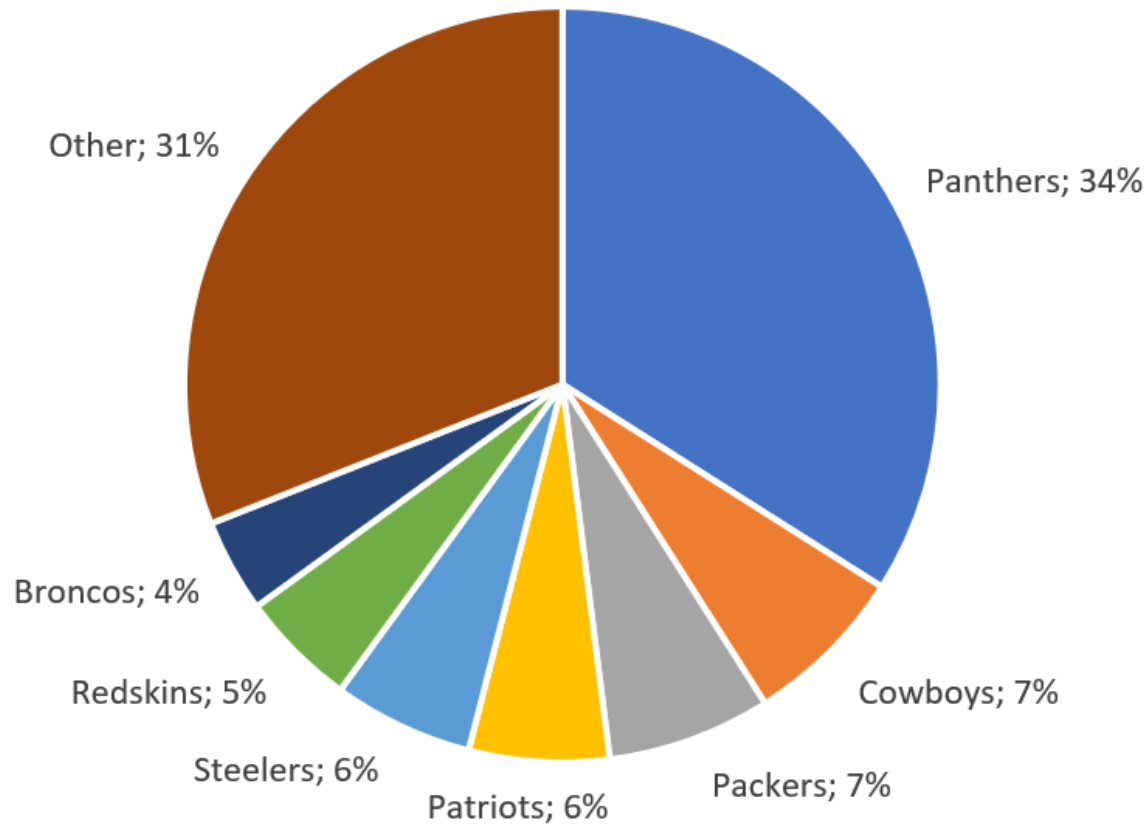
SOURCE: PUBLIC  
POLICY POLLING

WXII  
12

# Redesign #1

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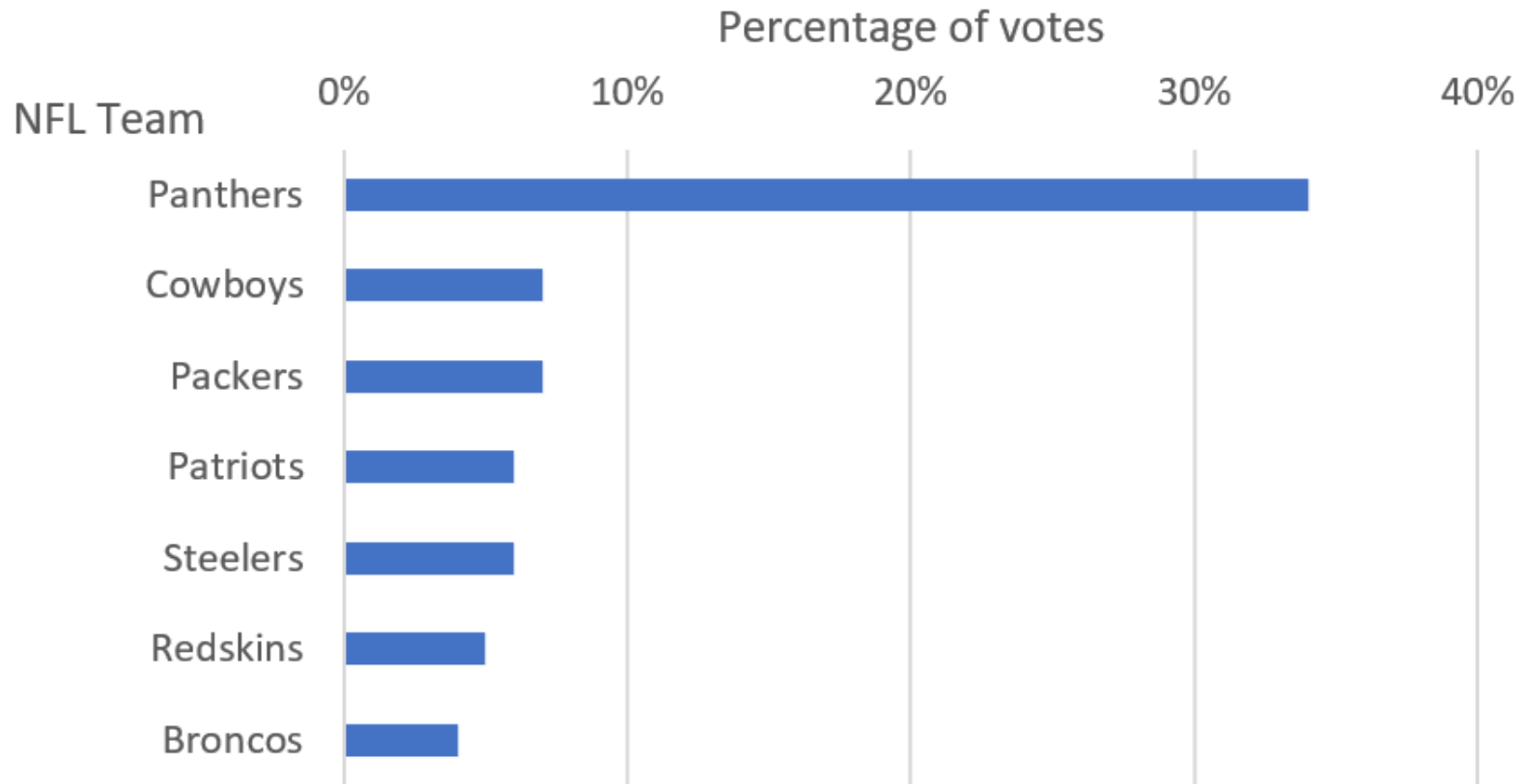
Preferences



# Redesign #2

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Favorite NFL teams in our audience

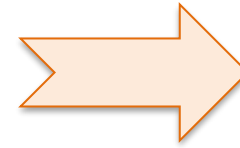


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# VISUALIZATION PIPELINE

# Visualization Pipeline

Knowledge



Decisions

**Information Understanding**

Visual Patterns, Trends, Exceptions

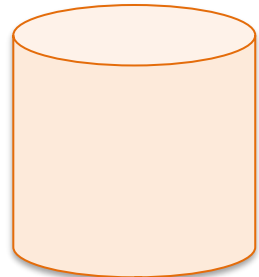
**Quantitative Reasoning**

Quantitative Relationship & Comparison

**Visual Perception**

Visual Properties & Objects

Data

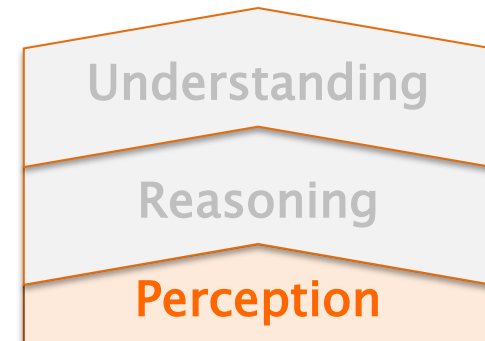


Representation/Encoding

# Visual Perception

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- Any variable (measure) must be **visually encoded**, i.e. we need to identify:
  - ◆ Visual object to represent entity
  - ◆ Visual attribute to represent the measure



# Example

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Votes received by four candidates in recent elections

Candidate	Votes	Proportion
Sergio	1 97800	50.09%
Alberto	1 40545	35.59%
Giorgio	53748	13.61%
Valter	2759	0.70%

<http://www.comune.torino.it/elezioni/2019/regionali/presidente/citta/>

# Encoding

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- Visual object: line
- Visual attribute: length

Alberto

- Giorgio  
Valter

Sergio

---

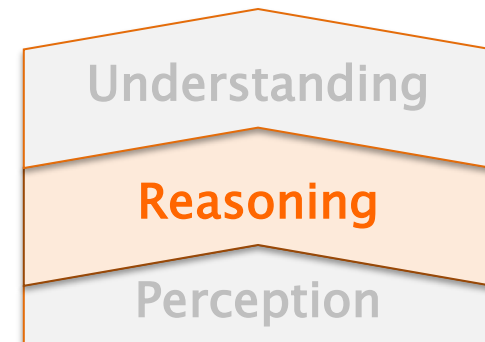


# Visual Reasoning

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Layout and visual attributes allow:

- **Discrimination**
  - ◆ Distinguish visual objects or group of –
- **Comparison**
  - ◆ Place visual objects in order
- **Magnitude assessment**
  - ◆ Evaluate the (relative) magnitude of visual objects



# Reasoning

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Alberto

- Giorgio  
Valter

Sergio

---

# Reasoning

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- Discrimination

Alberto 

---

Valter -

Giorgio 

---

Sergio 

---

# Reasoning

---

- Comparison



# Reasoning

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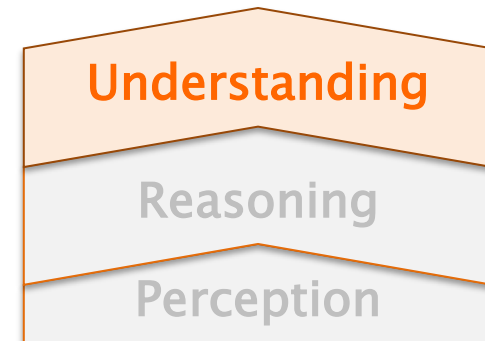
- Assessment



# Understanding

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- Variation within quantitative measures
  - ◆ Distribution
  - ◆ Deviation
  - ◆ Correlation
- Variation within category
  - ◆ Ranking
  - ◆ Part-to-whole
  - ◆ Time
  - ◆ Space
- Multivariate



# Understanding

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# Understanding

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- Ranking





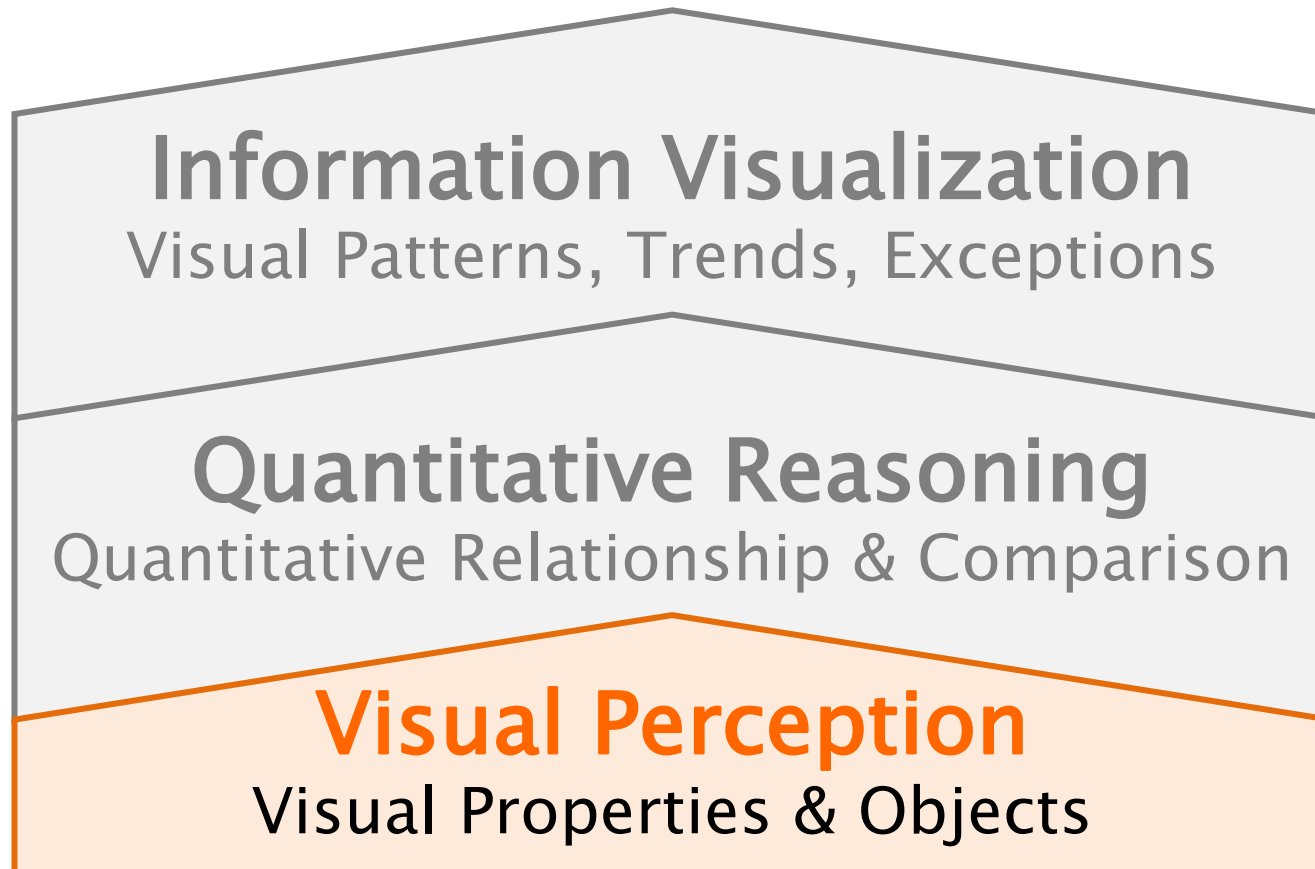
---

# VISUAL PERCEPTION

# Data Visualization

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## Understanding

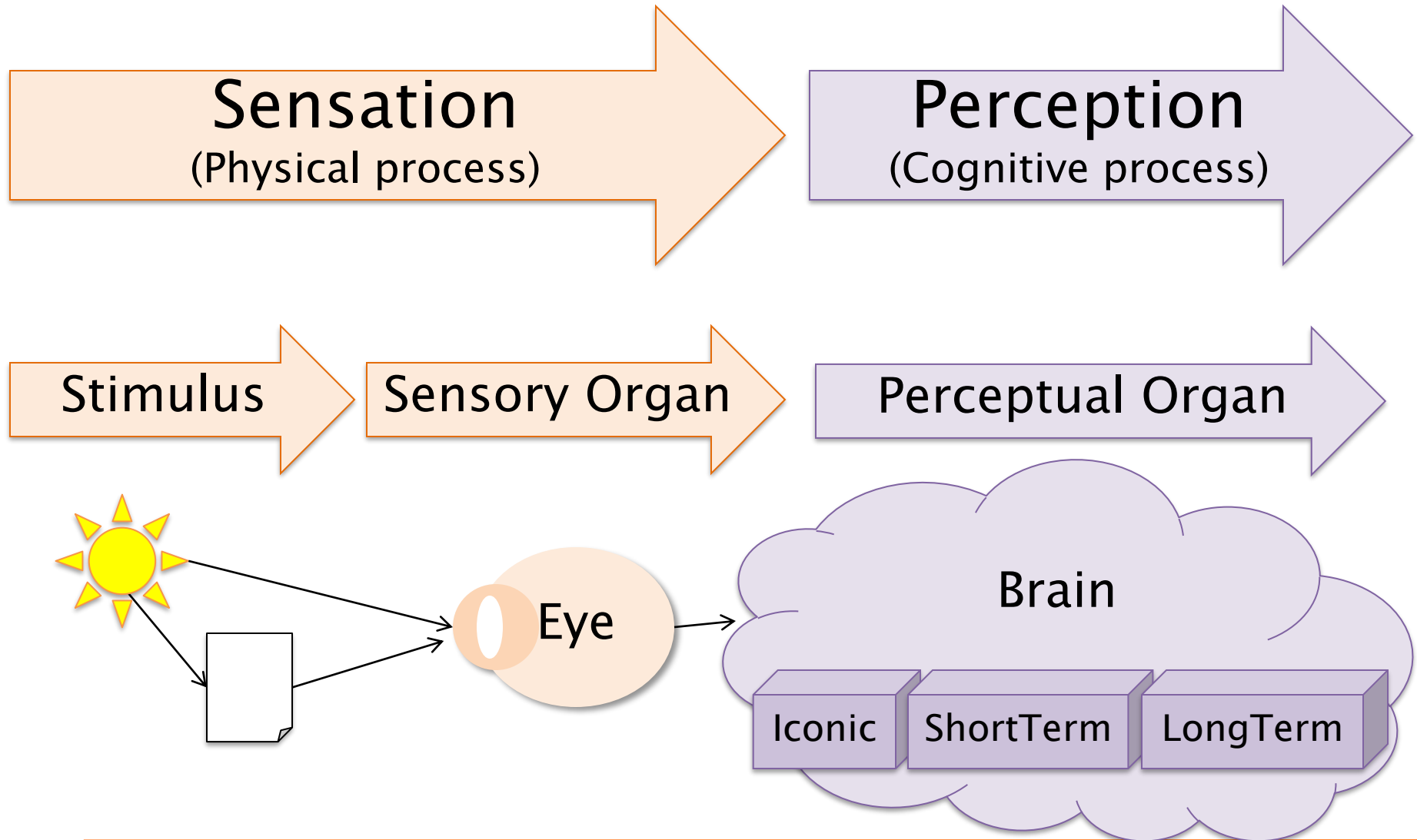


Data



Representation/Encoding

# Visual perception



# Memory Hierarchy

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- Iconic memory (visual sensory register)
  - ◆ Pre-attentive processing
  - ◆ Detects a **limited number of attributes**
- Short-term memory (working memory)
  - ◆ Store visual chunks
  - ◆ Limited number
- Long-term memory
  - ◆ Store high-level knowledge

# Simplified Model

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- The three levels of memory represent a simplified model
  - ◆ does not correspond to “real” physical brain structure
- Useful to explain a few phenomena
  - ◆ The  $7 \pm 2$  rule
  - ◆ Change blindness

# Change blindness





# Change blindness



# Pre-Attentive Attributes

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5	7	8	4	9	8	3	1	1	0	6	8	8	2	1	1	5	2	6	6	5
9	5	1	8	4	6	8	4	9	3	0	4	5	3	4	9	2	5	8	5	8
5	0	5	4	6	2	6	5	7	3	7	8	6	5	3	7	2	6	3	1	5
5	8	6	6	8	3	7	6	5	0	9	6	3	4	6	1	9	5	6	6	4
1	6	7	3	9	9	2	8	3	4	0	3	5	1	6	3	5	3	9	3	4
8	6	9	7	5	4	2	4	7	4	9	5	8	5	3	0	7	6	0	6	7
0	3	1	5	3	2	3	5	6	7	2	8	9	8	5	3	7	8	8	2	4
5	5	3	4	8	1	5	6	2	3	5	5	1	2	1	0	8	7	2	6	3
7	4	3	8	4	8	2	6	7	9	5	6	2	3	6	7	8	0	8	3	6
4	9	5	6	7	2	2	2	8	3	1	1	0	1	8	6	2	6	2	1	4



# Pre-Attentive Attributes

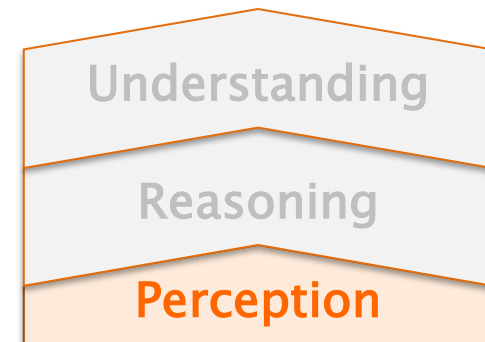
---

5	7	8	4	9	8	3	1	1	0	6	8	8	2	1	1	5	2	6	6	5
9	5	1	8	4	6	8	4	9	3	0	4	5	3	4	9	2	5	8	5	8
5	0	5	4	6	2	6	5	7	3	7	8	6	5	3	7	2	6	3	1	5
5	8	6	6	8	3	7	6	5	0	9	6	3	4	6	1	9	5	6	6	4
1	6	7	3	9	9	2	8	3	4	0	3	5	1	6	3	5	3	9	3	4
8	6	9	7	5	4	2	4	7	4	9	5	8	5	3	0	7	6	0	6	7
0	3	1	5	3	2	3	5	6	7	2	8	9	8	5	3	7	8	8	2	4
5	5	3	4	8	1	5	6	2	3	5	5	1	2	1	0	8	7	2	6	3
7	4	3	8	4	8	2	6	7	9	5	6	2	3	6	7	8	0	8	3	6
4	9	5	6	7	2	2	2	8	3	1	1	0	1	8	6	2	6	2	1	4

# Encoding

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- Encoding is the key to enable visual perception
  - ◆ Visual object to represent entity
  - ◆ Visual attribute to represent the measure
- Two main types
  - ◆ Quantitative (different properties)
  - ◆ Categorical (ordinal or not)



# Pre-Attentive attributes

Category	Attribute
Form	Orientation Length/distance Line width Size Shape Curvature Added marks Enclosure
Color	Hue Intensity
Spatial position	2-D position
Motion	Flicker Direction Speed

# Perception task

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Visual attributes allow:

- Discrimination
  - ◆ Distinguish visual objects
- Comparison
  - ◆ Place visual objects in order
- Magnitude assessment
  - ◆ Evaluate the (relative) magnitude of visual objects

# Just noticeable difference

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- Given a physical dimension (length, brightness, etc.)  $x$
- $d$  is the **just noticeable difference** if:
  - ◆ difference between  $x$  and  $x+d$  is perceivable
  - ◆ but not smaller differences
- $d$  depends on many factors:
  - ◆ Subject
  - ◆ Environment
  - ◆ Physical dimension

# Weber's law

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- Just noticeable difference  $d$  is:

$$d_p(x) = k_p \cdot x$$

- Where
  - ♦  $x$ : dimension
  - ♦  $d_p(x)$ : just noticeable difference
  - ♦  $k_p$ : constant
    - Subjective
    - Environmental

# Consequences of Weber's law

---

- It is easier to compare lengths that differ by a large percentage
- The same difference is easier to notice between smaller measures
  - ◆ More likely to be larger than just noticeable difference

$$x < y \implies d_p(x) < d_p(y)$$

- Length of non-aligned objects is harder to compare
  - ◆ Double comparison

# Non-aligned objects lengths

---

- Additional references may help comparison
  - ♦ They provide alternative possible comparisons
- If lengths range between 0 and a maximum ( $L$ ) e.g. percentages
- Comparing  $l_1$  and  $l_2$  (close to  $L$ ) that differ by a small amount  $d$ 
  - ♦ Difference  $L-l_1$  vs.  $L-l_2$  easier to notice than  $l_1$  vs.  $l_2$





# Stevens's law

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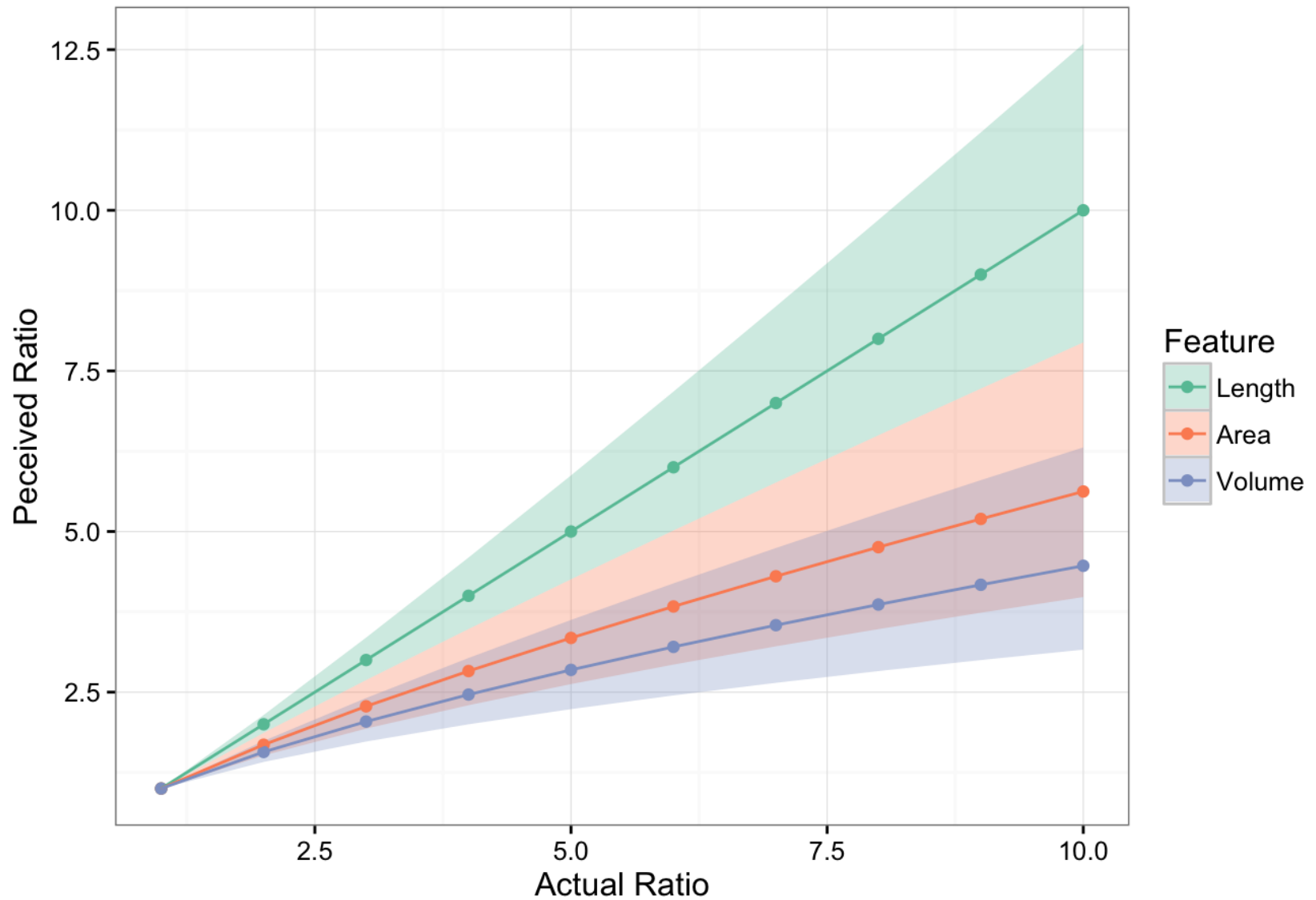
- Perceive scale (magnitude ratio)

$$p(x) = c \cdot x^{\beta}$$

- Where  $\beta$  depends on spatial dimension
  - ♦ 1D: Length  $\rightarrow \beta$  in  $[0.9, 1.1]$
  - ♦ 2D: Area  $\rightarrow \beta$  in  $[0.6, 0.9]$
  - ♦ 3D: Volume  $\rightarrow \beta$  in  $[0.5, 0.8]$

Stevens S. S. (1975). Psychophysics, John Wiley & Sons.

# Steven's law



# Steven's law

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# Consequences

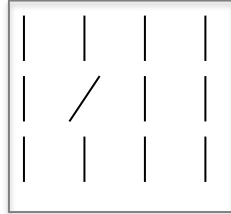
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- Prefer comparing lengths
- Avoid comparison between areas
  - ◆ Except for ordinal measures
- Never–ever make volume comparisons

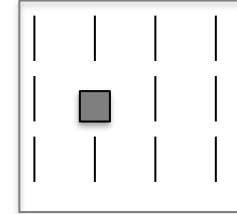
# Attributes of form

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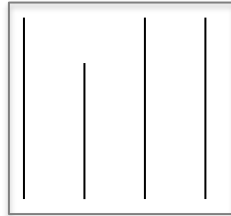
Orientation



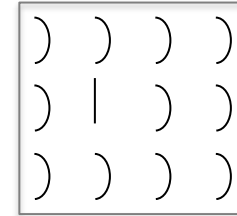
Shape



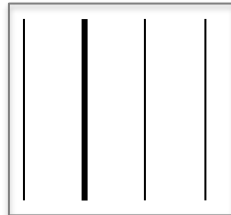
Line Length



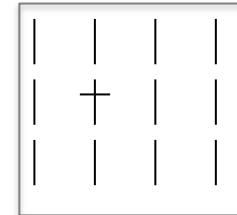
Curvature



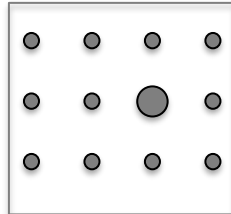
Line Width



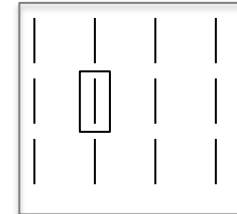
Added mark



Size



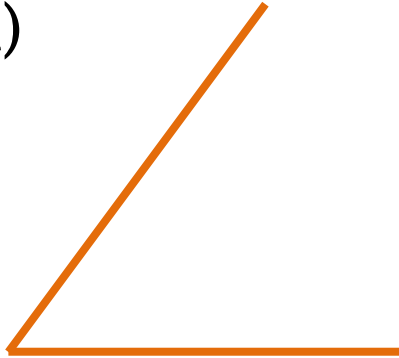
Enclosure



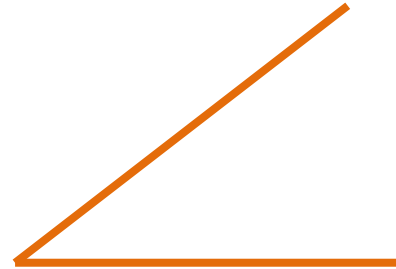
# Orientation (angle or slope)

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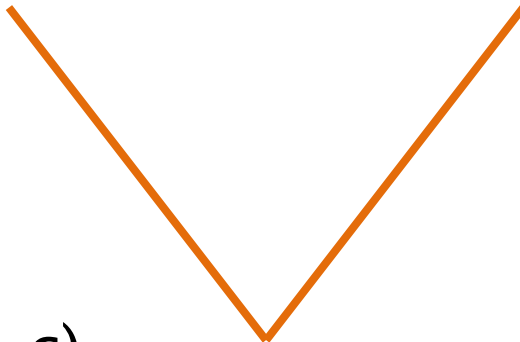
a)



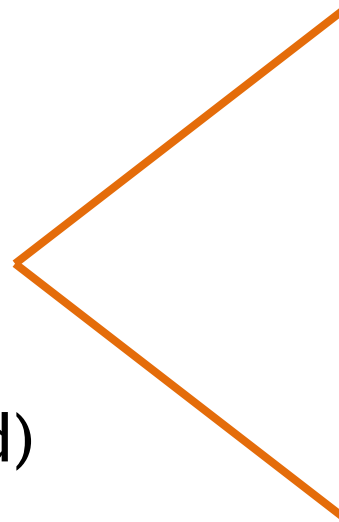
b)



c)

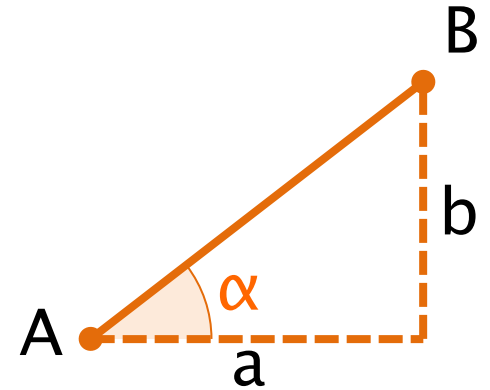


d)



# Angle vs. Slope

- Slope of A–B is  $b/a$ 
  - ♦  $\tan(\alpha)$
- Slope judgment typically falls back to an angle judgment
  - ♦ Given an error  $\epsilon$  in the angle judgment
  - ♦ It is reflected in a slope error



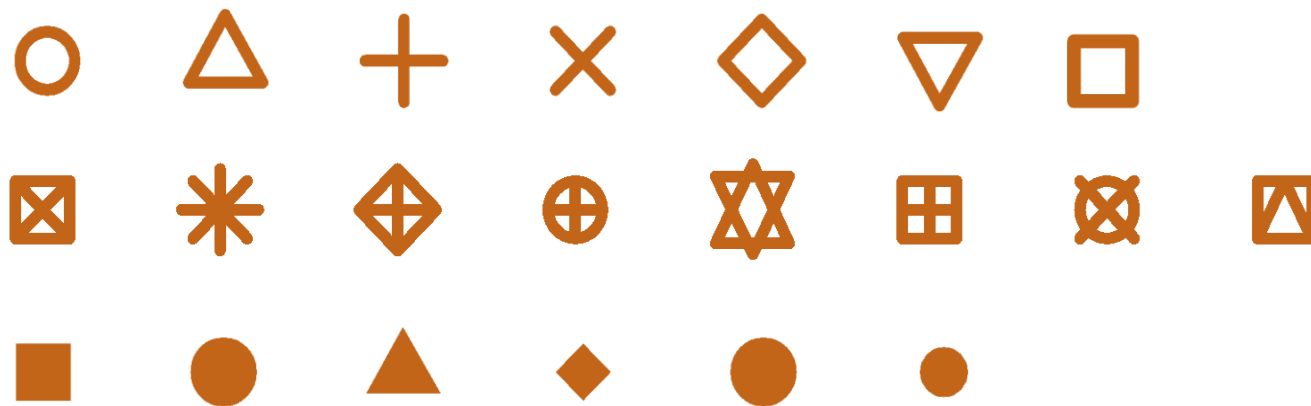
$$\tan(\alpha + \epsilon) - \tan(\alpha) = \epsilon \cdot \tan'(\alpha) = \frac{\epsilon}{\cos^2(\alpha)}$$

– Getting infinite as  $\alpha$  approaches to  $\pi/2$

# Shape

---

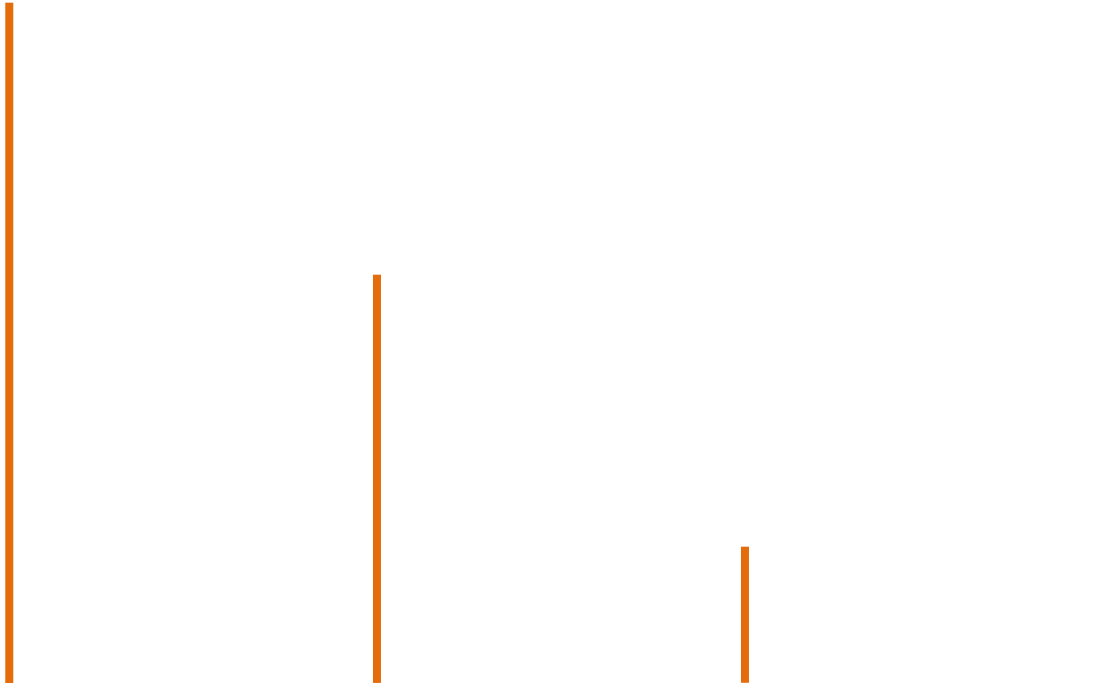
- There is no common quantitative semantics for the shapes
  - Unless they are characters...
- ◆ Fill textures are shapes too





# Length

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# Effect of context

---



# Curvature

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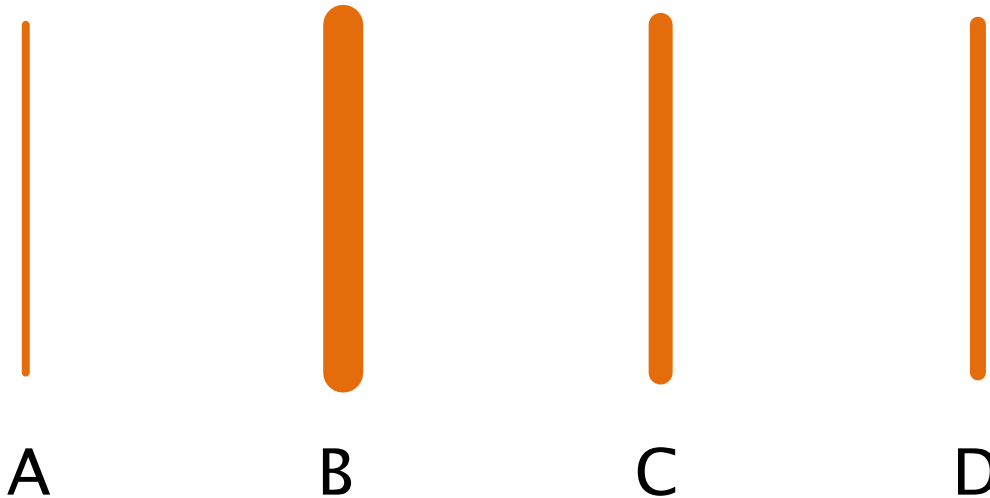
- There is no common magnitude assessment for the curvature



# Width

---

- Order can be identified
  - ♦ Difficult to appreciate actual magnitude



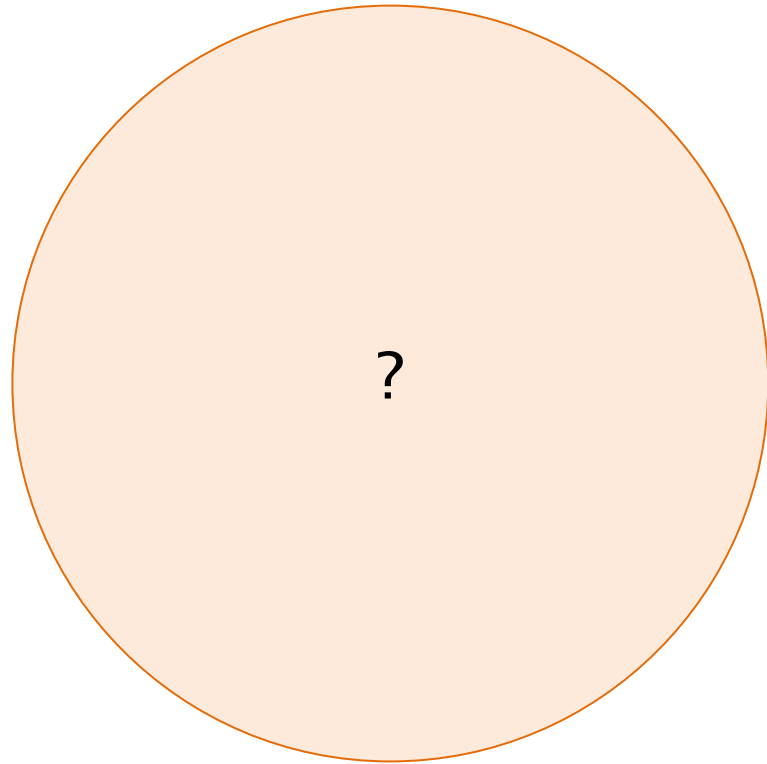
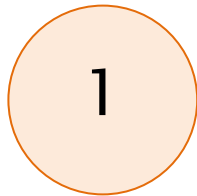
# Mark

---

- No common quantitative semantics of marks
- Number of marks could encode a natural number
  - ◆ Harder to read than a cipher

# Size / Area

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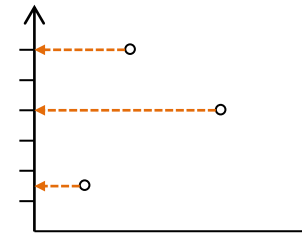
# Enclosure

---

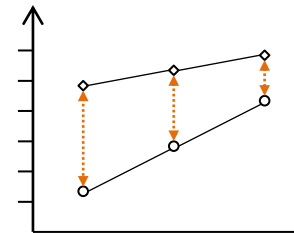
- No common quantitative semantics for enclosure
  - ◆ Except counting items enclosed

# Spatial Position

- Position along axis
  - ◆ Common scale
  - ◆ Distinct identical scales
    - Possibly un-aligned



- Distance

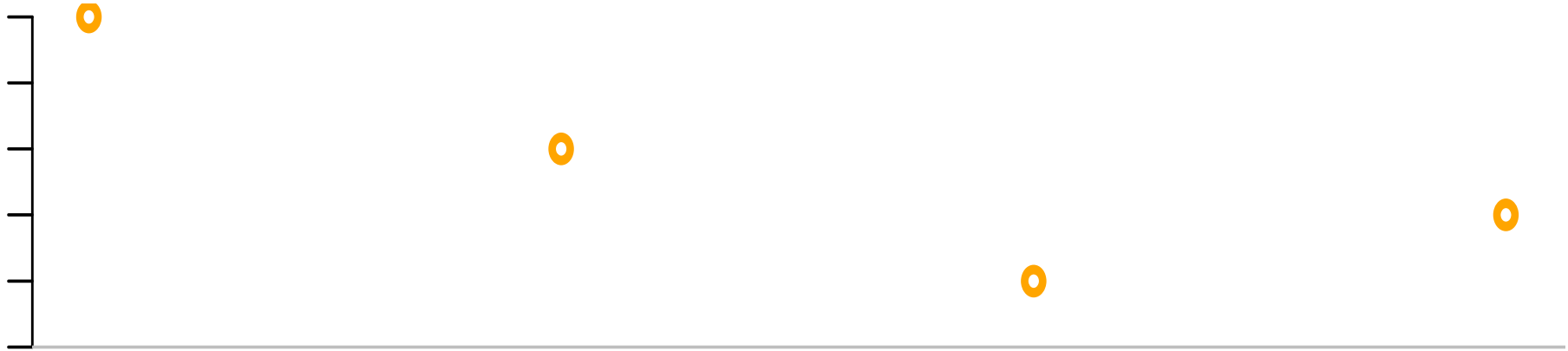




# Position

---

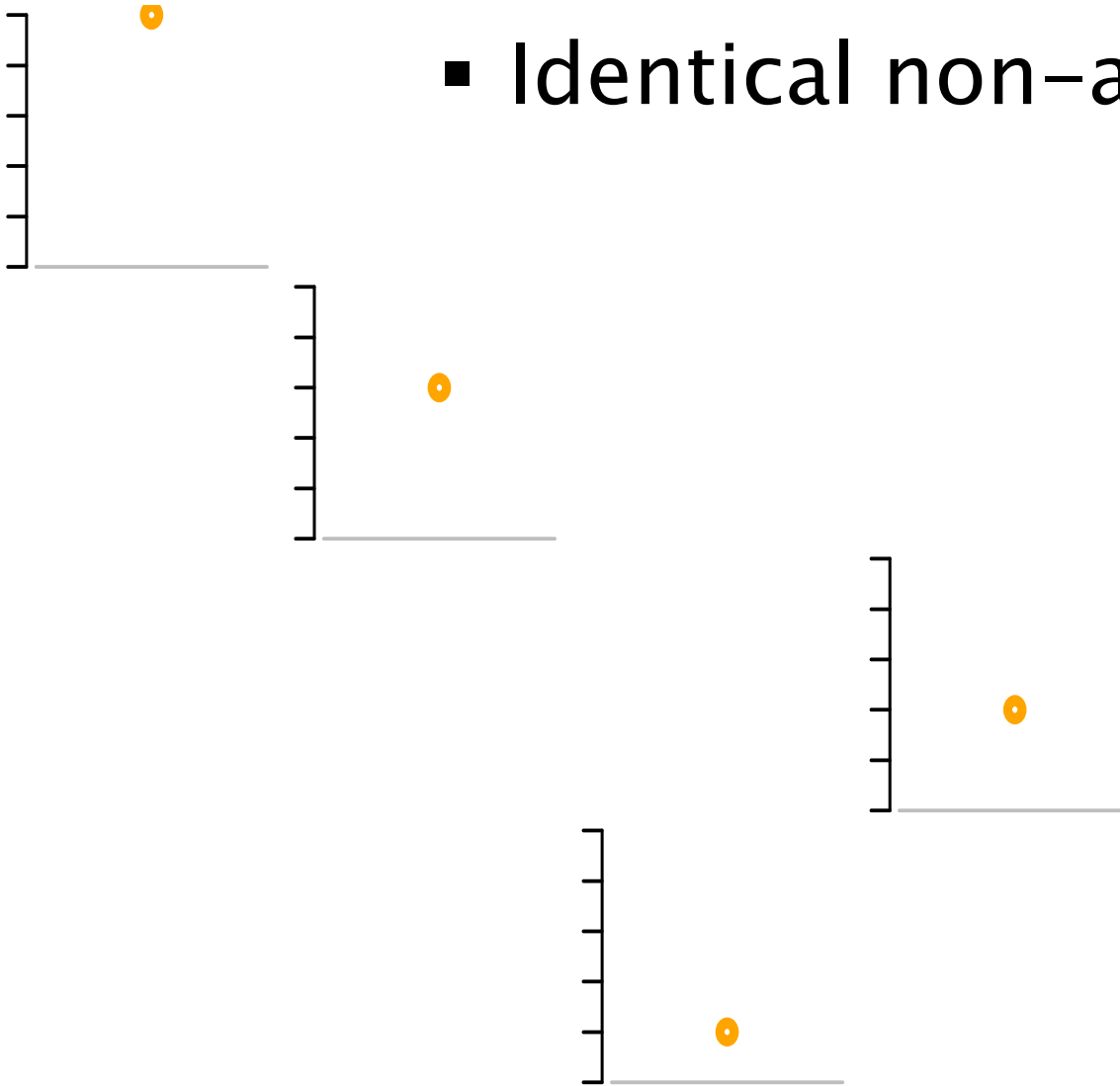
- A common scale



# Position

---

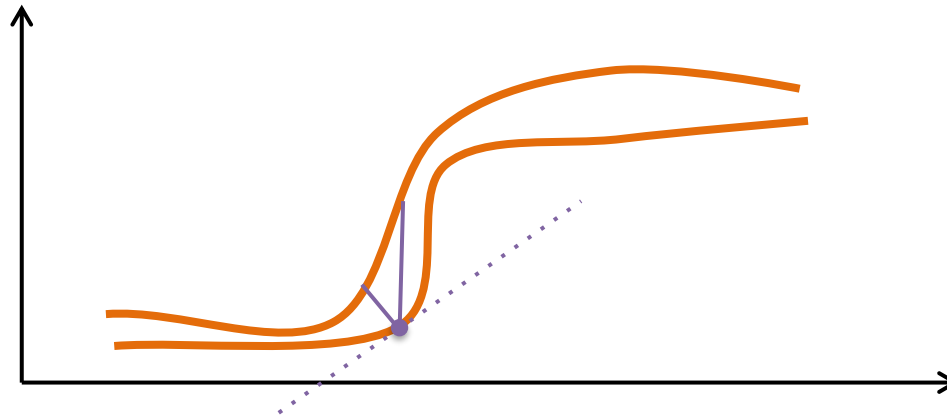
- Identical non-aligned scales



# Distance

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- Points
  - ◆ Use length of imaginary connecting lines
- Lines
  - ◆ Distance orthogonal to tangent
    - Not what is meant in xy plots



# Detection and Separation

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Comparison is affected by:

- Detection

- ◆ The capability to visually identify the objects that represent the data to be compared

- Separation

- ◆ The distance between the objects to be compared
  - affects negatively the accuracy

# Attributes of color

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- Hue



- Saturation



- Intensity



- ◆ Luminance

- ◆ Value

# Hue

---

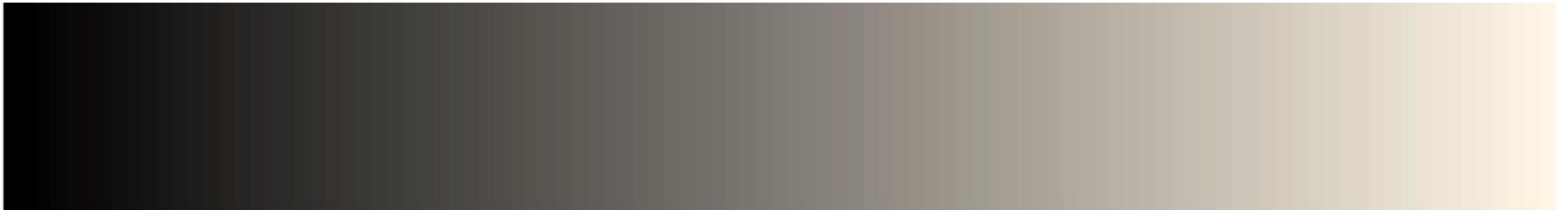
- There is no common ordering semantics for hues
  - ◆ High spatial frequencies are perceived through intensity changes
  - ◆ Often perceived as separated into bands of almost constant hue, with sharp transitions between hues
- Nominal values can be represented by suitably spaced values



# Intensity

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- ♦ a.k.a. Luminance, Value
- Provides a perceptually unambiguous ordering
  - ♦ Context can affect accuracy



# Saturation

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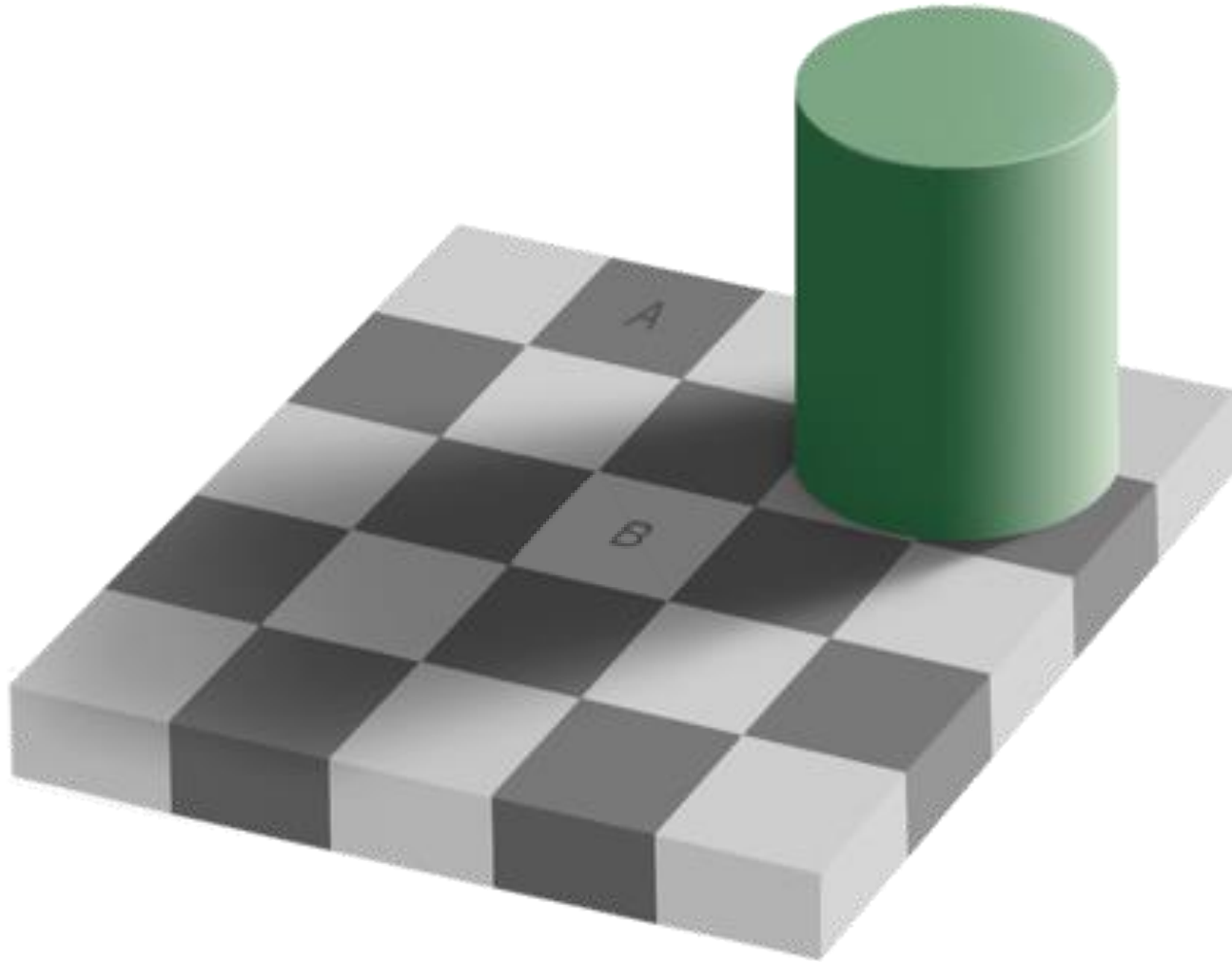
- Perceptually difficult to associate an ordered semantics
  - ◆ Can be combined with hue to increase discrimination





# Effect of Context

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# Effect of Context

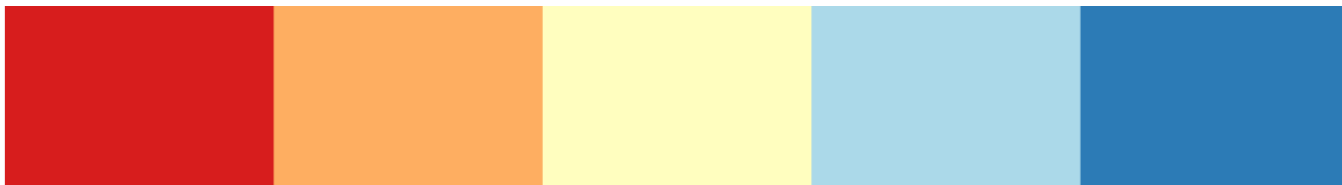
---

- Use uniform background
  - ◆ To make distinct visual objects for the same feature look the same
- Use a background color that is contrasting enough with the visual objects' color
  - ◆ To make visual objects easily seen
- Avoid non-uniform background

# Color usage

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- Ordinal measure should be mapped to increasing saturation **and** intensity
  - ♦ Avoid rainbow palette
- Use sequential or diverging palette
  - ♦ E.g.

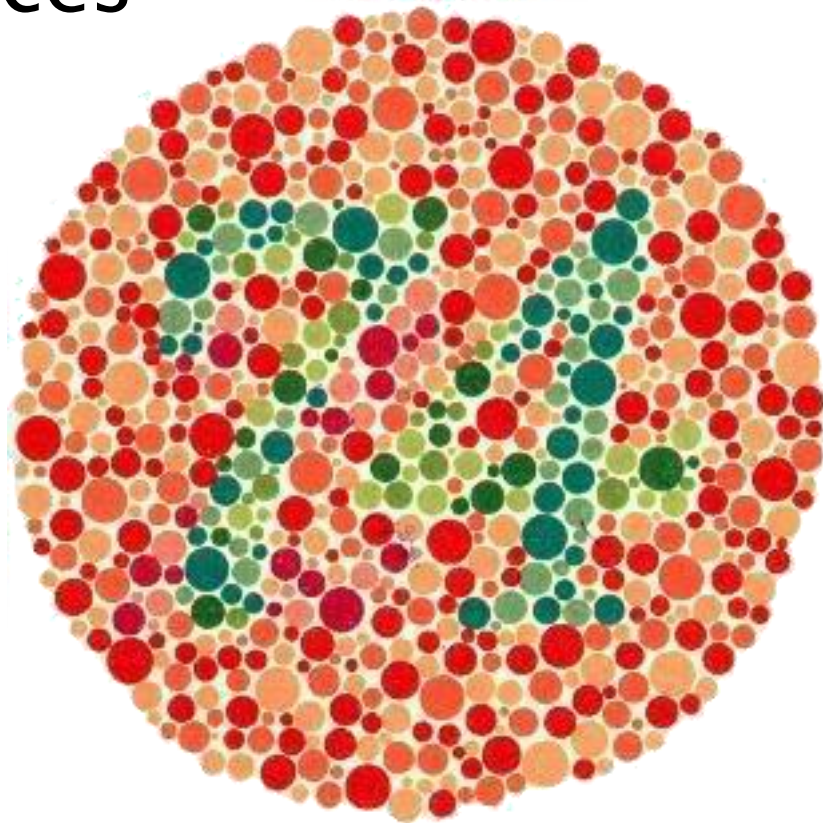


– <http://colorbrewer2.org/>

# Color Blindness

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- Inability to see colors or perceive color differences



<http://www.color-blindness.com>

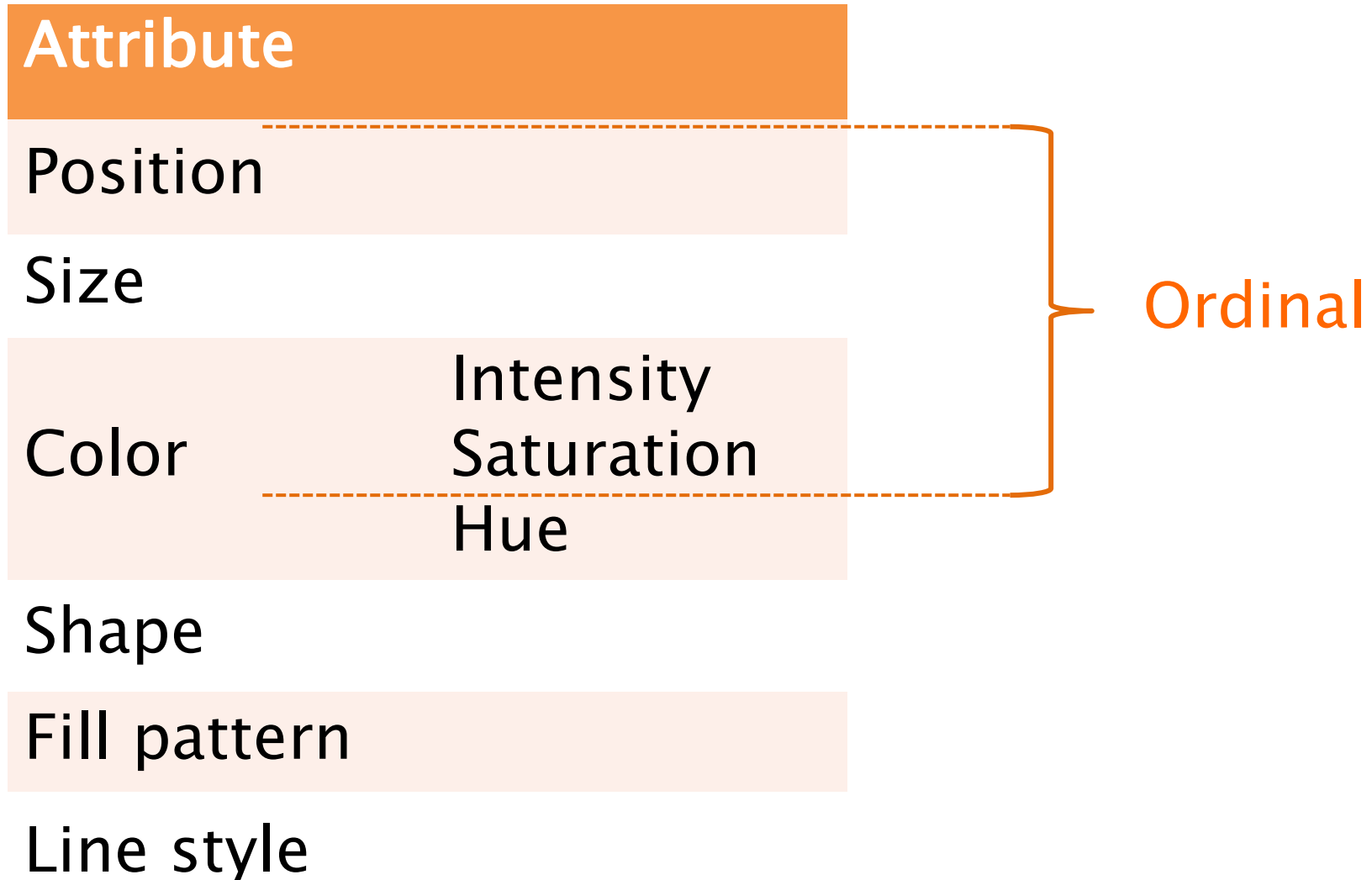
# Visual Encoding: Quantitative

---

Object	Attribute
Point	Position (w.r.t. axis/axes)
Line	Length Position (w.r.t. axis/axes) Slope
Bar	Length
Shape	Size (area) Count

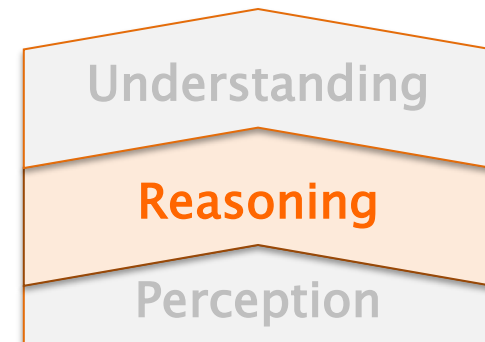
# Visual Encoding: Categorical

---



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# VISUAL REASONING

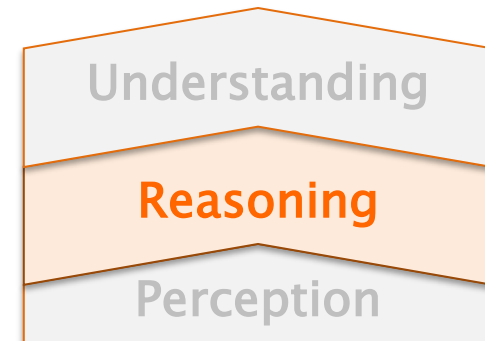


# Graph layout

---

Layout and visual attributes allow:

- **Discrimination**
  - ◆ Distinguish visual objects or group of –
- **Comparison**
  - ◆ Place visual objects in order
- **Magnitude assessment**
  - ◆ Evaluate the (relative) magnitude of visual objects





# Gestalt principles

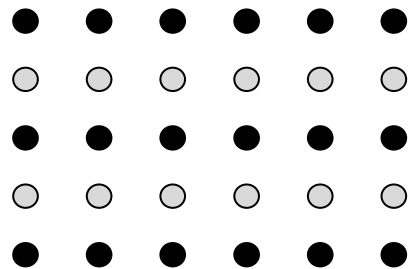
---

- Visual features that lead us to group visual objects together
  - ◆ Proximity
  - ◆ Similarity
  - ◆ Enclosure
  - ◆ Closure
  - ◆ Continuity
  - ◆ Connection

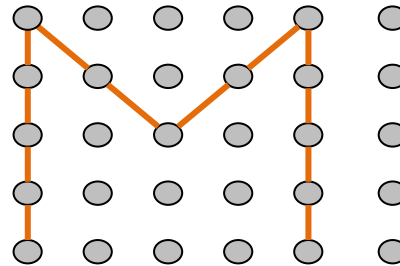
# Gestalt principles

- Visual features that lead the viewer to group visual objects together

Similarity



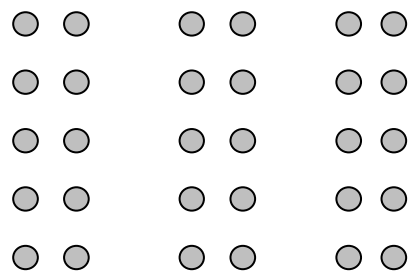
Connection



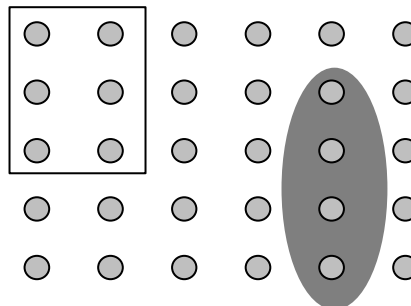
Closure



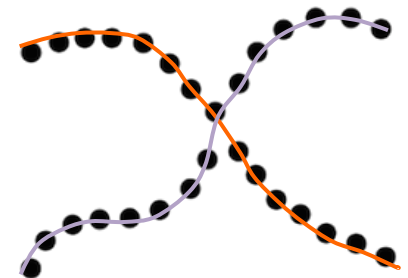
Proximity



Enclosure



Continuity



# Gestalt principles

---

- Visual attributes/patterns that lead observer to group objects together

- ◆ Proximity

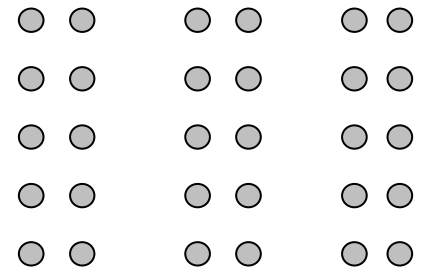
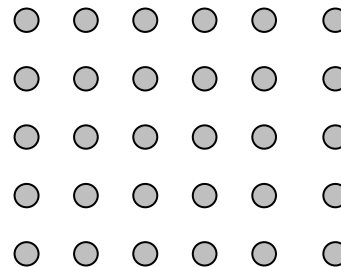
- ◆ Similarity

- ◆ Enclosure

- ◆ Closure

- ◆ Continuity

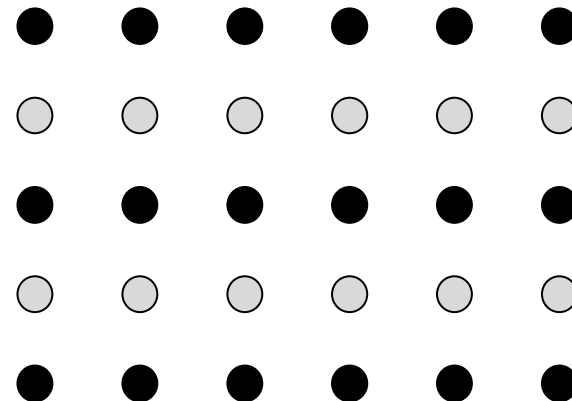
- ◆ Connection



# Gestalt principles

---

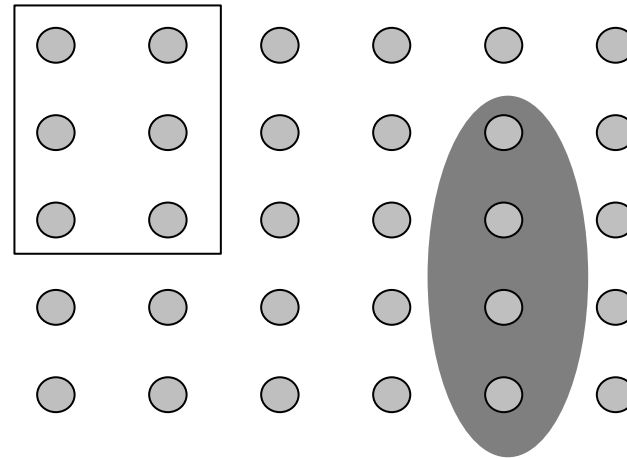
- Visual attributes/patterns that lead observer to group objects together
  - ◆ Proximity
  - ◆ **Similarity**
  - ◆ Enclosure
  - ◆ Closure
  - ◆ Continuity
  - ◆ Connection



# Gestalt principles

---

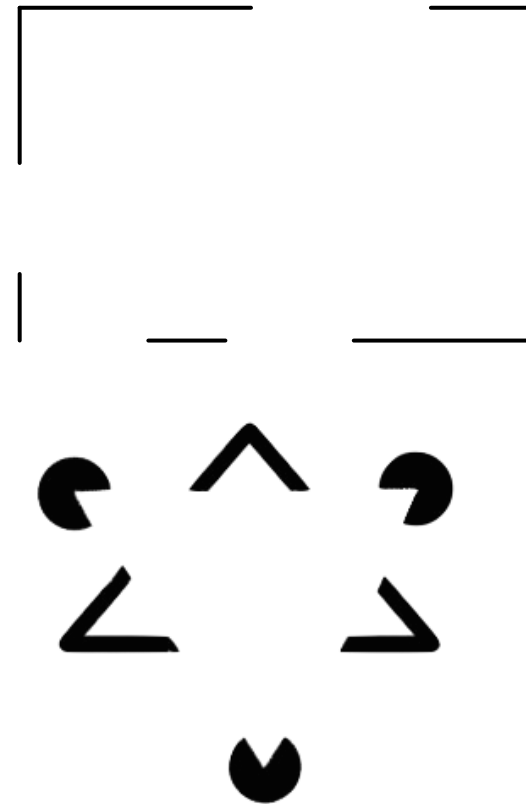
- Visual attributes/patterns that lead observer to group objects together
  - ◆ Proximity
  - ◆ Similarity
  - ◆ **Enclosure**
  - ◆ Closure
  - ◆ Continuity
  - ◆ Connection



# Gestalt principles

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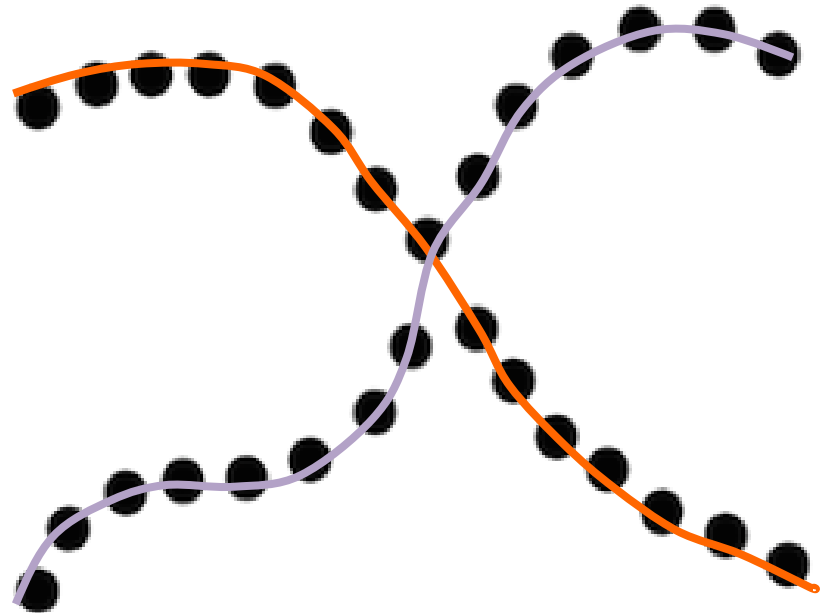
- Visual attributes/patterns that lead observer to group objects together
  - ◆ Proximity
  - ◆ Similarity
  - ◆ Enclosure
  - ◆ **Closure**
  - ◆ Continuity
  - ◆ Connection



# Gestalt principles

---

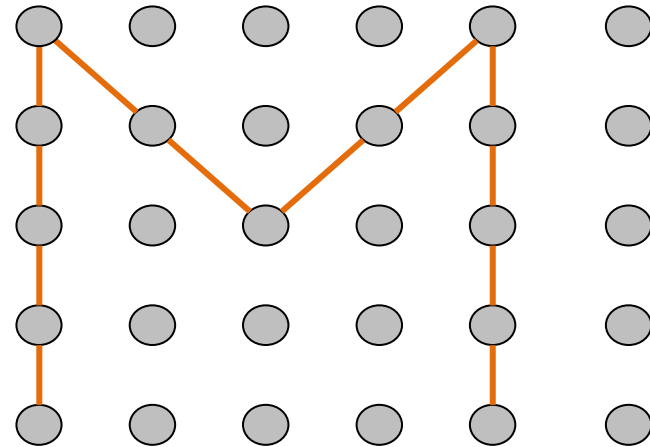
- Visual attributes/patterns that lead observer to group objects together
  - ◆ Proximity
  - ◆ Similarity
  - ◆ Enclosure
  - ◆ Closure
  - ◆ **Continuity**
  - ◆ Connection



# Gestalt principles

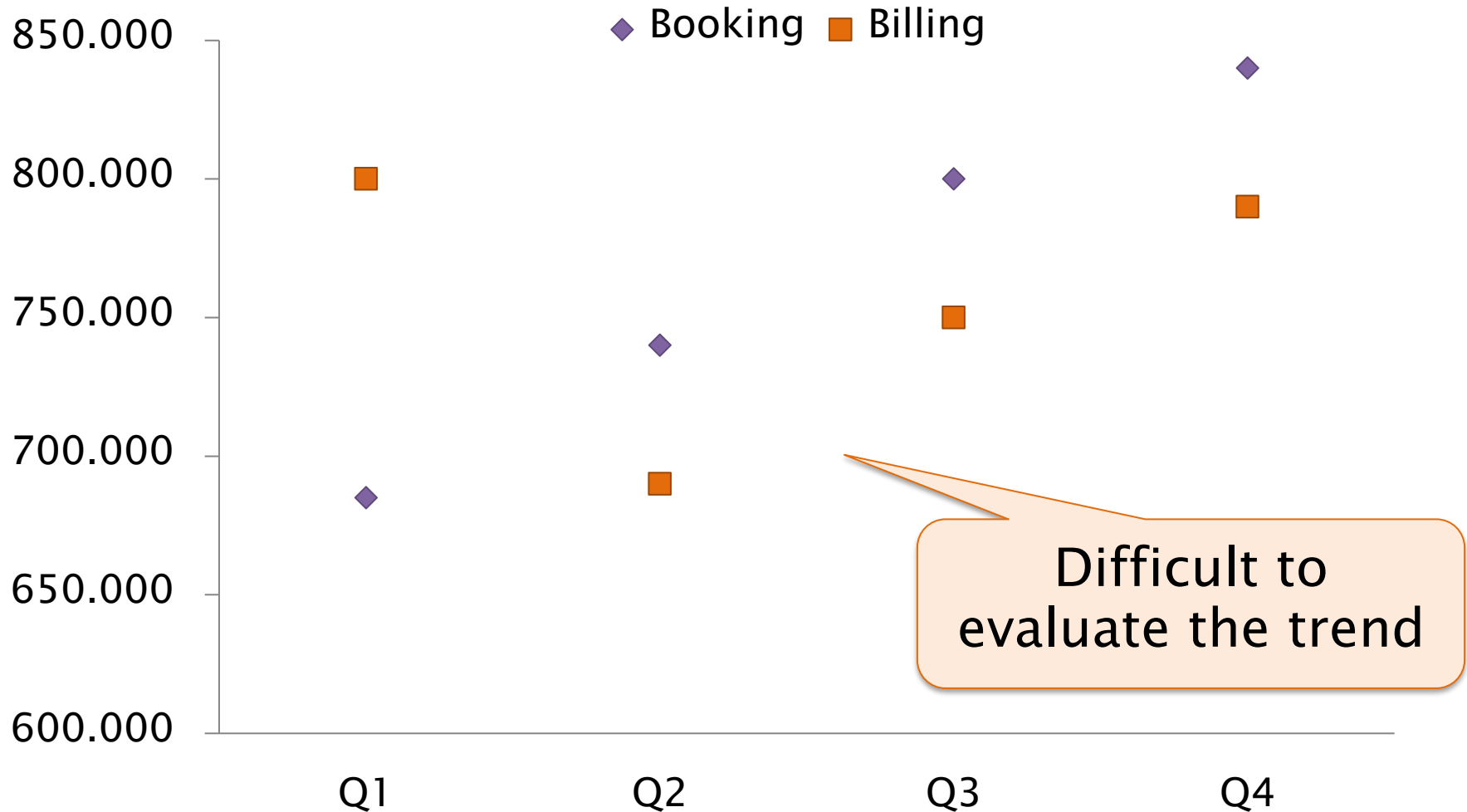
---

- Visual attributes/patterns that lead observer to group objects together
  - ◆ Proximity
  - ◆ Similarity
  - ◆ Enclosure
  - ◆ Closure
  - ◆ Continuity
  - ◆ **Connection**

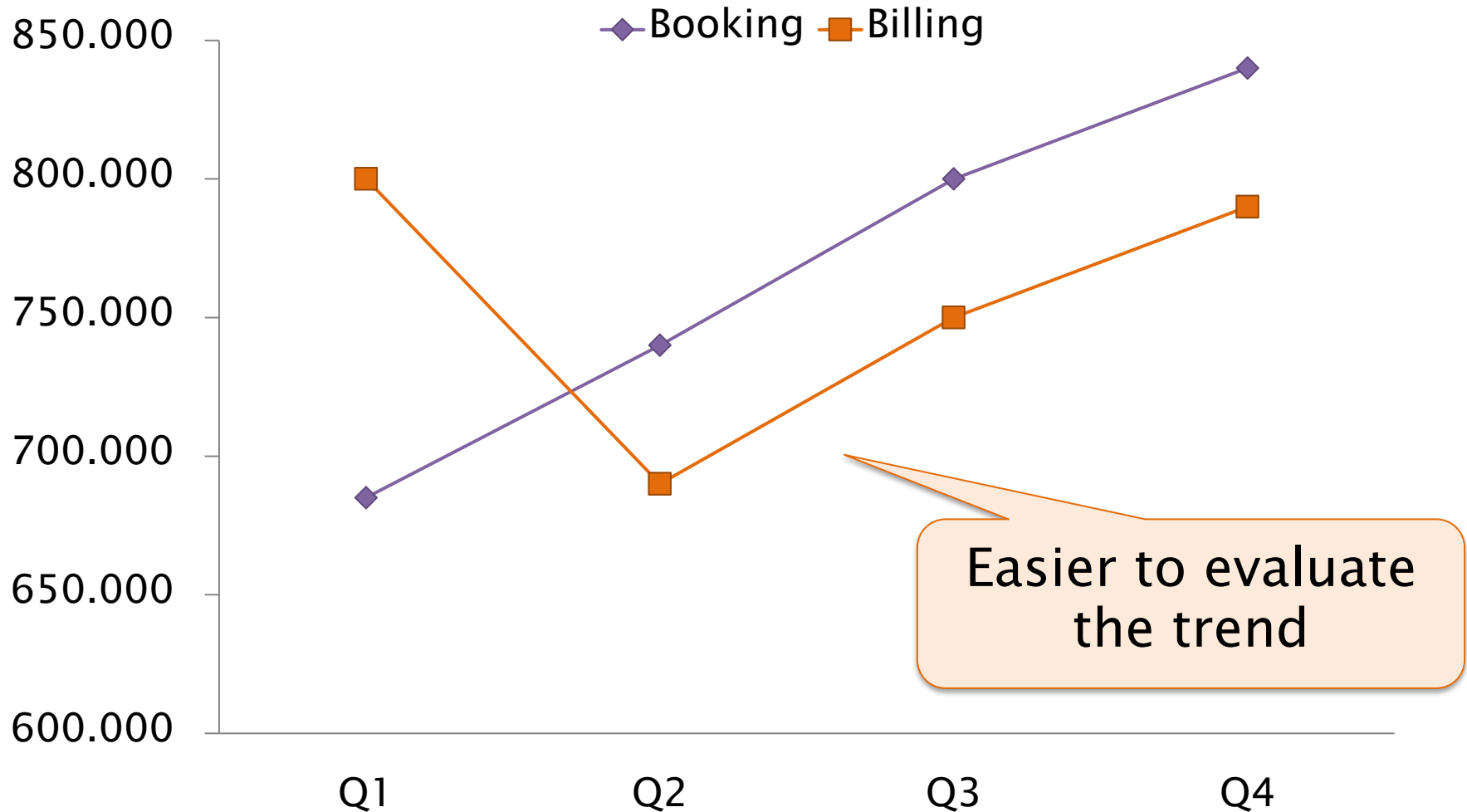




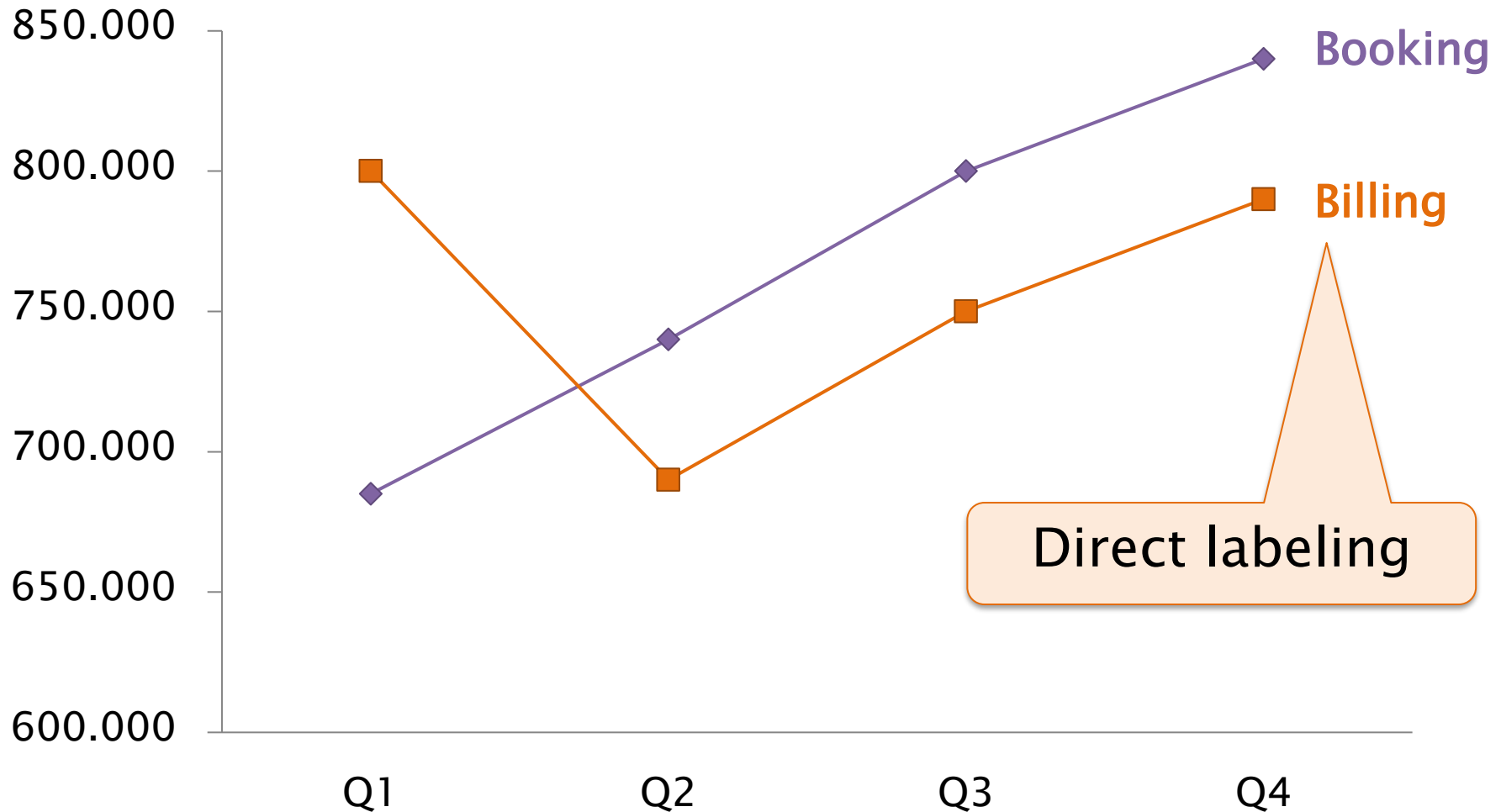
# Similarity in Shape & Color



# Similarity+Connection

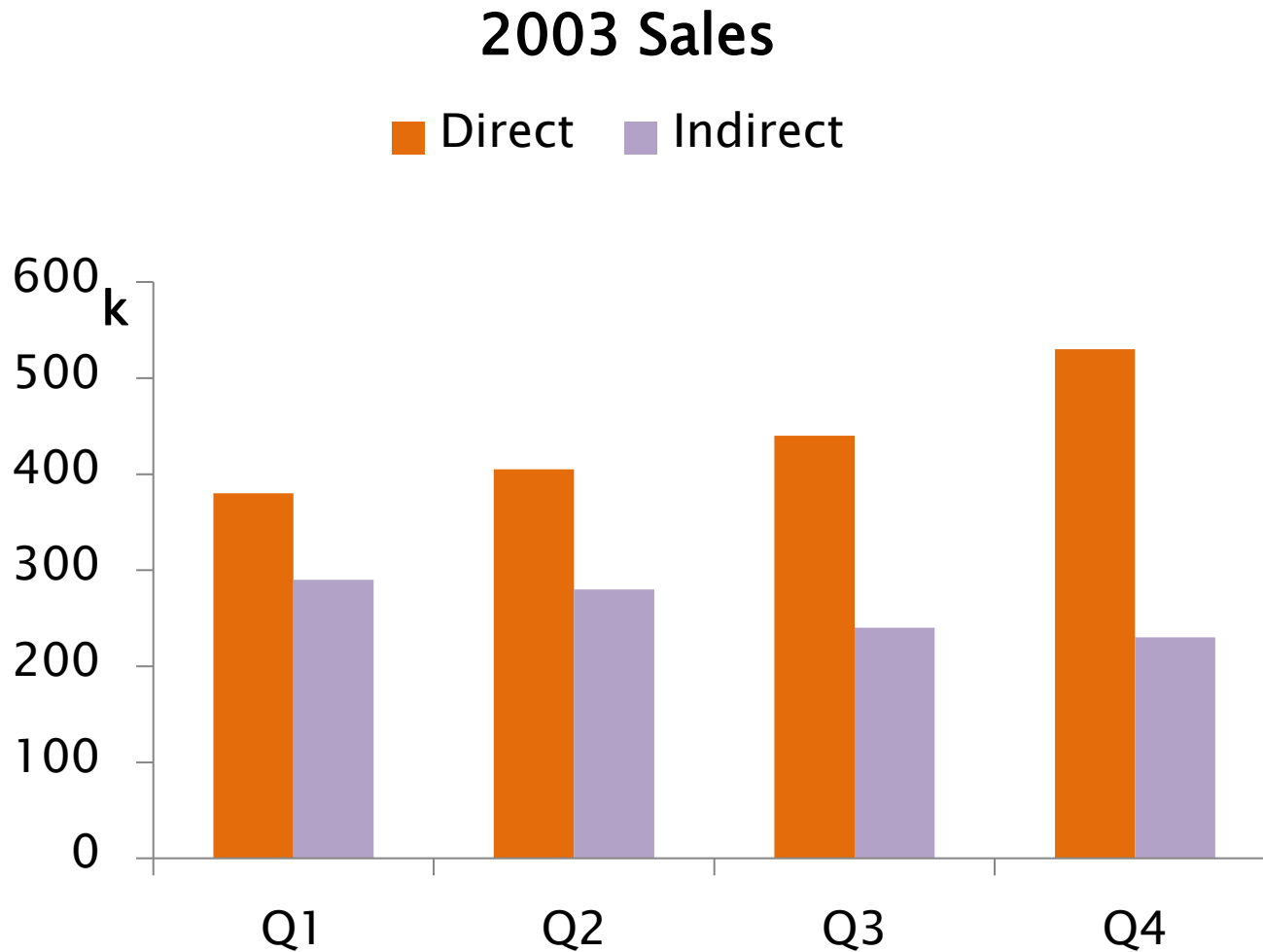


# Similarity+Connection+Proximity



# Similarity $\times$ Proximity

---

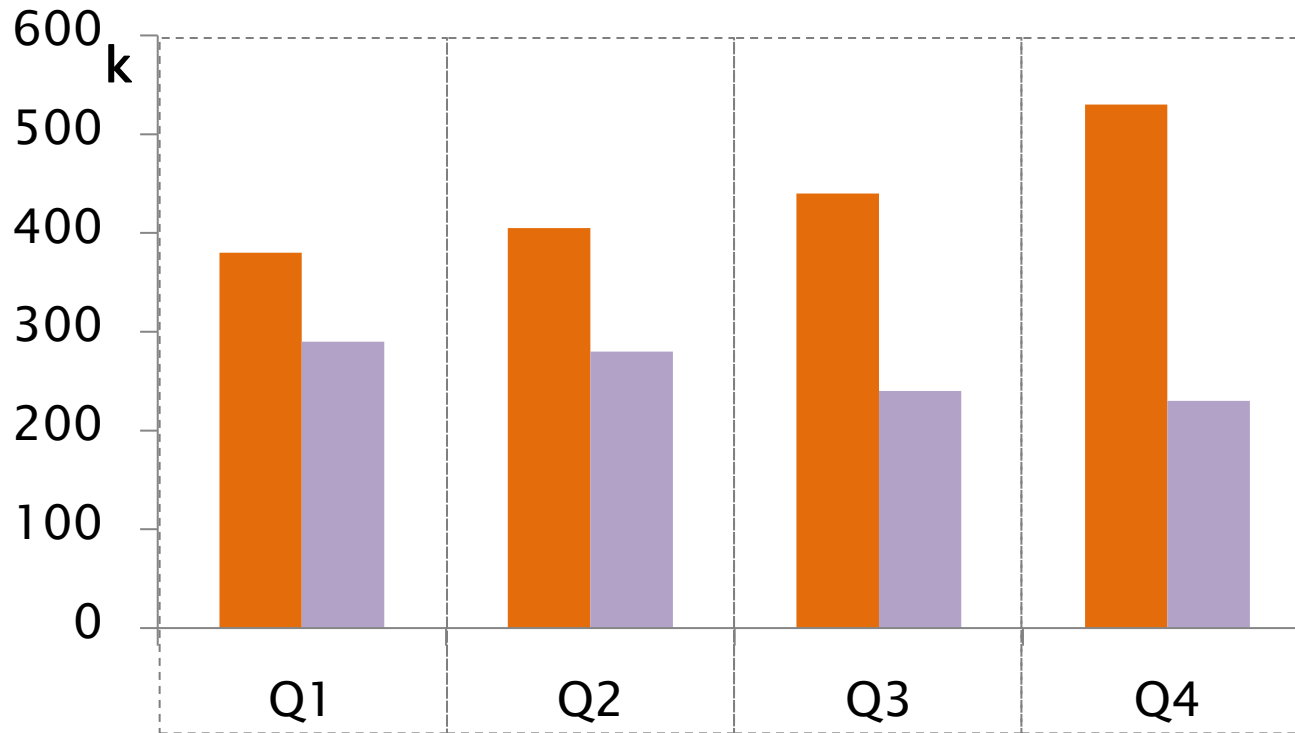


# Similarity $\times$ Proximity & Enclosure

---

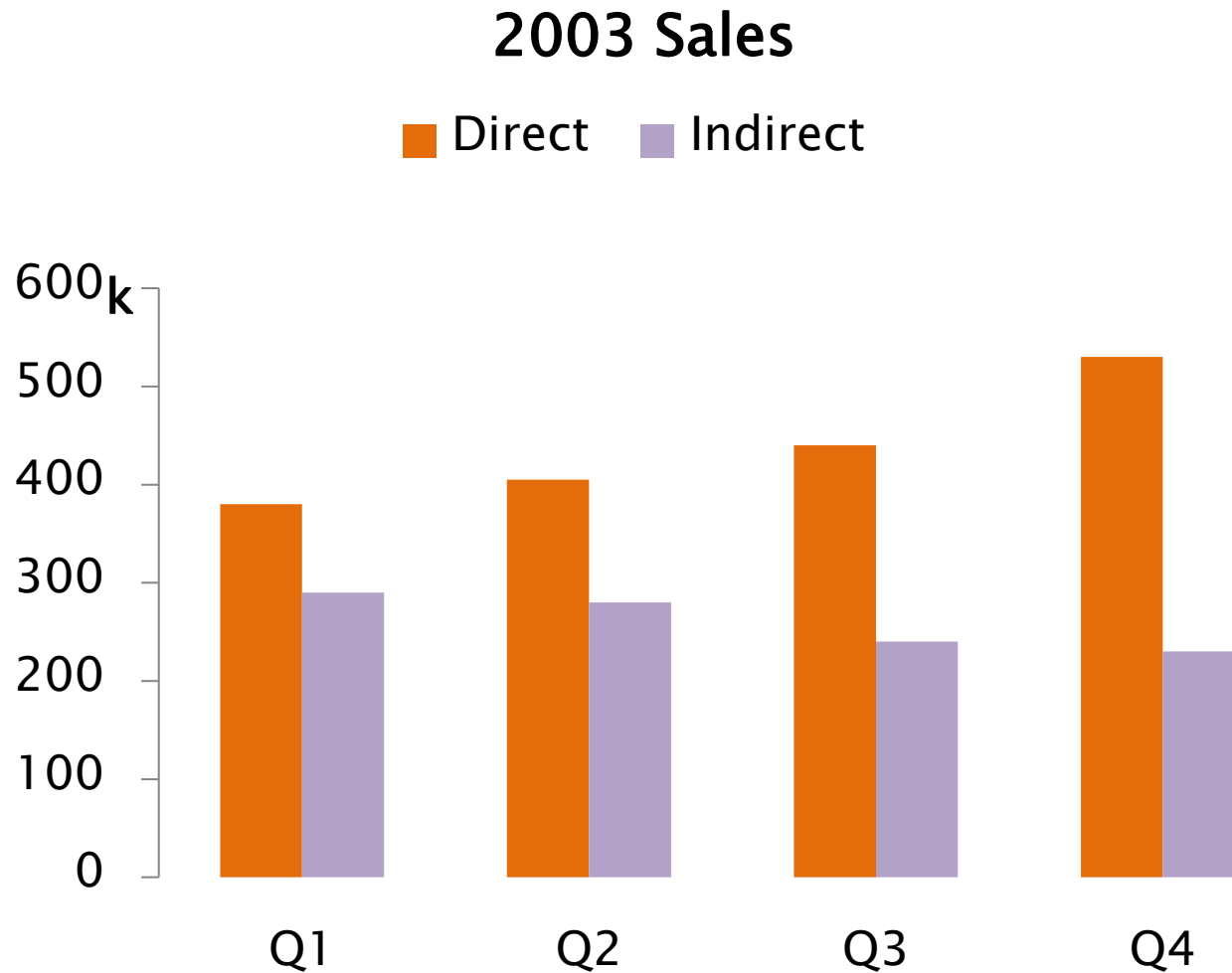
## 2003 Sales

Direct Indirect



# Continuity replaces axis

---

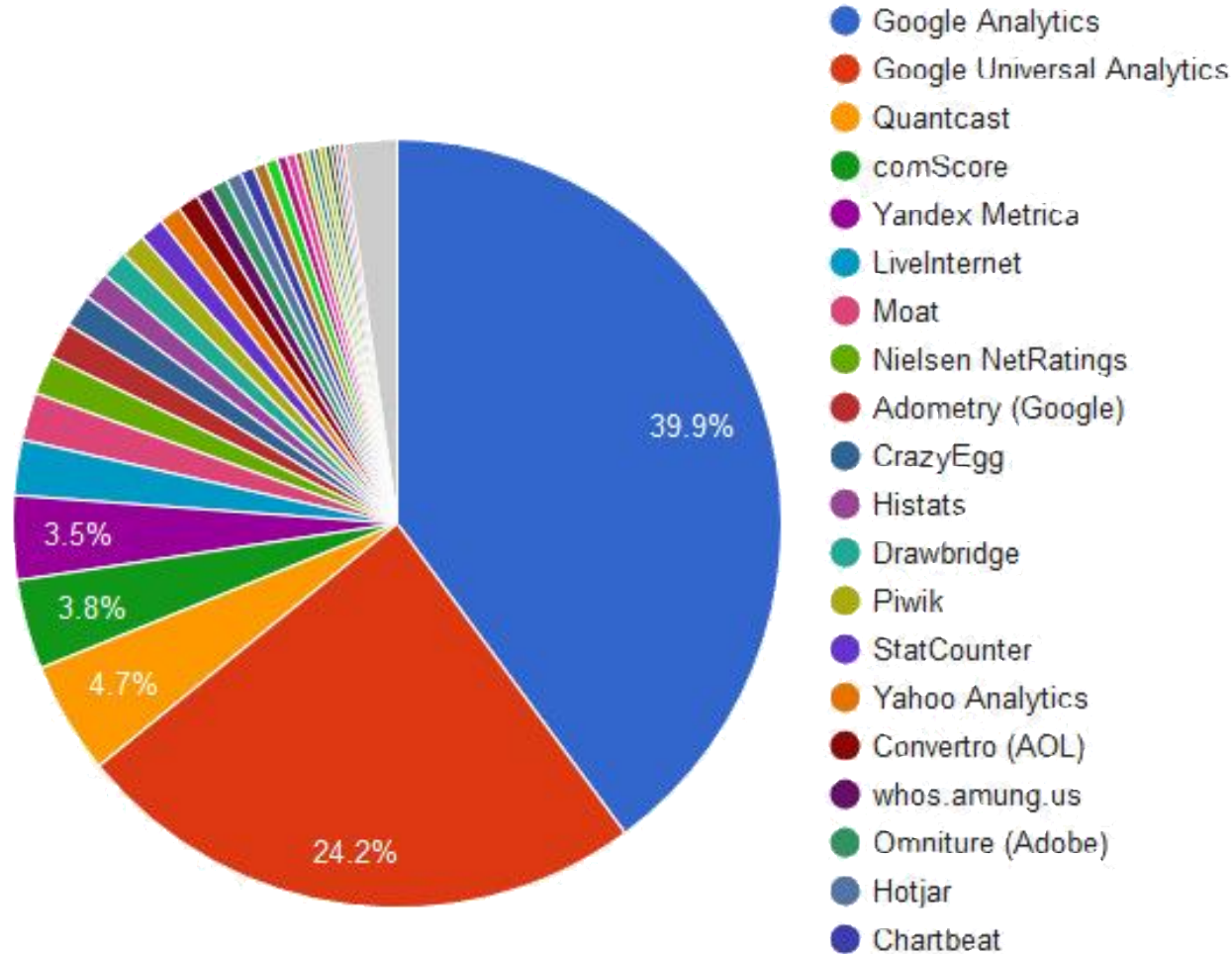


# Distinct perceptions

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- The immediacy of any pre-attentive cue declines as the variety of alternative patterns increases
  - ◆ Even if all the distracting patterns are individually distinct from the target
  - ◆ For each single attribute no more than **four** distinct levels are discernible

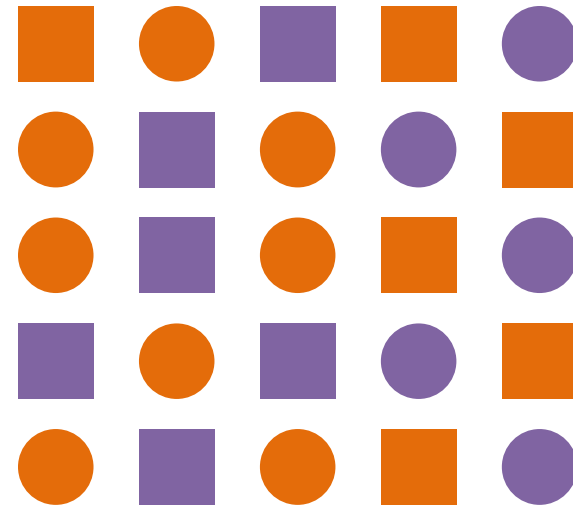
# Rainbow Pies





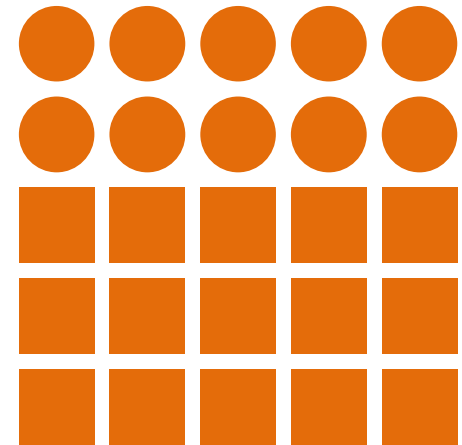
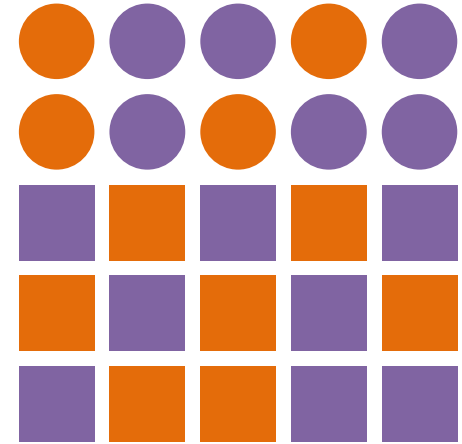
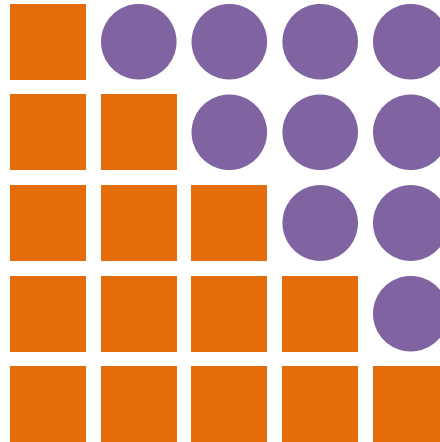
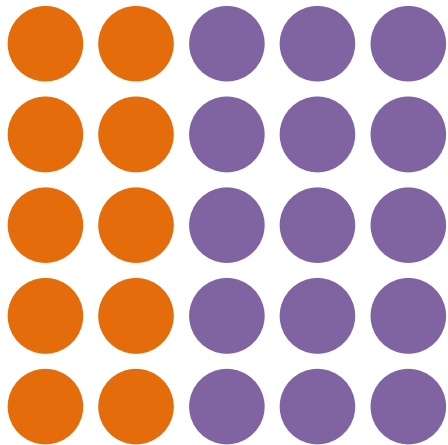
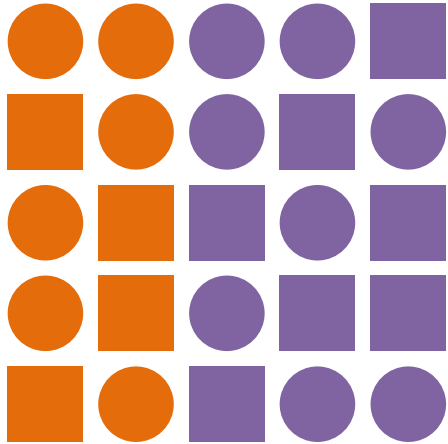
# Attribute Interference

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# Attribute Interference

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# Cultural conventions

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- Reading proceed from left to right and from top to bottom
  - ◆ At least in western culture
- What is at the top (on the left) precedes what is at the bottom (on the right) in terms of
  - ◆ Importance
  - ◆ Ordering
  - ◆ Time

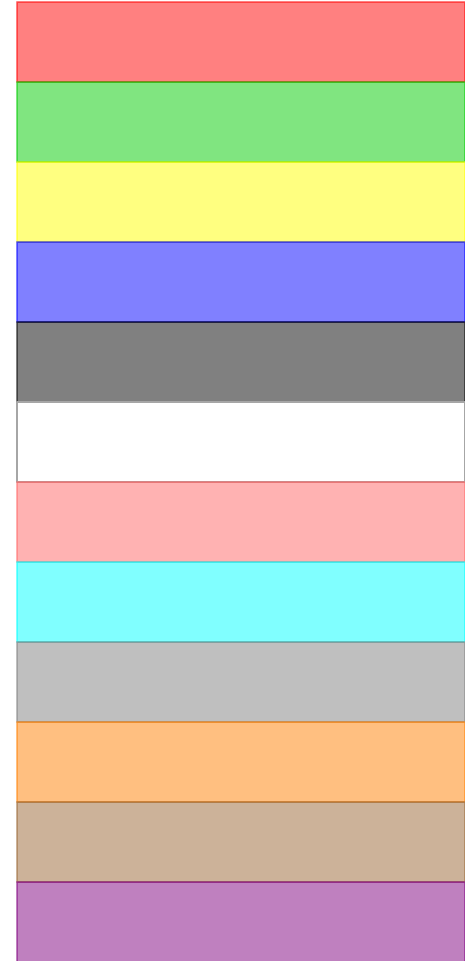
# Emphasis

Attribute	Tables	Graphs
Line width	Boldface text	Thicker lines
Size	Bigger tables Larger fonts	Bigger graphs Wider bars Bigger symbols
Color intensity	Darker or brighter colors	
2-D position	Positioned at the top Positioned at the left Positioned in the center	

# Colors

---

- Red
- Green
- Yellow
- Blue
- Black
- White
- Pink
- Cyan
- Gray
- Orange
- Brown
- Purple



# References

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- C. Ware. *Information Visualization: Perception for Design*. Morgan Kaufmann Publishers, Inc., San Francisco, California, 2000
- C. Healey, and J. Enns. *Attention and Visual Memory in Visualization and Computer Graphics*. IEEE Transactions on Visualization and Computer Graphics, 18(7), 2012
- I. Inbar, N. Tractinsky and J. Meyer. Minimalism in information visualization: attitudes towards maximizing the data-ink ratio.
  - ♦ <http://portal.acm.org/citation.cfm?id=1362587>

# References

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- S.Few, “Practical Rules for Using Color in Charts”
  - ♦ [http://www.perceptualedge.com/articles/visual\\_business\\_intelligence/rules\\_for\\_using\\_color.pdf](http://www.perceptualedge.com/articles/visual_business_intelligence/rules_for_using_color.pdf)
- D. Borland and R. M. Taylor II, "Rainbow Color Map (Still) Considered Harmful," in *IEEE Computer Graphics and Applications*, vol. 27, no. 2, pp. 14–17, March–April 2007.
  - ♦ [http://ieeexplore.ieee.org/xpls/abs\\_all.jsp?arnumber=4118486](http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=4118486)
- <http://www.color-blindness.com>
- <http://www.csc.ncsu.edu/faculty/healey/PP/index.html>